M. H. Elkin
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★Eliot Norton

OF

UNIT

1

INCREASED BY THE SCALE OF TENS.

&c. 10000 1000 100 10

DIVIDED, ACCORDING TO SCALE OF TENS.

.1 .01 .001 .0001 .00001 &c.

1

INCREASED BY VARYING SCALES.

		£	s.	d.	far.
		1	1	1	1
T.	cwt.	qr.	lb.	oz.	dr.
1	1	1	1	1	1

&c., for all denominate numbers.

DIVIDED, ACCORDING TO VARYING SCALES.

$\frac{1}{4}$ $\frac{6}{9}$ $\frac{8}{17}$ $\frac{9}{47}$ } Simple.

$\frac{1}{6}$ *lb.* $\frac{1}{8}$ *oz.* $\frac{1}{7}$ *cwt.* } Denominate.

PROPORTION OF **1** TO ALL NUMBERS.
and of the numbers to each other.

APPLICATIONS.

For explanation see "GRAMMAR of the Diagram, OF ARITHMETIC."

ARITHMETICAL DIAGRAM.

NEW

UNIVERSITY ARITHMETIC,

EMBRACING THE

SCIENCE OF NUMBERS,

AND THEIR

APPLICATIONS ACCORDING TO THE MOST IMPROVED METHODS

OF

ANALYSIS AND CANCELLATION.

BY

CHARLES DAVIES, LL. D.,

AUTHOR OF PRIMARY, INTELLECTUAL, AND SCHOOL ARITHMETICS; ELEMENTARY ALGEBRA; ELEMENTARY GEOMETRY; PRACTICAL MATHEMATICS; ELEMENTS OF SURVEYING; ELEMENTS OF ANALYTICAL GEOMETRY; DESCRIPTIVE GEOMETRY; SHADES, SHADOWS, AND PERSPECTIVE; DIFFERENTIAL AND INTEGRAL CALCULUS; AND LOGIC AND UTILITY OF MATHEMATICS.

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PREFACE.

SCIENCE, in its popular signification, means knowledge reduced to order; that is, knowledge so classified and arranged, as to be easily remembered, readily referred to, and advantageously applied.

ARITHMETIC is the science of numbers. It is the foundation of the exact and mixed sciences and a knowledge of it is an important element either of a liberal or practical education. While Arithmetic is a science in all that concerns the properties of numbers, it is an art in all that relates to their practical application.

It is the first subject in a well-arranged course of instruction to which the reasoning powers of the mind are applied, and is the guide-book of the mechanic and man of business. It is the first fountain at which the young votary of knowledge drinks the pure waters of intellectual truth.

It has seemed, to the author, of the first importance that this subject should be well treated in our Elementary Text Books. In the hope of contributing something to so desirable an end, he has prepared a series of arithmetical works, embracing four books, entitled, Primary Arithmetic; Intellectual Arithmetic; School Arithmetic; and University Arithmetic—the latter of which is the present volume.

PRIMARY ARITHMETIC. This first book is adapted to the capacities and wants of young children. Sensible objects are employed to illustrate and make familiar the simple combinations and relations of numbers. Each lesson embraces one combination of numbers, or one set of combinations.

Intellectual Arithmetic. This work is designed to present a thorough analysis of the science of numbers, and to form a complete course of mental arithmetic. It is thought to be accessible to young pupils by the simplicity and gradation of its methods, and to be particularly adapted to the wants of advanced students, as the attempt has been faithfully made to give the subjects of which it treats a scientific arrangement and logical connection in all the higher methods of arithmetical analysis.

School Arithmetic. Great pains has been taken in the preparation of this book to combine *theory* and *practice* ; to explain and illustrate principles, and to apply them to the common business transactions of life—to make it *emphatically a practical* work. The student is required to demonstrate every principle laid down, by a course of *mental* reasoning, before deducing a proposition or making a practical application of a rule to examples. He is required to fix upon the *unit* or *unity* as the *base* of all numbers, whether integral or fractional—to reason with constant reference to this base, and thus make it the *key* to the solution of all arithmetical questions. It is thought, that the language used in the statement of principles, in the definitions of terms, and in the explanation of methods, will be found to be clear, exact, brief and comprehensive.

University Arithmetic. This work is designed to answer another object. Here, the entire subject is treated as a *science*. The scholar is supposed to be familiar with the simple operations in the four ground rules, and with the first principles of fractions, these being now taught to small children either orally or from elementary treatises. This being premised, the language of figures, which are the representatives of numbers, is carefully taught, and the different significations of which the figures are susceptible, depending on the manner in which they are written, are fully explained. It is shown, for example, that the simple numbers in which the value of the unit

increases from right to left according to the scale of tens, and the Denominate or Compound numbers in which it increases according to a varying scale, belong to the same class of numbers, and that both may be treated under the same rules. Hence, the rules for Notation, Addition, Subtraction, Multiplication and Division, have been so constructed as to apply equally to all numbers. This arrangement, which the author has not seen elsewhere, is deemed an essential improvement in the science of Arithmetic.

In developing the properties of numbers, from their elementary to their highest combinations, great labor has been bestowed in classification and arrangement. It has been a leading object to present the entire subject of arithmetic as forming a *series of dependent and connected propositions:* so that the pupil, while acquiring useful and practical knowledge, may at the same time be introduced to those beautiful methods of reasoning, which science alone teaches.

Great care has been taken to demonstrate every proposition—to give a complete analysis of all the methods employed, from the simplest to the most difficult, and to explain fully, the reason of every rule. A full analysis of the science of Numbers has developed but *one law;* viz., *the law which connects all the units of arithmetic with the unit one,* and *which points out the relations of these units to each other.*

In the Appendix, which treats of Units, Weights and Measures, &c., the methods of determining the Arbitrary Unit, as well as the general law which prevails in the formation of numbers, are fully explained. I cannot too earnestly recommend this part of the work to the special attention of Teachers and pupils.

In fine, the attention of teachers is especially invited to this work, because *general methods* and *general rules* are employed to abridge the common arithmetical processes, and to give to them a more *scientific* and *practical* character. In the present edition the matter is presented in a new form; the arrange-

ment of the subjects is more natural and scientific; the methods have been carefully considered; the illustrations abridged and simplified; the definitions and rules thoroughly revised and corrected; and a very large number and variety of practical examples have been added. The subjects of Fractions, Proportion, Interest, Percentage, Alligation, Analysis, and Weights and Measures, present many new and valuable features, which are not found in other works.

A Key to the present work has also been published for the use of such Teachers as may desire it,—prepared with great care, containing not only the answers and solutions of all the examples, but a full and comprehensive *analysis* of the more difficult ones.

The author has great pleasure in acknowledging the interest which Teachers have manifested in the success of his labors: they have suggested many improvements, both in rules and methods, not only in his elementary, but also in his advanced works. The school room is the tribunal, and the intelligent and practical teacher the judge, before whom all text-books must stand or fall.

This general acknowledgment is not a sufficient recognition of the valuable labors of Mr. D. W. FISH, a practical Teacher of great acuteness and intelligence. He has aided me materially in the preparation of this work; and I have great pleasure in saying to the teachers of the country that they are indebted to him for some of the best practical methods in the applications of numbers.

FISHKILL LANDING,
July, 1856.

CONTENTS.

FIRST FIVE RULES.

COMMON FRACTIONS.

DECIMAL FRACTIONS.

CONTINUED FRACTIONS.

RATIO AND PROPORTION.

PERCENTAGE.

APPLICATIONS.

POWERS AND ROOTS.

ARITHMETICAL PROGRESSION.

GEOMETRICAL PROGRESSION.

ANALYSIS.

MENSURATION.

APPENDIX

UNIVERSITY ARITHMETIC.

DEFINITIONS.

1. A single thing, is called *one* or a *unit*. A number is a unit, or a collection of units.

2. A single thing of a collection, is called the *unit* or *base* of the collection. The *primary* base of every number is the unit one.

3. SCIENCE treats of the properties and relations of things. ART is the practical application of the principles of Science.

4. ARITHMETIC treats of numbers. It is a *science* when it determines the properties and relations of numbers; and an *art* when it applies principles of science to practical purposes.

5. A PROPOSITION is something to be *done*, or *demonstrated.*

6. An ANALYSIS is an examination of the separate parts of a proposition.

7. An OPERATION, in Arithmetic, is the act of doing something with numbers. The number obtained by an operation is called a *result*, or *answer*.

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1. What is a single thing called? What is a number?
2. What is a single thing of a collection called? What is the primary base of every number?
3. Of what does science treat? What is art?
4. Of what does Arithmetic treat? When is it a science? When an art?
5. What is a proposition?
6. *What is an analysis?*
7. *What is an operation?* What is the number obtained called?

8. A RULE is a direction for performing an operation, and may either be inferred from an analysis, or deduced from a demonstration.

9. There are five fundamental processes of Arithmetic: Notation and Numeration, Addition, Subtraction, Multiplication, and Division.

EXPRESSING NUMBERS.

10. There are three methods of expressing numbers:

1st. By words, or common language;
2d. By letters, called the Roman method;
3d. By figures, called the Arabic method.

BY WORDS.

11. A single thing is called	- - - -	*One.*
One and one more	- - - -	*Two.*
Two and one more	- - - -	*Three.*
Three and one more	- - - -	*Four.*
Four and one more	- - - -	*Five.*
Five and one more	- - - -	*Six.*
Six and one more	- - - -	*Seven.*
Seven and one more	- - - -	*Eight.*
Eight and one more	- - - -	*Nine.*
Nine and one more	- - - -	*Ten,*
&c.	&c.	&c.

Each of the words, or terms, *one, two, three, four,* &c., denotes how many units are taken. These terms are generally called numbers; though, in fact, they are but the *names* of numbers.

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8. What is a rule? How may it be deduced?

9. How many fundamental processes are there in Arithmetic? What are they?

10. How many methods are there of expressing numbers? What are they?

11. What does each of the words, *one, two, three,* &c., denote? What are these words generally called? What are they, in fact?

NOTATION.

12. NOTATION is the method of expressing numbers either by letters or figures. The method by letters, is called *Roman Notation;* the method by figures is called *Arabic Notation.*

ROMAN NOTATION.

13. In the Roman Notation, seven capital letters are used, viz.: I, stands for *one;* V, for *five;* X, for *ten;* L, for *fifty;* C, for *one hundred;* D, for *five hundred;* and M, for *one thousand.* All other numbers are expressed by combining these letters according to the following

ROMAN TABLE.

I. - - - - -	One.	LXX. -	Seventy.
II. - - - -	Two.	LXXX.	Eighty.
III.- - - -	Three.	XC. -	Ninety.
IV.- - - -	Four.	C. - -	One hundred.
V. - - - -	Five.	CC.- -	Two hundred.
VI.- - - -	Six.	CCC. -	Three hundred.
VII. - - -	Seven.	CCCC.-	Four hundred.
VIII. - - -	Eight.	D. - -	Five hundred.
IX. - - -	Nine.	DC. -	Six hundred.
X. - - - -	Ten.	DCC. -	Seven hundred.
XX. - - -	Twenty.	DCCC.-	Eight hundred.
XXX. - -	Thirty.	DCCCC	Nine hundred.
XL. - - -	Forty.	M. - -	One thousand.
L. - - - -	Fifty.	MD. -	Fifteen hundred.
LX. - - -	Sixty.	MM. -	Two thousand.

NOTE.—This Notation was used by the Romans: hence, its name. It is still used for dates, numbering of chapters, pages, &c.

The principles of this Notation are these:

1. Every time a letter is repeated, the number which it denotes is also repeated.

---

12. What is Notation? What is the method by letters called? What is the method by figures called?

13. How many letters are used in the Roman Notation? What are they? What does each stand for?

Note.—What are the three principles of this Notation?

2. If a letter denoting a *less* number is written on the right of one denoting a *greater*, their sum will express the number.

3. If a letter denoting a less number is written on the *left* of one denoting a greater, their *difference* will express the number.

EXAMPLES IN ROMAN NOTATION.

Express the following numbers in the Roman Notation:

1. Sixteen.
2. Fourteen.
3. Eighteen.
4. Sixty-nine.
5. Seventy-eight.
6. One hundred and fifteen.
7. Four hundred and nine.
8. Seven hundred and fifty-one.
9. One thousand and sixty.
10. Two thousand and ninety-one.
11. Five hundred and sixty-nine.
12. Seven hundred and forty-five.
13. Nine hundred and sixty-one.
14. Six hundred and ninety-nine.
15. Nine hundred and fifty-seven.
16. One thousand two hundred and six.
17. Four hundred and ninety-five.
18. Seven hundred and fifty-five.
19. Eighteen hundred and forty-seven.
20. Two thousand five hundred and twenty.

ARABIC NOTATION.

14. Arabic Notation is the method of expressing numbers by figures. Ten figures are used, and they form the *alphabet of the Arabic Notation.*

---

14. What is Arabic Notation? How many figures are used? What *do they form*? Name the figures? What does the 0 express? What *are the other figures* called?

They are,	0 called zero, cipher, or	Naught.
	1 - -	One.
	2 - - -	Two.
	3 - - -	Three.
	4 - - -	Four.
	5 - - -	Five.
	6 - - -	Six.
	7 - - -	Seven.
	8 - - -	Eight.
	9 - -	Nine.

The cipher 0, expresses no value. It is used to denote the *absence of a thing*. The nine other figures are called *significant figures*, or *Digits*.

15. We have no single figure for the number ten. We therefore *combine* the figures already known. This we do by writing 0 on the right hand of 1, thus:

10, which is read, *ten*.

This 10 is equal to *ten* of the units expressed by 1. It is, however, but a *single ten*, and may be regarded as a *unit*, the value of which is *ten times* as great as the unit 1. It is called a unit of the *second order*.

16. When two figures are written by the side of each other, the one on the right is in the *place of units*, and the other in the *place of tens*, or of *units of the second order*. *Each unit of the second order is equal to ten units of the first order.*

When units simply are named, *units of the first order are always meant.*

---

15. Have we a separate character for ten? How do we express ten? To how many units 1 is 1 ten equal? May ten be regarded as a single unit? Of what order?

16. When two figures are written by the side of each other, what place does the right hand figure occupy? The figure on the left? When units simply are *named*, what units are meant?

17. In order to express *ten units of the second order*, or *one hundred*, we form a new combination:

It is done thus, - - - - - 100,

by writing two ciphers on the right of 1. This number is read, one hundred.

Now, this one hundred expresses 10 *units of the second order*, or 100 *units of the first order*. The one hundred is but *an individual hundred*, and, in this light, may be regarded as a unit of the *third order*.

We can now express any number less than one thousand.

For example, in the number two hundred and fifty-five, there are 5 units, 5 tens, and 2 hundreds. Write, therefore, 5 units of the first order, 5 units of the second order, and 2 of the third; and read from the right, *units*, *tens*, *hundreds*.

huns.	tens.	units.
2	5	5

In the number five hundred and ninety-five, there are 5 units of the first order, 9 of the second, and five of the third; and it is read from the right, *units*, *tens*, *hundreds*.

huns.	tens.	units.
5	9	5

In the number six hundred and four, there are 4 units of the first order, 0 of the second, and 6 of the third.

huns.	tens.	units.
6	0	4

The right hand figure always expresses units of the first order; the second, units of the second order; and the third, units of the third order.

18. To express *ten* units of the third order, or one thousand, we form a new combination by writing three ciphers on the right of 1; thus, - - - - - - 1000.

Now, this is but *one single thousand*, and may be regarded as a unit of the fourth order.

---

17. How do you write one hundred? To how many units of the second order is it equal? To how many of the first order? How may it be regarded? Of what order? How many units of the third order in 200? In 600? In 900?

18. To what are ten units of the third order equal? How do you write it? How do you write a single unit of the first order? How do you write a unit of the second order? Of the third? Of the fourth?

Thus, we may form as many orders of units as we please:
a single unit of the first order is expressed by - - 1,
a unit of the second order by 1 and 0; thus, - - 10,
a unit of the third order by 1 and two 0's; - - 100,
a unit of the fourth order by 1 and three 0's; - - 1000,
a unit of the fifth order by 1 and four 0's; - - 10000;
and so on, for units of higher orders:

19. Therefore,

1st. *The same figure expresses different units according to the place which it occupies:*

2d. *Units of the first order occupy the place at the right; units of the second order, the second place; units of the third order, the third place, and so on for places still to the left:*

3d. *Ten units of the first order make one of the second; ten of the second, one of the third; ten of the third, one of the fourth; and so on for the higher orders:*

4th. *When figures are written by the side of each other, ten units in any one place make one unit of the place next at the left.*

EXAMPLES IN WRITING THE ORDERS OF UNITS.

1. Write 7 units of the 1st order.
2. Write 8 units of the 2d order.
3. Write 9 units of the 4th order.
4. Write 3 units of the 1st order, with 9 of the 2d.
5. Write 9 units of the 3d order, with 6 of the 2d, and 1 of the 1st.
6. Write 0 units of the 2d order, 8 of the 1st, with 4 of the 3d, and 7 of the 4th.

---

19. On what does the unit of a figure depend? What is the unit of the place on the right? What is the unit of the second place? What of the third place? What of the fourth? &c.

How many units of the first order make one of the second? How many of the second make one of the third? How many of the third one of the fourth? &c. When figures are written by the side of each other, how many *units of any place* make one unit of the place next at the *left?*

7. Write 8 units of the 6th order, 7 of the 4th, 9 of the 5th, 0 of the 3d, 2 of the 2d, and 1 of the 1st.

8. Write 8 units of the 8th order, 6 of the 7th, 0 of the 1st, 3 of the 2d, 4 of the 3d, 9 of the 4th, 0 of the 6th, and 2 of the 5th.

9. Write 4 units of the 10th order, 8 of the 7th, 3 of the 9th, 2 of the 8th, 0 of the 6th, 3 of the 1st, 6 of the 2d, 0 of the 3d, 1 of the 4th, and 2 of the 5th.

10. Write 3 units of the 2d order, 2 of the 1st, 9 of the 3d, 0 of the 4th, 9 of the 9th, 6 of the 8th, 7 of the 7th, 0 of the 6th, and 4 of the 5th.

11. Write 3 units of the 11th order, 0 of the 10th, 8 of the 4th, 0 of the 5th, 2 of the 6th, 0 of the 7th, 3 of the 8th, 4 of the 9th, 1 of the 3d, 2 of the 2d, and 3 of the 1st.

12. Write 3 units of the 12th order, 6 of the 11th, 3 of the 8th, 7 of the 6th, 2 of the 4th, and 1 of the 2d.

13. Write 5 units of the 13th order, 8 of the 12th, 0 of the 9th, 6 of the 7th, 8 of the 3d, and 12 of the 1st.

14. Write 7 units of the 14th order, 5 of the 13th, 6 of the 12th, 5 of the 10th, 7 of the 8th, 9 of the 6th, 5 of the 4th, and 8 of the 1st.

15. Write 9 units of the 15th order, 4 of the 13th, 8 of the 9th, 2 of the 6th, 7 of the 3d, and 2 of the 2d.

16. Write 6 units of the 16th order, 9 of the 12th, 7 of the 9th, 4 of the 7th, 0 of the 6th, 8 of the 4th, 9 of the 5th, and 2 of the 2d.

17. Write 8 units of the 20th order, 5 of the 18th, 6 of the 13th, 4 of the 11th, 9 of the 9th, 1 of the 17th, 4 of the 5th, and 9 of the 3d.

18. Write 6 units of the 10th order, 5 of the 8th, 9 of the 6th, 0 of the 4th, and 1 of the 1st.

19. Write 9 units of the 18th order, and then diminish the figure of each order by 1 till you come to and include 0; then increase the figure of each order by 1, till you reach the first order; and then read each order.

20. Write the number which has 20 units of the 17th order,

0 of the 14th, 8 of the 16th, 4 of the 13th, 0 of the 8th, 0 of the 9th, and one in each of the other places.

NUMERATION.

20. **Numeration** is the art of reading correctly any number expressed by figures or letters.

The pupil has already been taught to read all numbers from one to one thousand. The Numeration Table will teach him to read any number whatever; that is, to express numbers in words.

TABLE.

7th Period. Quintillions.			6th Period. Quadrillions.			5th Period. Trillions.			4th Period. Billions.			3d Period. Millions.			2d Period. Thousands.			1st Period. Units.		
Hundreds of Quintillions.	Tens of Quintillions.	Quintillions.	Hundreds of Quadrillions.	Tens of Quadrillions.	Quadrillions.	Hundreds of Trillions.	Tens of Trillions.	Trillions.	Hundreds of Billions.	Tens of Billions.	Billions.	Hundreds of Millions.	Tens of Millions.	Millions.	Hundreds of Thousands.	Tens of Thousands.	Thousands.	Hundreds.	Tens.	Units.
3	7	0	8	9	4	2	1	6	6	3	6	8	0	6	3	0	4	6	2	5

Notes.—1. Numbers expressed by more than three figures are written and read by periods, as shown in the above table.

2. Each period always contains three figures, except the left hand period, which may contain one, two or three figures.

3. The unit of the first, or right-hand period, is 1; of the second period, 1 thousand; of the third, 1 million; of the fourth, 1 billion; and so on, for periods, still to the left.

4. To Quintillions succeed Sextillions, Septillions, Octillions, Nonillions, Decillions, Undecillions, Duodecillions, &c.

5. The pupil should be required to commit, thoroughly, the names

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20. What is Numeration? What is the unit of the first period? What is the unit of the second? Of the third? Of the fourth? Fifth? Sixth? Seventh? &c. Give the rule for reading numbers. Give the rule for *writing numbers.*

of the periods, so as to repeat them in their regular order from left to right, as well as from right to left.

6. Formerly, in the English Notation, *six* places were given to Millions, Trillions, Quadrillions, &c. They were read, Millions, Tens of Millions, Hundreds of Millions, *Thousands* of Millions, *Tens of Thousands* of Millions, *Hundreds of Thousands* of Millions; and the same for Billions, Trillions, Quadrillions, &c. This method produced great irregularity in the Notation, as it gave *three* places to the units of the first two periods, (viz.: units and thousands,) and six places to each of the others. The French method, which gives *three places to the unit* of each period, is fully adopted in this country, and must soon become universal.

RULE FOR READING NUMBERS.

I. *Divide the number into periods of three figures each, beginning at the right hand.*

II. *Name the unit of each figure, beginning at the right hand.*

III. *Then, beginning at the left hand, read each period as if it stood alone, naming its unit.*

EXAMPLES IN READING NUMBERS.

Let the pupil point off and read the following numbers—then write them in words.

1.	97	6.	32045607	11.	784236704
2.	326	7.	90464213	12.	7403026054
3.	3302	8.	47364291	13.	21704080495
4.	65042	9.	4037902169	14.	21896720421
5.	742604	10.	91046302	15.	8140290308097

16.	8504680467023	19.	30467214302704
17.	90403040720156	20.	167320410341204
18.	172304736893210	21.	2164032189765421

Let each of the above examples, after being written on the black board, be analyzed as a class exercise; thus:

1. In how many ways may the number 97 be read?

1st. The common way, 97.

2d. We may read, 9 tens, and 7 units.

2. In how many ways may 326 be read?

1st. By the common way, three hundred and twenty-six.

2d. Three hundreds, 2 tens, and 6 units.

3d. Thirty-two tens, and six units.

3. In how many ways may the number 5302 be read?

1st. Five thousand three hundred and two.

2d. Five thousand, three hundreds, 0 tens, and 2 units.

3d. Fifty-three hundreds, 0 tens, and 2 units.

4th. Five hundred and thirty tens, and 2 units.

4. In 65042, how many ten thousands? How many thousands? How many hundreds? How many tens? How many units?

5. In 742604, how many hundred thousands? How many ten thousands? How many thousands? How many hundreds? How many tens? How many units?

RULE FOR WRITING NUMBERS, OR NOTATION.

I. *Begin at the left hand and write each period in order, as if it were a period of units.*

II. *When the number, in any period except the left-hand period, can be expressed by less than three figures, prefix one or two ciphers; and when a vacant period occurs, fill it with ciphers.*

EXAMPLES IN NOTATION.

Express the following numbers in figures:

1. Six hundred and twenty-one.
2. Five thousand seven hundred and two.
3. Eight thousand and one.
4. Ten thousand four hundred and six.
5. Sixty-five thousand and twenty-nine.
6. Forty millions two hundred and forty-one.
7. Fifty-nine millions three hundred and ten.
8. Eleven thousand eleven hundred and eleven.
9. Three hundred millions, one thousand and six.
10. Sixty-nine billions, three millions, two hundred and eleven.

11. Forty-seven quadrillions, sixty-nine billions, four hundred and sixty-five thousands, two hundred and seven.

12. Eight hundred quintillions, four hundred and twenty-nine millions, six thousand and nine.

13. Ninety-five sextillions, eighty-nine millions, eighty-nine thousands, three hundred and six.

14. Six quintillions, four hundred and fifty-one billions, sixty-five millions, forty-seven thousands, and one hundred and four.

15. Write, in figures, nine hundred and ninety-nine billions, sixty-five millions, eight hundred and forty-one thousands, four hundred and eleven.

16. Four hundred and seventy nonillions, forty octillions, four millions, six thousands, two hundred and four.

17. Sixty-five sextillions, eight hundred quadrillions, seven hundred and fifty billions, seven hundred and fifty-one millions, nine hundred and seventy-five thousands, three hundred and ten.

FORMATION AND NATURE OF NUMBERS.

21. The term, *one*, may refer to *any* single thing: it has no reference to kind or quality: it is called an *abstract* unit.

22. The term, *one foot*, refers to a single foot, and is called a *concrete* or *denominate* unit.

23. An *abstract* number is one whose unit is abstract: thus, three, four, six, &c., are abstract numbers.

24. A *concrete* or *denominate* number, is one whose unit is *concrete* or *denominate:* thus, three feet, four dollars, five pounds, are denominate numbers.

25. A Simple Number is a single unit, or a single collection of units, either abstract or denominate.

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21. Does the term, *one*, refer to the *kind* of thing to which it is applied? What is it called?

22. To what does one foot refer? What is it called?

23. What is an abstract number?

24. *What is a concrete*, or denominate number?

25. *What is a simple* number?

26. QUANTITY is anything which can be measured by a unit.

27. Numbers which have the same unit are of the same denomination: and numbers having different units are of different denominations. Thus, 4 yards and 6 yards are of the same denomination; but 4 yards and 6 feet are of different denominations.

28. If two or more denominate numbers, having different units, are connected together, forming a single expression, this is called, a *compound* denominate number. Thus, 3 yards 2 feet and 6 inches, is a *compound denominate number.*

29. We have seen (Art. 19) that when figures are written by the side of each other, thus,

678904,

the language implies that ten units of any place make one unit of the place next to the left.

30. When figures are written to express English Currency, thus:

£	*s.*	*d.*	*far.*
4	17	10	3

the language implies, that four units of the lowest denomination make one of the second; twelve of the second, one of the third; and twenty of the third, one of the fourth.

31. When figures are written to express Avoirdupois weight, thus:

T.	*cwt.*	*qr.*	*lb.*	*oz.*	*dr.*
27	17	2	24	11	10

the language implies, that 16 units of the lowest denomination

---

26. What is quantity?

27. When are numbers said to be of the same denomination? When of different denominations?

28. What is a compound denominate number?

29. When several figures are simply written by the side of each other, what does the language imply?

33. In the English Currency, how many units of the lowest denomination make one of the second? How many of the second one of the third? How many of the third one of the fourth?

31. In the Avoirdupois weight, how many of the lowest make one of the second? How many of the second one of the third?

make one of the second; 16 of the second, one of the third 25 of the third, one of the fourth; 4 of the fourth, one of the fifth; and 20 of the fifth, one of the sixth.

All the other compound denominate numbers are formed on the same principle; and in all of them, *we pass from a lower to the next higher denomination by considering how many units of the lower make one unit of the next higher.**

32. A SCALE expresses the relations between the different units of a number. There are two kinds of scales—*uniform* and *varying*. In the uniform scale, the number of units which make 1 of the next higher is 10. In English Currency, 4, 12, and 20, make up the *varying scale*; and 16, 16, 25, 4 and 20, in Avoirdupois weight.

SCALE OF TENS.

33. If we write a row of 0's, thus:

1 Hundred Billion.	1 Ten Billion.	1 Billion.	1 Hundred Million.	1 Ten Million.	1 Million.	1 Hundred Thousand.	1 Ten Thousand.	1 Thousand.	1 Hundred.	1 Ten.	1 Unit.
0	0	0,	0	0	0,	0	0	0,	0	0	0,

The language of figures determines the *unit of each place*, and also, the *law of change* in passing from one place to another. This is called the *decimal* system, in which the units change according to the scale of *tens*.

If it be required to express a given number of units, of any

* For the Tables of Denominate Numbers, see Appendix, page 383.

32. What is a scale? How many kinds of scales are there? What are they? What is the scale in the common system of numbers? What is the scale in English Currency? What in Avoirdupois weight?

33. If a row of 0's be written, what does the language of figures determine? What is such a system called? How does the unit change? How do you express a given number of units of any order?

order, we first select from the arithmetical alphabet the figure which designates the number, and then write it in the place corresponding to the order. Thus, to express three millions, we write

3000000;

and similarly for all numbers.

UNITED STATES MONEY.

34. United States money affords an example of a system of denominate units, increasing according to the scale of tens: thus,

Eagle,	Dollar,	Dime,	Cent,	Mill,
1	1	1	1	1

may be read 11 thousand 1 hundred and 11 *mills;* or, 1111 *cents* and 1 mill; or, 111 dimes, 1 cent, and 1 mill; or, 11 dollars, 1 dime, 1 cent, and 1 mill; or, 1 eagle, 1 dollar, 1 dime, 1 cent, and 1 mill. Thus, we may read the number with any one of its units as a base, or we may name them all; as 1 eagle, 1 dollar, 1 dime, 1 cent, 1 mill. Generally, in United States money, we read in the denominations of dollars cents and mills; and say, 11 dollars 11 cents and 1 mill.

United States money is denoted by the character, $. The figures expressing dollars are separated from those which denote cents and mills by a comma; thus,

$11,111

is read, 11 dollars 11 cents 1 mill; the figures on the *left* of the comma always denote dollars; the first two on the right denote cents, and the third, mills.

ALIQUOT PARTS.

One number is said to be an *aliquot* part of another, when

34. Are the numbers used in United States money abstract or denominate? According to what scale do the units change? How are dollars separated from cents and mills? What is an aliquot part? Name the aliquot parts of a dollar?

it is contained in that other an exact number of times. Thus; 50 cents, 25 cents, &c., are aliquot parts of a dollar: so also, 2 months, 3 months, 4 months and 6 months are aliquot parts of a year. The parts of a dollar are sometimes expressed fractionally, as in the following

TABLE.

$1	=	100 cents.	$\frac{1}{8}$ of a dollar	=	$12\frac{1}{2}$ cents.
$\frac{1}{2}$ of a dollar	=	50 cents.	$\frac{1}{10}$ of a dollar	=	10 cents.
$\frac{1}{3}$ of a dollar	=	$33\frac{1}{3}$ cents.	$\frac{1}{16}$ of a dollar	=	$6\frac{1}{4}$ cents.
$\frac{1}{4}$ of a dollar	=	25 cents.	$\frac{1}{20}$ of a dollar	=	5 cents.
$\frac{1}{5}$ of a dollar	=	20 cents.	$\frac{1}{2}$ of a cent	=	5 mills.

VARYING SCALES.

35. If we write the well-known signs of the English currency, and place 1 under each denomination, we shall have

£	*s.*	*d.*	*far.*
1	1	1	1

Now, the signs £. *s.* *d.* and *far.* fix the value of the unit 1 in each denomination; and they also determine the relations between the different units. For example, this simple language expresses the following ideas:

1*st.* That the unit of the right hand place is 1 farthing—of the place next at the left, 1 penny—of the next place, 1 shilling—of the next place, 1 pound; and

2*d.* That 4 units of the lowest denomination make one unit of the next higher; 12 of the second, one of the third; and 20 of the third, one of the fourth. Hence, 4, 12 and 20, make up the scale.

36. If we take the denominate numbers of Avoirdupois weight, we have

Ton	*cwt.*	*qr.*	*lb.*	*oz.*	*dr.*
1	1	1	1	1	1;

---

35. In English currency, is the scale uniform or varying? How does it vary?

36. Name the units of the scale in Avoirdupois weight.

in which the units increase in the following manner; viz.: counting from the right, 16 units of the lowest denomination make 1 unit of the second; 16 of the second, 1 of the third; 25 of the third, 1 of the fourth; 4 of the fourth, 1 of the fifth; 20 of the fifth, 1 of the sixth. The scale, therefore, for this class of denominate numbers, varies according to the above law.

37. If we take any other class of denominate numbers, as Troy weight, or any of the systems of measures, we shall have different scales for the formation of the different numbers. But in all the formations, we shall recognize the application of the same general principles.

There are, therefore, two general methods of forming the different systems of integral numbers from the unit one. The first consists in preserving a uniform law of relation between the different units. If that law of relation is expressed by 10, we have the system of common numbers.

The second method consists in the application of known, though varying laws of change in the units. These changes in the units, produce different systems of denominate numbers, each of which has its appropriate scale.

INTEGRAL UNITS OF ARITHMETIC.

38. The Integral units of Arithmetic may be arranged into eight classes:

1*st.* Abstract units: 2*d.* Units of currency: 3*d.* Units of length: 4*th.* Units of surface: 5*th.* Cubic units, or units of volume: 6*th.* Units of weight: 7*th.* Units of time: 8*th.* Units of circular measure.

First among the units of arithmetic stands the abstract unit 1. This is the primary base of all abstract numbers, and becomes the base, also, of all denominate numbers, by merely naming, in succession, the particular thing to which it is applied.

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37. How many general methods are there of forming numbers from the unit one? What is the first? What is the second?

38. Into how many general classes may the units of Arithmetic be arranged? What are they?

OF THE SIGNS.

39. The sign $=$, is called the sign of *equality*. When placed between two numbers it denotes that they are *equal;* that is, that each contains the same number of units.

The sign $+$, is called *plus*, which signifies *more*. When placed between two numbers it denotes that they are to be added together: thus, $3 + 2 = 5$.

The sign $-$, is called *minus*, a term signifying *less*. When placed between two numbers it denotes that the one on the right is to be taken from the one on the left: thus, $6 - 2 = 4$.

The sign $\times$, is called the sign of *multiplication*. When placed between two numbers it denotes that they are to be multiplied together; thus, 12×3, denotes that 12 is to be multiplied by 3.

The parenthesis is used to indicate that the sum of two or more numbers is to be multiplied by a single number: thus,

$$(2 + 3 + 5) \times 6$$

shows, that the sum of 2, 3 and 5 is to be multiplied by 6.

The parenthesis is also used to denote that the difference between two numbers is to be multiplied by a third; thus,

$$(5 - 3) \times 6,$$

denotes that the difference between 5 and 3 is to be multiplied by 6.

The sign $\div$, is called the sign of division. When placed between two numbers it denotes that the one on the left is to be divided by the one on the right: thus, $4 \div 5$, denotes that 4 is to be divided by 5.

PROPERTIES OF THE 9'S.

40. In any number, written with a single significant figure, as 4, 40, 400, 4000, &c., the excess over exact 9's is equal to

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39. What is the sign of equality? What is the sign of addition? What of subtraction? What of multiplication? For what is the parenthesis used? What is the sign of division?

40. What will be the excess over exact 9's in any number expressed by a single significant figure? How may the excess over exact 9's be found in any number whatever?

the number of units in the significant figure. For, any such number may be written,

$$4 = 4.$$

Also, - - - - $40 = (9 + 1) \times 4$,

" - - - - $400 = (99 + 1) \times 4$,

" - - - - $4000 = (999 + 1) \times 4$,

&c., &c., &c.

Each of the numbers 9, 99, 999, &c., contains an exact number of 9's; hence, when multiplied by 4, the several products will contain an exact number of 9's: therefore,

The excess over exact 9's, *in each number, is* 4; *and the same may be shown for each of the other significant figures.*

If we write any number, as

6253,

we may read it 6 thousands 2 hundreds 50 and 3. Now, the excess of 9's in the 6 thousands is 6; in 2 hundreds it is 2; in 50 it is 5; and in 3 it is 3: hence, in them all, it is 16, which is one 9 and 7 over: therefore, 7 is the excess over exact 9's in the number 6253. Hence,

The excess over exact 9's *in any number whatever, is found by adding together the significant figures and rejecting the exact* 9's *from the sum.*

NOTE.—It is best to reject or drop the 9 as soon as it occurs: thus we say, 3 and 5 are 8 and 2 are 10; then, dropping the 9, we say, 1 to 6 is 7, which is the excess; and the same for all similar operations.

1. What is the excess of 9's in 48701? In 67498?
2. What is the excess of 9's in 9472021? In 2704962?
3. What is the excess of 9's in 87049612? In 4987051?

REDUCTION.

CHANGE OF UNITIES.

41. REDUCTION is the operation of changing the unit of a number without altering its value. Thus, if we have 4 yards,

---

41. What is Reduction? How do you change yards to feet? How do you change feet to inches? How do you change inches to feet? How do you change feet to yards?

in which the unit is 1 yard, and wish to change to feet, the units of the scale will be 3, since 3 feet make 1 yard: therefore, the number of feet will be

$$4 \times 3 = 12 \text{ feet.}$$

If it were required to reduce 12 feet to inches, the units of the scale would be 12, since 12 inches make 1 foot. Hence,

$$4 \text{ yards} = 4 \times 3 = 12 \text{ feet} = 12 \times 12 = 144 \text{ inches.}$$

If, on the contrary, we wish to change 144 inches to feet, and then to yards, we should first divide by 12, the units of the scale in passing from inches to feet; and then by 3, the units of the scale in passing from feet to yards. Hence, Reduction is of two kinds:

1*st.* To reduce a number from a higher unit to a lower:

Multiply the units of the highest denomination by the number of units in the scale which connects it with the next lower, and then, add to the product the units of that denomination: Proceed in the same manner through all the denominations till the unit is brought to the required denomination.

2*d.* To reduce a number from a lower unit to a higher:

Divide the given number by the number of units in the scale which connects it with the next higher denomination; and set down the remainder, if there be one. Divide the quotient thus obtained, and each succeeding quotient in the same manner, till the unit is reduced to the required denomination: the last quotient with the several remainders annexed, will be the answer.

EXAMPLES.

1. Reduce £3 14*s.* 4*d.* to pence. We first multiply the £3 by 20, which gives 60 shillings. We then add 14, making 74 shillings: we next multiply by 12, and the product is 888 pence: to this we add 4*d.* and we have 892 pence, which are of the same value as £3 14*s.* 4*d.*

If, on the contrary, we wished to change 892 pence to pounds shillings and pence, we should first divide by 12: the quotient is 74 shillings, and 4*d.* over. We next divide by 20, and the

quotient is £3, and 14*s*. over: hence, the result is £3 14*s*. 4*d*., which is equal to 892 pence.

The reductions, in all the denominate numbers, are made in the same manner.

2. In £5 5*s*., how many shillings, pence, and farthings?

```
 £    s.
 5    5
 20
----
 105  5 shillings added.
 12
----
1260
   4
----
5040
```

Here the reduction is from a greater to a less unit.

3. In 5040 farthings, how many pence, shillings, and pounds?

```
   4)5040 farthings.
  12)1260 pence.
  2|0)10|5 shillings.
     £5 5s.
```

In this example, the reduction is from a less to a greater unit.

4. In 34*T*., 16*cwt*., 3*qrs*., 19*lb*., how many pounds?

```
  34
  20
 ----
  696   16cwt. added.
    4
 ----
 2787   3qr. added.
   25
 -----
13954   19lb. added.
5574
-----
69694lbs.
```

5. In 69674*lb*., how many *tons*, *cwt*., *qr*., and *lb*.

```
25)69674
   4)2787qr. . . . 19lb.
   20)696cwt. . . . 3qr.
      334T. . . . 16cwt.
```

Ans. 34*T*. 16*cwt*. 3*qr*. 19*lb*.

6. In $426, how many cents? How many mills?

7. In 36 eagles 8 dollars and 6 dimes, how many cents?

8. In 8750 mills, how many dollars and cents?

9. In 43 eagles 3 dollars and 5 mills, how many mills?

10. In £37 9*s*. 8*d*., how many pence?

11. In 1569 farthings, how many pounds, shillings, pence, and farthings?

12. In 7*T.* 14*cwt.* 1*qr.* 20*lbs.*, Avoirdupois, how many pounds?

13. In 15445*lb.*, Avoidupois, how many tons, cwts., qrs., and lbs.

14. How many grains of silver in 4*lb.*, 6*oz.*, 12*dwt.* and 7*grs.*?

15. How many pounds, ounces, pennyweights, and grains of gold, in 704121 grains?

16. In 5℔, 1℥, 1ʒ, 1℈, 2*gr.*, Apothecaries' weight, how many grains?

17. In 174947 grains, how many pounds, ounces, drachms, scruples and grains?

18. In 6 yards 2 feet 9 inches, how many inches?

19. In 5 miles, how many rods, yards, feet and inches?

20. In 2730 inches, how many yards feet and inches?

21. In 56 square feet, how many square yards?

22. In 355 perches, or square rods, how many acres, roods and perches?

23. In 456 square chains, how many acres?

24. In 3*A.*, 2*R.*, 8*P.*, how many perches?

25. In 14 tons of round timber, how many cubic inches?

26. In 31 cords of wood, how many cubic feet?

27. In 56320 cubic feet, how many cords?

28. In 157 yards of cloth, how many nails?

29. In 192 Ells Flem., how many yards?

30. 97*yd.*, 3*qr.*, how many Ells English?

31. In 4*hhd.* Wine measure, how many quarts?

32. In 7560 pints, Wine measure, how many hogsheads?

33. In 7 hogsheads of ale, how many pints?

34. In 74304 half pints of ale, how many barrels?

35. In 31 bushels, Dry measure, how many pints?

36. In 2110 pints, Dry measure, how many bushels?

37. In 2 years of 365*d.* 5*h.* 48*m.* 48*sec.*, each, how many seconds?

38. How many months, weeks and days in 254 days, reckoning the month at 30 days?

ADDITION.

42. THE SUM *of two or more numbers is a number containing as many units as all the numbers taken together.*

ADDITION *is the operation of finding the sum of two or more numbers.*

1. What is the sum of 769 and 437

	OPERATION.
ANALYSIS.—Write the numbers thus:	769
draw a line beneath them, - - - -	487
sum of the units, - - - - - - -	16
sum of the tens, - - - - - - -	14
sum of the hundreds, - - - - - -	11
Entire sum, - - - -	1256

The example may be done in another way, thus: Set down the number as before: then say, 7 and 9 are 16: set down 6 in the units place, and the 1 ten under the 8 in the column of tens. Then say, 1 to 8 are 9, and 6 are 15. Set down the 5 in the column of tens, and the 1 hundred in the column of hundreds. We then add the hundreds, and find their sum to be 12: hence, the entire sum of 1256.

OPERATION.
769
487
116
1256

NOTE 1.—Observe, that units of the *same value* are always written in the same column, since every *collection must contain units of the same kind.*

2. When the sum in any column, exceeds 9, it produces a unit of a higher order, which belongs to the next column at the left. In that case, write down the excess over tens, and add the tens to the next column. This is called *carrying to the next column.* The number to be carried, should not, in *practice,* be written under the column at the left, but added *mentally.*

Beginners, however, should set down the numbers to be carried,

---

42. What is the sum of two or more numbers? What is Addition? How are numbers written down for Addition? What do you do in simple numbers when the sum of any column exceeds 9? What is this called? What *is the general rule for* the addition of numbers?

each under its proper column, as in the examples below.

(2)	(3)	(4)
85468	672143	4783614
9104	79161	504126
379	8721	872804
94951	760025	6160544
1012	12110	211101

5. What is the sum of 35 dollars 4 dimes 6 cents 5 mills, 4 dollars 7 mills, and 97 cents 3 mills?

NOTE.—Write the units of the same value in the same column, separating the dollars from the cents and mills by a comma (Art. 40): then add the columns as in simple numbers.

OPERATION.

$35,465
4,007
,973
$40,445
111

6. Let it be required to find the sum of £14 7*s.* 8*d.* 3*far.*, and £6 18*s.* 9*d.* 2*far.*

ANALYSIS.—Write the numbers, as before, so that units of the same value shall fall in the same column. Beginning with the lowest denomination, we find the sum to be 5 farthings. But since 4 farthings make a penny, we set down 1 farthing, and carry one penny to the column of pence. The sum of the pence then becomes 18, which is 1 shilling and 6 pence over. Set down the 6 pence, and carry the 1 shilling to the column of shillings, the sum of which becomes 26; that is, 1 pound and 6 shillings. Setting down the 6 shillings and carrying 1 to the column of pounds, we find the entire sum to be £21 6*s.* 6*d.* 1*far.*

OPERATION.

£	*s.*	*d.*	*far.*
14	7	8	3
6	18	9	2
21	6	6	1

Hence, for the addition of all numbers,

I. *Write the numbers so that units of the same value shall fall in the same column.*

II. *Add the units of the lowest denomination, and divide their sum by so many as make one unit of the denomination next higher. Set down the remainder and carry the quotient to the next higher denomination; proceed in the same manner through all the denominations and set down the entire sum of the last column.*

PROOF.

43. The proof of an operation, in Addition, consists in showing that the answer contains as many units as there are in all the numbers added. There are three methods of proof:

I. *Begin at the top of the units column and add, in succession, all the columns downwards. If the two results agree, the work is supposed to be right; for, it is not likely that the same mistake will have been made in both additions.*

II. *Divide the given numbers into parts, and add the parts separately: then add together the partial sums; if the results agree, the work is supposed to be right; for, a whole is equal to the sum of all its parts.*

III. *Find the excess of 9's in each number, and place it at the right* (Art. 20). *Add these numbers and note the excess of 9's in their sum. This excess should be equal to the excess of 9's in the sum of the numbers.*

1st. Method.

	182796
	143274
	32160
	47047
Sum	405277

2d. Method.

	182796	32160
	143274	47047
Partial sums	326070	79207

	326070	1st partial sum.
	79207	2d "
Sum	405277	

3d Method.

- 182796	- -	6 excess of 9's,
- 143274	- -	3 " "
- 32160	- -	3 " "
- 47047	- -	4 " "
Sum 405277 ··· 7		16 - - - 7 excess of 9's.

READING.

44. The pupil should be early taught to omit the *intermediate*

---

43. What is the proof of an operation in Addition? How many methods of proof are there? Explain each separately?

44. *What is the reading* process, in Addition?

words in the addition of columns of figures. Thus, in the above example, instead of saying 7 and 0 are 7; 7 and 4 are eleven; 11 and 6 are seventeen; he should simply say, seven, eleven, seventeen. Then, in the column of tens he should say, five, eleven, eighteen, twenty-seven; and similarly, for the other columns at the left. This is called *reading* the columns. Let the pupils be often practised in the readings, both separately and in concert in the class.

EXAMPLES.

(1.)	(2.)	(3.)	(4.)
94201	80032	98800	10304
46390	4291	10926	67491
37467	2376	321	1324
4572	840	479	46
182630	87539	110526	

5. What is the sum of 1376, 38940, 8471, 23607, 891?

6. What is the sum of 3480902, 3271, 567321, 91243, 6001, 169?

7. What is the sum of 42300, 6000, 347001, 525, 47?

(8.)	(9.)	(10.)	(11.)	(12.)
days.	*bushels.*	*rods.*	*minutes.*	*gallons.*
1276	47917	9003	67321	760324
3718	12031	1881	4702	18720
9024	5672	6035	1067	5762
6357	8321	7810	456	359
1028	728	3176	377	1082
9131	47	2004	99	47269

(13.)	(14.)	(15.)	(16.)	(17.)
miles.	*furlongs.*	*pounds.*	*dollars.*	*casks.*
1600	47468	76389	1602	40506
2588	59012	1036	9614	37219
9101	23419	2671	4732	50170
6793	15760	5132	5675	32614
8267	27900	6784	8211	78462
4572	12317	1672	4455	10001

(18.)	(19.)	(20.)	(21.)	(22.)
$175,365	$30,365	$180,000	$300,40	$4802,279
278,056	28,779	489,007	167,275	1642,107
420,96	10,101	76,119	18,197	3026,267
76,125	9,08	16,423	29,94	125,092
41,04	7,14	9,011	10,08	42,75
				[illegible]

(23.)				(24.)			(25.)			(26.)		
£	*s.*	*d.*	*far.*	*lb.*	*oz.*	*dwt.*	℔	℥	ʒ	*lb.*	*oz.*	*dr.*
14	11	3	1	174	11	19	17	11	7	17	15	12
17	18	10	2	75	10	13	94	10	6	29	32	10
29	7	6		642	3	10	60	9	2	84	10	9
42	14	11	3	125	7	5	42	3	9	14	3	7
17	10	00	1	62	0	16	12	0	6	40	9	9
84	00	1	0	39	1	4	98	7	5	76	4	7
16	19	8	2	176	10	15	127	1	0	18	11	15

(27.)			(28.)			(29.)			(30.)		
cwt.	*qr.*	*lb.*	*yd.*	*qr.*	*na.*	*E. E.*	*qr.*	*na.*	*L.*	*mi.*	*fur*
174	2	20	74	3	3	14	4	3	17	2	7
320	1	14	60	1	2	75	1	2	10	1	4
136	3	23	14	0	1	84	3	1	7	0	6
47	0	12	45	2	3	17	2	0	5	2	3
84	1	24	69	1	0	10	0	2	25	1	0
90	2	9	11	0	0	19	1	1	36	2	2
7	3	5	36	3	1	29	3	2	40	1	0

(31.)			(32.)			33.)			(34.)		
yrds.	*ft.*	*in.*	*A.*	*R.*	*P.*	*Tun.*	*hhd.*	*gal.*	*gal.*	*qt.*	*pt.*
174	11	1	77	3	39	714	3	56	14	3	1
260	2	0	64	2	37	626	1	48	74	2	1
150	10	2	16	1	29	320	0	29	96	1	0
126	9	1	72	0	18	156	2	31	47	2	1
96	7	0	36	2	20	225	1	42	22	0	1
72	4	1	42	2	14	84	0	17	65	1	0
8	6	2	11	3	7	96	1	34	19	0	0

(35.)			(36.)			(37.)			(38.)		
chal.	*bu.*	*qt.*	*yr.*	*mo.*	*wk.*	*da.*	*hr.*	*min.*	*qr.*	*lb.*	*oz.*
14	31	6	127	9	2	140	12	27	44	21	14
25	14	2	320	10	3	340	16	40	14	16	12
36	29	7	146	8	1	227	20	56	22	10	11
42	24	3	75	6	0	102	13	25	36	19	7
39	32	1	70	11	2	67	21	37	51	13	9
56	19	5	54	7	1	14	9	10	30	22	11
14	20	4	27	4	3	10	19	46	16	15	15

39. The population of the United States and territories, in 1850, was as follows: White population, 19553068; Free Colored population, 434495; Slave population, 3204313; Indians, 400674; what was the entire population?

40. In the year 1850, the expenditures of the United States, amounted to 43002168 dollars; in 1851, to 48905879 dollars; in 1852, to 46007893 dollars: what were the expenditures of the United States for these three years?

41. A man of fortune bequeathed to each of his three sons, 10492 dollars; to each of his two daughters, 5976 dollars; to his wife, the remainder of his property, which exceeded the amount bequeathed to his children by twelve hundred dollars: find the amount of his property?

42. A stage goes in one day 27 miles 3 furlongs 36 rods; the next, 32 miles 10 rods; the next, 36 miles 2 furlongs; the next, 25 miles 6 furlongs 38 feet: how far did it go in four days?

43. The population of Boston, in 1854, was 178000; Providence, 60,000; Buffalo, 75000; New Orleans, 139190; Louisville, 55000; Sacramento, 12000: what was the entire population of these cities?

44. The population of New York city, in 1850, was 515547; Brooklyn, 127618; Baltimore, 169054; Washington, 40000: Cincinnati, 115436; Chicago, 29963; St. Louis, 77860; Milwaukie, 20061; Detroit, 21119; Indianapolis, 8091: what was the entire population of these cities?

45. Bought a barrel of flour for eight dollars and seventy-five cents; a ton of plaster for five dollars sixty-two and a-half cents; a hat for three dollars twelve cents and five mills; fifty pounds of sugar, for four dollars fifty cents and nine mills: what was the amount of my bill?

46. A lady bought a bonnet for $5,375; some silk for $12,03; some ribbon for $0,875; a shawl for $9,46: what did the whole amount to?

47. A wine-merchant taking an invoice of his liquors, finds that he has 5*hhds.* 36 *gals.* 2*qts.* of wine; 3*hhds.* 15*gals.* 1*qt.* 1*pt.* of rum; 1*hhd.* 2*qts.* of gin; 40*gals.* 1*pt.* of whisky: how much liquor in all?

48. Tea was imported into the United States in the year 1851, to the value of $4798005; in 1852, $7285817; in 1853, $8224853: what was the value of the tea imported during these three years?

49. The United States exported tobacco, in the year 1851, to the amount of $9219251; in 1852, $10031283; in 1853, $11319319: what was the entire value of tobacco exported in these three years?

50. A man sold his house and lot for $25840, which was $3186 less than he gave for them; how much did they cost him?

51. A speculator bought three city lots, for the first he paid $2870,43; for the second, $2346,75; for the third, $1563,82. He sold the same at an average profit upon each of $476,25; what amount did he receive for the lots?

52. The population of England, in 1851, was 16921888; of Ireland, 6515794; of Scotland, 2888742: what was the entire population of the three?

53. The churches of the United States and territories in 1850, were, Baptists, 9375; Congregationalists, 1706; Presbyterians, 4824; Methodists, 13280; Universalists, 529: what was the whole number of churches belonging to these five denominations?

54. In the same year, the value of the church property

the first; and the third put in as much as the other two: what was the whole amount of capital invested?

73. A farmer raised in one field 240*bush.* 3*pks.* 2*qts.* of wheat; in another 97*bush.* 6*qts.*; in another 42*bush.* 1*pk.*: how much did he raise in the three fields?

74. Add together three hundred dollars, ten eagles, forty dimes, ninety-six cents, seven mills, nine dollars, forty-seven cents, five mills, four eagles, three dollars, and nine dimes.

75. What is the sum of £17 10*s.* 6*d.*; £25 4*s.* 10½*d.*; 18*s.* 6*d.* 3*far.*; £11 9¼*d.*; £1 18*s.*; 21*s.* 6¼*d.*?

76. The Deluge, according to Chronology, occurred 1656 years after the creation; the call of Abraham 427 after the Deluge; the departure of the Israelites, 430 after the call of Abraham; the foundation of the temple, 479 after the departure of the Israelites; the end of the captivity, 476 after the foundation of the temple; and the birth of Christ, 536 years after the end of the captivity: how many years from the creation to the present time, it being the year 1856?

SUBTRACTION.

45. THE DIFFERENCE *between two numbers is such a number as, added to the less, will give the greater.*

If the numbers are unequal, the larger is called the *minuend*, and the less the *subtrahend.* If they are equal, either is the minuend and the other the subtrahend. Their difference, whether they are equal or unequal, is called the *remainder.*

SUBTRACTION *is the operation of finding the difference between two numbers.*

1. From 869 take 327; that is, from 8 hundreds 6 tens and 9 units, take 3 hundreds 2 tens and 7 units.

---

45. What is the difference between two numbers? When the numbers are unequal, what is the larger number called? What is the less called? What is their difference called? What is Subtraction? Give the rule for finding the difference between two numbers.

ANALYSIS.—Beginning with the right hand figure, we take units from units; then, tens from tens; then, hundreds from hundreds, and find the remainder to be 542.

OPERATION.

869	minuend.
327	subtrahend.
542	remainder.

2. From 624 take 395.

ANALYSIS.—Having written down the numbers, we subtract 3 from 4, and find a remainder 1. At the next step we meet a difficulty, for we cannot subtract 9 tens from 2 tens.

Take 1 hundred = 10 tens, from 6 hundreds and add it to 2 tens. Then 9 tens from 12 tens leaves 3 tens, and 3 hundreds from 5 leaves 2 hundreds, and the remainder is 231.

OPERATION.

		hun.	tens.	units.
10				
624	-	5	12	4
393	-	3	9	3
1				
231	-	2	3	1

The remainder can be found by adding, mentally, 10 to 2 tens, and then saying, 9 from 12 leaves 3 tens; then adding 1 to 3 hundreds, and say, 4 from 6 leaves 2 hundreds.

The process of adding 10 to a figure of the *minuend* and returning 1 to the next figure of the *subtrahend*, at the left, is called *borrowing*.

3. From 6*T.* 14*cwt.* 2*qr.* 20*lb.* 12*oz.*, take 4*T.* 17*cwt.* 1*qr.* 21*lb.* 10*oz.*

ANALYSIS.—Taking 10*oz.* from 12*oz.*, 2*oz.* remain. At the next step we find a difficulty, for 21*lb.* cannot be taken for 20*lb.* We then take 1*qr.* = 25*lb.* from 2*qr.* and add it to 20*lb.*, making 45*lb.*; then say, 21*lb.* from 45*lb.* leaves 24*lb.*; we then add 1 to the next left hand figure of the subtrahend, and say 2*qr.* from 2*qr.* leaves 0; then 17*cwt.* from 34*cwt.* leaves 17*cwt.*, and 5 from 6 leaves 1 ton.

T.	*cwt.*	*qr.*	*lb.*	*oz.*
	20		25	
6	14	2	20	12
4	17	1	21	10
1		1		
2	17	0	24	2
5	34	1	45	12
4	17	1	21	10

Hence, to find the difference between two numbers,

I. *Set down the less number under the greater, so that units of the same value shall fall in the same column.*

II. *Begin with the units of the lowest denomination and subtract each from the units above it.*

III. *When the units of any denomination in the subtrahend exceed those of the same denomination in the minuend, suppose*

so many units to be added as make one unit of the next higher denomination; after which add 1 *to the next denomination of the subtrahend, and subtract as before.*

FIRST PROOF.

46. The *difference* or *remainder*, is such a number as added to the subtrahend, will give a sum equal to the minuend (45); hence,

Add the remainder to the subtrahend. If the work is right, the sum will be equal to the minuend.

SECOND PROOF.

Since the remainder added to the subtrahend is equal to the minuend (Art. 45), it follows that the excess of 9's in these two numbers is equal to the excess of 9's in the minuend; hence,

Find the excess of 9's *in the minuend, in the subtrahend and in the remainder; if the work is right, the excess of* 9's *in the two last numbers will be equal to the excess of* 9's *in the first.*

What is the difference between 874136 and 45302?

1st. Method.	2d. Method.		
874136	874136	- -	2 excess of 9's in the first.
45302	45302	- -	5 " " 2d.
828834	828834	- -	$6+5=11$: 2 excess.

READING.

47. What is the difference between 426 and 295?

By the common method, which is spelling, we say, 5 from 6 leaves 1; 9 from 12 leaves 3; 1 to carry to 2 are 3; 3 from 4 leaves 1.

OPERATION.

426
295
131

By reading the words which express the final result, we make the operations mentally, and say, *one*, *three*, *one*.

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46. What is the difference or remainder? How do you prove Subtraction? How do you prove Subtraction by the second method?

47. Explain the process of reading in Subtraction.

TIME BETWEEN DATES.

48. What time elapsed between the inauguration of Mr. Jefferson, March 4th, 12 o'clock, M., 1801, and July 4th, 3 P. M., 1855?

OPERATION.

yr.	*mo.*	*da.*	*hr.*
1855	7	4	15
1801	3	4	12
54	4	0	3

ANALYSIS.—Place the earlier date under the later, writing the number of the year, reckoned from the beginning of the Christian Era, on the left. Then, write in the same line the number of the month, reckoned from the first of January, the number of the day, reckoned from the first of the month, the number of the hour, reckoned from 12 at night, and write the number of minutes and seconds, if there are any, still to the right. Hence, to find the time between two dates,

Write down as above, and subtract the earlier date from the latter.

NOTE 1.—In finding the difference between dates, as in casting interest, the month is regarded as the twelfth part of the year and as containing 30 days.

2. The civil day begins and ends at 12 o'clock at night.

EXAMPLES.

	(1.)	(2.)	(3.)	(4.)
From	472567	103796	900372	1760134
Take	109271	47217	167301	48207

	(5.) *rods.*	(6.) *dollars.*	(7.) *mills.*	(8.) *barrels.*
From	74623457	8600000	162347	8462
Take	32700169	761820	56321	4071

	(9.) *bushels.*	(10.) *inches.*	(11.) *minutes.*
From	100000	200763194	3601789412
Take	37214	2142079	10031761

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48. How do you find the difference of time between two dates? In this computation, what part of a year is a month? How many days are reckoned to the month?

	(12.)	(13.)	(14.)
	cords.	*gallons.*	*pounds.*
From	4200000	8888777	100000000
Take	325	9999	23

	(15.)	(16.)	(17.)
From	$8475,656	$1000,759	$4871036,008
Take	32,015	194,375	17362,25

	(18.)				(19.)				(20.)		
	£	*s.*	*d.*	*far.*	*ton.*	*cwt.*	*qrs.*	*lb.*	*yd.*	*qr.*	*na.*
From	25	12	6	2	5	17	3	21	137	1	3
Take	10	14	3	1	2	9	1	14	19	3	2

	(21.)				(22.)					(23.)		
	L.	*mi.*	*fur.*	*rd.*	*tun.*	*hhd.*	*gal.*	*qt.*	*pt.*	*A.*	*R.*	*P.*
From	75	2	7	37	14	1	26	2	1	100	2	27
Take	16	1	4	9	5	3	35	3	1	10	3	30

	24.)			(25.)			(26.)		
	bush.	*pk.*	*qt.*	*cords.*	*ft.*	*in.*	*E.E.*	*qr.*	*na.*
From	1000	3	4	225	42	1242	42	1	2
Take	25	1	6	100	112	720	16	4	3

(27.)			(28.)			(29.)			(30.)		
℔	℥	ʒ	℥	ʒ	℈	*E.E*	*qr.*	*na.*	*E.F.*	*qr.*	*na.*
144	10	5	27	4	1	174	3	1	171	1	3
64	11	7	14	7	2	49	4	2	74	3	2

(31.)			(32.)			(33.)			(34.)		
T.	*cwt.*	*qr.*	*cwt.*	*qr.*	*lb.*	*qr.*	*lb.*	*oz.*	*lb.*	*oz.*	*dr.*
14	12	2	17	1	21	143	22	12	174	11	10
1	14	3	14	2	24	74	19	14	39	12	13

(35.)			(36.)			(37.)			(38.)		
A.	*R.*	*P.*	*A.*	*R.*	*P.*	*da.*	*hr.*	*min.*	*hr.*	*min.*	*sec.*
12	1	32	112	1	31	167	21	50	147	50	51
1	3	14	74	2	37	19	23	54	94	59	57

39. From \$10000 take \$1240,37$\frac{1}{2}$.

40. From 183701289 take 34627.

41. From 17*yr.* 9*mo.* 1*wk.* 16*da.* take 10*y.* 11*mo.* 2*wk.* 5*da.*

42. From 144℔ 7℥ 5ʒ 1℈ take 56℔ 6℥ 7ʒ 1℈.

43. From two eagles seven dimes, take twelve dollars and fifty cents.

44. From forty dollars twelve and a half cents, take twenty-five cents and seven mills.

45. From one eagle five dollars six dimes and ten cents, take five dollars seven cents and four mills.

46. What sum added to £11 14*s.* 9$\frac{1}{4}$*d.* will make £133 11*s.* 9$\frac{1}{2}$*d.*?

47. An apprentice, who is 14 years 11 months 3 weeks, 14 hours 58 minutes old, is to serve his master until he is 21 years of age. How long has he to serve?

48. The greater of two numbers is seven millions three hundred and four thousand and ten; the less is nine hundred and fifty thousand one hundred and forty. What is their difference?

49. Mont Blanc, the highest mountain in Europe, is 15680 feet high; Chimborazo, the highest in America, is 21427 feet. What is the difference in their heights?

50. A man sold his farm for seven thousand five hundred and thirty dollars, which was fifteen hundred and ten dollars more than he gave for it. How much did he give for it?

51. The revenue collected at the port of New York for the year ending 30th June, 1853, was \$38289341,58; at Philadelphia, \$4537046,16; at Boston, \$7203048,52; at Baltimore, \$836437,99. How much more was collected at the port of New York than at the other three?

52. A man engaging in trade found at the end of five years that he had increased his capital ten thousand three hundred and ten dollars, and that his whole capital amounted to forty-six thousand five hundred dollars. How much did he commence with?

53. The minuend exceeds the remainder by 683021, and the remainder is 902563. What is the subtrahend?

54. The amount of tea consumed in the United States in the year 1846, was 16891020 pounds; the amount of coffee, 124336054 pounds. How much more coffee than tea was consumed?

55. What number is that to which, if you add 3726, the sum will be ten thousand?

56. From a stack of hay containing 9*tons* 3*qr.* 20*lb.*, I sold 4*tons* 17*cwt.* 22*lb.* How much was then left?

57. A owes B £25; after paying him £5 $9\frac{1}{4}$*d.*, how much will he still owe him?

58. If the distance from New York to Liverpool be 3100 miles, after a ship has sailed 800*m.* 5*fur.* 36*rods*, what distance remains?

59. Bought a farm for three thousand five hundred dollars and fifty cents; sold the same for three thousand three hundred dollars and eighty-seven and a half cents: how much did he lose by the bargain?

60. If a lot of goods are bought for $750, and sold for $925,87½, what will be gained?

61. If I buy a bushel of wheat for $1,87½; ten gallons of molasses for $2,50; five yards of cloth for $12,37½: how much change must I receive back for two ten dollar bills?

62. The population of the United States in the year 1850 was 23191876, of which 3204313 were slaves: what was the white population?

63. England contains 50922 square miles; Scotland, 31324 square miles; Wales 7398 square miles; the United States contain 2988892 square miles. How many more square miles does the United States contain than the whole of Great Britain?

64. A gentleman of fortune owning an estate of two hundred thousand dollars, bequeathed thirty thousand dollars to objects of charity; twenty-five thousand two hundred and fifty dollars to each of his three sons; twenty thousand five hundred

and seventy-five dollars to his daughter; and the remainder to his widow. How much did the widow receive?

65. The population of New Orleans in 1850 was 116375; in 1854 it was 139190: what was the increase in four years?

66. Having deposited $1500 in a bank, I drew out at one time $475,12$\frac{1}{2}$; at another time $300; at another $526,25: how much remained?

67. If the Declaration of Independence was made at precisely 12 o'clock, on the 4th day of July, 1776; how much time will have passed to the 4th day of March, 1857, at 30 minutes past 3 o'clock, P. M.?

68. If I borrow $1576 of a friend, and afterwards pay him $920,87$\frac{1}{2}$, how much would I still owe him?

69. The first settlement made in the United States was at Jamestown, in Virginia, May 23, 1607: how many years from that time to the 4th of July, 1856?

70. The sum of two numbers is 36804, and the less number is eighteen thousand nine hundred and twenty-seven: what is the greater number?

71. The revenue of the United States in the year 1853 was $61337574; the expenditures $54026818: how much did the revenue exceed the expenditures?

72. The exports of the State of New York in the year 1853 were valued at $66030355; those of the State of Virginia, for the same year, $3302561: how much did the exports of New York exceed those of Virginia?

73. A ship-builder sold a vessel for $50376, which cost him $42978: how much did he gain?

74. A farmer sold his farm for six thousand three hundred and seventy-five dollars; after paying his debts, he has four thousand and fifteen dollars left: what was the amount of his debts?

75. Gunpowder was invented in the year 1330: how many years from that time to the year 1856?

76. What number is that to which if you add 3726 the sum will be ten thousand?

77. A speculator bought a quantity of flour for \$2084,50; of bacon for \$760,87$\frac{1}{2}$; of hops for \$1836,25. He sold the flour for \$2375,60; the bacon for \$912,375; the hops for \$1750: what did he gain or lose on the whole?

78. A farmer has two pastures, one containing 9*A.* 3*R.* 32*P.*; the other 12*A.* 29*P.* He has also two meadows, one containing 10*A.* 2*R.*; the other 15*A.* 1*R.* 20*P.*: how much more meadow than pasture has he?

79. From a pile of wood containing 76 cords and 6 cord feet, was taken at one time 20 cords and 48 cubic feet; at another time 14 cords 1 cord foot and 80 cubic feet: how much remained in the pile?

80. A gentleman purchased a house worth \$9436; a carriage for \$475,50; a span of horses for \$840,40. He paid at one time \$5260; at another \$1275,37$\frac{1}{2}$; at another \$936,42: how much remained unpaid?

81. If a ship and cargo are valued at \$47568,487, and the cargo alone at \$3406,50, what is the value of the ship without the cargo?

82. A gentleman dying left an estate of \$50000; after paying his debts, which amounted to \$5647,50, he desired that each of his two sons should receive \$15000, and his widow the remainder: how much did the widow receive?

83. A note on interest, dated July 1st, 1853, was to be paid March 20th, 1856: how long was it on interest?

84. Bought a hogshead of wine, from which was drawn 32*gals.* 1*qt.* 1*pt.*; how much remained in the cask?

85. The population of Chicago in 1850 was 29963; in 1855 it was 80025: what was the increase in three years?

86. A land speculator owning twenty-five thousand acres of land, sells at one time fifteen hundred acres; at another four thousand seven hundred; at another twenty-five hundred acres; at another seven hundred and fifty acres: what number of acres has he left?

87. The latitude of New Orleans is 29° 57′ 30″; that of Boston, 42° 21′ 23″: what is the difference in the latitude of these two places?

88. A person bought a span of horses for three hundred dollars; a carriage for \$410,50; a harness for \$50,675; he sold the whole for six hundred dollars: did he gain or lose, and how much?

89. The population of Great Britain and its adjacent islands in the year 1841, was 18664761; in 1851 it was 20936468: what was the increase of population in ten years?

90. From a piece of cloth containing 47 yards, a tailor cut 14*yds.* 3*qrs.* 2*na.*: how much was left?

91. A tradesman failing in business, was indebted to A £105 19*s.* 11*d.*; to B, £127 10*s.* 9½*d.*; to C, £34 18*s.* 10*d.*; to D, £500 19*s.*; to E, £700 14*s.* 6½*d.* When this took place, he had in cash £50, in goods to the amount of £350 14*s.* 9*d.*, his household furniture was worth £24 11*s.*, his book accounts amounted to £94 14*s.* 8*d.* If all these were given up to the creditors, how much would they lose?

MULTIPLICATION.

48. MULTIPLICATION *is the operation of taking one number as many times as there are units in another.*

The number to be taken is called the *multiplicand.*

The number denoting how many times the multiplicand is taken, is called the *multiplier.*

The result of the operation is called the *product.*

The multiplicand and multiplier are called *factors*, or *producers of the product.*

NOTE.—Since, when the multiplier is an integral number, the *product* may be obtained by adding the multiplicand to itself as many times less 1 as there are units in the multiplier, *Multiplication is sometimes called a short method of addition.*

---

48. What is Multiplication? What is the number to be taken called? What does the multiplier denote? What is the result called? What are the multiplier and multiplicand called? Why is Multiplication called a *short method of Addition*?

49. There are three parts in every operation of multiplication. First, the *multiplicand:* second, the *multiplier:* and third, the *product.*

From the definition of Multiplication, we see that,

1st. If the multiplier is 1, the product will be equal to the multiplicand.

2d. If the multiplier is greater than 1, the product will be as many times greater than the multiplicand, as the multiplier is greater than 1.

3d. If the multiplier is less than 1, that is, if it is a proper fraction, then the product will be such a part of the multiplicand as the multiplier is of 1.

50. Let it be required to multiply any two numbers together, say 6 to 4.

ANALYSIS.—If we write, in a horizontal line, as many stars as there are units in the multiplicand, and write as many such lines as there are units in the multiplier, it is evident that, *all the stars* will represent the number of units which arise from taking the multiplicand as many times as there are units in the multiplier.

$$4\left\{\begin{array}{cccccc} \multicolumn{6}{c}{\overbrace{\qquad\qquad\qquad\qquad}^{6}} \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \end{array}\right.$$

Change now the multiplier into the multiplicand: that is, multiply 4 by 6.

Make in a vertical line, as many stars as there are units in the new multiplicand, (4), and as many vertical lines are there are units in the new multiplier, (6), when it is again evident that, all the stars will represent the number of units in the product. Hence,

The product of two factors is the same whichever factor is used as the multiplier.

$3 \times 7 = 7 \times 3 = 21$: also, $6 \times 3 = 3 \times 6 = 18$.

$9 \times 5 = 5 \times 9 = 45$: also, $8 \times 6 = 6 \times 8 = 48$.

and, $8 \times 7 = 7 \times 8 = 56$: also, $5 \times 7 = 7 \times 5 = 35$.

---

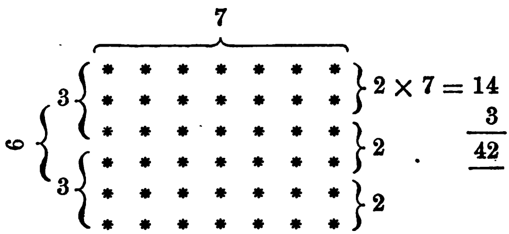
49. How many parts are there in every operation of Multiplication? What are they? How many principles follow from the definition of Multiplication? What are they?

50. In how many ways may 6 and 4 be multiplied together? How do the two products compare with each other? What does this prove?

52. A Composite Number is one that may be produced by the multiplication of two or more numbers, called *factors.* Thus, $2 \times 3 = 6$, in which 6 is the composite number, and 2 and 3 the factors. Also, $16 = 8 \times 2$, in which 16 is a composite number, and 8 and 2 the factors; and since $4 \times 4 = 16$, we may also regard 4 and 4 as factors of 16.

A Prime Number is one which cannot be produced by multiplication, and is divisible only by itself and 1.

53. Let it be required to multiply 7 by the composite number 6, of which the factors are 2 and 3.



$$\begin{array}{r} 7 \\ \underline{3} \\ 21 \\ \underline{2} \\ \underline{42} \end{array}$$

If we write 6 horizontal lines with 7 units in each, it is evident that the product of $7 \times 6 = 42$, will express the number of units in all the lines.

Let us first connect the lines in sets of two each, as at the right; the number of units in each set will then be expressed by $7 \times 2 = 14$. But there are 3 sets; hence, the number of units in all the sets, is $14 \times 3 = 42$.

Again, if we divide the lines into sets of 3 each, as at the left, the number of units in each set will be equal to $7 \times 3 = 21$, and since there are two sets, the whole number of units will be expressed by $21 \times 2 = 42$.

---

52. What is a composite number? Give an example of a composite number? What are its factors? What are the factors of 16? What is a prime number?

53. If several factors are multiplied together, will the product be altered by changing their order? How do you multiply by a composite number?

Since the product of either two of the three factors 7, 3 and 2, will be the same whichever be taken for the multiplier (Art. 50), and since the same principle will apply to that product and to the other factor, as well as to any additional factor, if introduced, it follows that,

The product of any number of factors will be the same in whatever order they are multiplied:

Hence, to multiply by a composite number,

I. *Separate the composite number into its factors:*

II. *Multiply the multiplicand and the partial products by the factors, in succession, and the last product will be the entire product sought.*

NOTE.—Any number whatever, as 440, ending with 0, is a composite number of which 10 is a factor: for, $440 = 44 \times 10$. If there are two 0's on the right of the significant figures, then 100 is a factor, and so on for a greater number of ciphers. Hence, when there are ciphers on the right of significant figures, either in the multiplicand or multiplier, or both,

Multiply the significant figures together, and then annex the ciphers to the product.

54. 1. Multiply 627 by 214.

ANALYSIS.—The multiplicand 627 is to be taken 214 times; that is, 4 units times, 1 ten times, and 2 hundred times. Taking it 4 units times, gives 2508; taking it 1 ten times gives 627, of which the *lowest unit* is 1 *ten*; hence, 7 is written in the *tens place*: taking it 2 hundred times, gives 1254, the *lowest unit* of which is 1 hundred. Adding, we have 134178 for the product.

OPERATION.
```
   627
   214
  2508
  627
 1254
134178
```

NOTE.—What is one factor of a number ending in 0? What is one factor of a number ending in two 0's? In three 0's? &c. How do you multiply by such a number when there are ciphers in one or both factors?

54. Explain the operation of multiplying 627 by 214. Explain the five principles which come from this analysis. What is a partial product? Give the general rule for multiplication. What must be observed in the multiplication of United States money?

It is seen, from the preceding analysis, that

1. *If units be multiplied by units, the unit of the product will be* 1.

2. *If tens be multiplied by units, the unit of the product will be* 1 *ten.*

3. *If hundreds be multiplied by units, the unit of the product will be* 1 *hundred; and so on:*

And since the product of the factors is the same whichever is taken for the multiplier (Art. 50), it follows that,

4. *If units of the first order be multiplied by units of a higher order, the units of the product will be the same as that of the higher order.*

5. *If units of any order be multiplied by units of any other order, the unit of the product will be of an order one less than the sum of the units denoting the two orders.*

NOTE.—When the multiplier contains more than one figure, the product obtained by multiplying the multiplicand by a single figure, is called a *partial product.* In the last example there are three partial products, 2508, 627, and 1254. The *sum* of the partial products is equal to the product sought:

2. Multiply £3 8*s.* 6*d.* 3*far.* by 6.

OPERATION.

£	s.	d.	far.
3	8	6	3
			6
20	11	4	2

ANALYSIS.—Multiplying 3 farthings by 6, we have 18 farthings, equal to 4*d.* and 2*far.*: set down the 2*far.*; then, 6 times 6*d.* are 36*d.*, and 4 pence to carry are 40*d.*, equal to 3 shillings and 4*d.*: then, 6 times 8*s.* are 48*s.* and 3*s.* to carry are 51 shillings, equal to £2 and 11 shillings; then, 6 times £3 are £18 and £2 to carry are £20, which set down.

NOTE.—The unit of each product will be the same as the unit of the multiplicand. Hence, for the multiplication of all numbers, we have the following

RULE.—*Multiply every order of units in the multiplicand, in succession, beginning with the lowest, by each figure in the multiplier, and divide each product so formed by so many units as make one unit of the next higher denomination: write down each*

remainder under the units of its own order, and carry the quotient to the next product.

NOTE.—In multiplying United States money, care must be taken to point off as many places for cents and mills as there are in the multiplicand.

1. Multiply 14 dollars 16 cents and 8 mills, by 5, 6, and 7.

\$14,168	\$14,168	\$14,168
5	6	7

(2.)	(3.)	(4.)
\$870,46	\$894,120	\$2141,096
9	$14 = 7 \times 2$	$36 = 6 \times 6.$

PROOFS OF MULTIPLICATION.

55. There are three methods of proof for multiplication:

I. Write the multiplier in the place of the multiplicand, and find the product as before: if the two products are the same, the work is supposed to be right (Art. 50).

II. Divide the product by one of the factors, and the quotient will be the other factor.

III. By the method of casting out the 9's.

FIRST METHOD.

```
Multiply  -  -   80432        506
      by  -  -     506      80432
                ------     ------
                482592     4048
               402160       2024
               --------      1518
               40698592       1012
                           --------
                           40698592
```

NOTE 1.—Although we generally begin the multiplication by the figure of the lowest unit, yet we may multiply in any order, if we only preserve the *places of the different orders of units.* In the example at the right, we began with the order of tens of thousands or 5th order.

55. How many proofs are there for multiplication? What is the first? What is the second? What is the third?

2. Although either factor may be used as the multiplier (Art. 50), still it is best to use that one which contains the fewest figures. For, if we change the process and use the multiplicand as the multiplier, there will be more multiplications, as shown in the last example.

PROOF BY THE 9'S.

56. Let it be required to multiply *any two numbers together*, as 641 and 232.

ANALYSIS.—We first find the excess over exact 9's in both factors, and then separate each factor into two parts, one of which shall contain exact 9's, and the other the excess, and unite the two by the sign plus. It is now required to take $639 + 2 = 641$, as many times as there are units in $225 + 7 = 232$.

Every partial product, in this multiplication, contains exact 9's, except 14, which contains one 9 and 5 over; and as the same may be shown for any two numbers, we see that,

OPERATION.

$$\begin{array}{r} 641 = 639 + 2 \\ 232 = 225 + 7 \\ \hline 4473 + 14 \\ 450 \\ 3195 \\ 1278 \\ 1278 \\ \hline 148698 + 14 \end{array}$$

If we find the excess of 9's in each of two factors, and then multiply them together, the excess of 9's in their product will be equal to the excess of 9's in the product of the factors.

	(1.)	*Ex.*	(2.)	*Ex.*
Multiply	87603 - -	6	818327 - -	2
by	9865	1	9874	1
Prod.	864203595 - -	6	8080160798 - -	2

3. By multiplication we have

Ex. 4. Ex. 8. Ex. 4. Ex. of product, 2.

$$7285 \times 143 \times 976 = 1016752880.$$

NOTE.—Is it *necessary* to commence the multiplication with the lowest unit? Which factor is it most convenient to use as a multiplier?

56. How do you find the excess of 9's in the product of two factors? If the excess of 9's in any factor is 0, what is the excess of 9's in the product?

Ex. 5. Ex. 4. Ex. 0. Ex. 0.

4. We also have $869 \times 49 \times 36 = 1532916$.

NOTE.—When the excess of 9's in any factor is 0, the excess of 9's in the product is always 0.

EXAMPLES.

(1.)	(2.)	(3.)	(4.)
847046	9807602	570409	216987
8	7	6	9

(5.)	(6.)	(7.)
103672	8163021	90031746
42	126	274

(8.)	(9.)	(10.)
\$14,168	\$894,126	\$20034,645
5	14	48

(11.)	(12.)	(13.)
47321809	1237506	437024
4261	3460	400

(14.)	(15.)	(16.)
8703600	107030	30671200
34600	5700	482

(17.)	(18.)	(19.)
£ s. d.	T. qr. lb. oz.	yd. ft. in.
20 6 8	3 3 21 14	16 2 9
4	8	7

(20.)	(21.)	(22.)
	hhd. gal. qt. pt.	E.F. qrs. na.
12° 42′ 55″	4 42 2 1	24 2 3
9	12	24

23. Multiply 18*tons* 2*qrs.* 16*lbs.* 9*oz.* by 48.

24. Multiply 5*yr.* 8*mo.* 2*wk.* 3*da.* 42*m.* by 56.

25. Multiply 68 by the factors 9 and 8 of the composite number 72.

26. Multiply 3657 by the factors of 64.
27. Multiply 37046 by the factors of 121.
28. Multiply 2187406 by the factors of 144.
29. Multiply 430714934 by 743.
30. Multiply 37157437 by 14972.
31. Multiply 47157149 by 37049.
32. Multiply 57104937 by 40709.
33. Multiply 79861207 by 890416.
34. Multiply 9084076 by 9908807.
35. Multiply 2748 by 200.
36. Multiply 67046 by 10: also by 100.
37. Multiply 57049 by 100: also by 1000.
38. Multiply 4980496 by 1000: also by 10000.
39. Multiply 90720400 by 100: also by 10000.
40. Multiply 74040900 by 1: also by 10.
41. Multiply 674936 by 100: also by 100000.
42. Multiply 478400 by 270400.
43. Multiply 367000 by 37409000.
44. Multiply 7849000 by 84694000.
45. Multiply 89999000 by 97770400.
46. Multiply 9187416300 by 274987650000.
47. Multiply 86543291213456 by 12637482965.
48. Multiply 76729835645873 by 217834569.
49. If it costs 2479 dollars to build one mile of plank road, how much would it cost to build 25 miles?
50. How far would a vessel sail in 9 days, of 24 hours each, at the rate of 15 miles an hour?
51. A man bought two farms, one of 125 acres, at 26 dollars an acre; another of 96 acres, at 32 dollars an acre; he paid at one time 2500 dollars; at another time 1725 dollars: what remained to be paid?
52. In 9 pieces of kersey, each containing 14*yd.* 3*qr.* 2*na.*, how many yards?*

---

* Note.—When the multiplicand is a compound denominate number, and the multiplier a composite number, it is best always to multiply by the factors of the composite number.

53. What will 15 gallons of wine cost at *5s.* $3\frac{1}{2}$*d.* per gallon?

54. What will be the value of 416 sheep at $2,48 a head?

55. Bought 40 barrels of flour at $8,75 a barrel, and sold them for $9,12$\frac{1}{2}$ a barrel: what was the whole gain?

56. What is the weight of 11 hogsheads of sugar, each weighing *7cwt. 2qr. 18lb.*, and what would be its value at 6 cents a pound?

57. A merchant bought 36 pieces of broadcloth, each piece containing 44 yards, at 4 dollars a yard: what did the whole cost?

58. A gentleman whose annual income is $3479, expends for pleasure and travelling $600; for books and clothing $570; for board and other expenses $1200: how much will he save in 5 years?

59. The number of milch cows in the state of New York in 1850 was 931324: what would be their value at 18 dollars each?

60. If a man travel *20mi. 5fur. 16rd.* in one day, how far will he travel in 24 days?

61. How long will it take a man to mow 14 acres of grass, allowing 10 working hours a day, if he mow one acre in *4hr. 45mi. 30sec.*?

62. If a man spend six cents a day for segars, how much will he spend in thirty years, allowing three hundred and sixty-five days to the year?

63. A farmer sold 118 bushels of barley for 62$\frac{1}{2}$ cents a bushel, and receives 5 barrels of flour at $9,87$\frac{1}{2}$ a barrel, and the remainder in cash: how much cash did he receive?

64. Two persons start at the same point and travel in opposite directions, one at the rate of 34 miles a day, the other at the rate of 28 miles a day: how far apart will they be at the end of 14 days?

65. An apothecary sold 8 bottles of laudanum, each containing 10 ℥ 6 ʒ 2 ℈ 14*gr.*: what was the weight of the whole?

66. A farmer took 7 loads of oats to market, each load

having 20 bags, and each bag containing *2bush. 3pk. 6qt.*: how many bushels of oats did he take to market?

67. If in a woollen factory 468 yards of cloth are made in one day, how many yards will be made in 313 days?

68. The greatest number of whales ever captured in the northern seas, in one season, was 2018. Estimating the oil produced from each to have been 212 barrels, what was the amount of oil produced?

69. What will be the value of an ox weighing *7cwt. 2qr. 16lb.*, at 11 cents a pound?

70. What will be the cost of 245 hogsheads of sugar, each weighing 984 pounds, at 7 cents a pound?

71. Bought 6 loads of hay, each weighing *18cwt. 3qrs. 21lb.*; after letting a neighbor have *2tons 15cwt. 1qr. 5lb.*, how much will there be left?

72. In an orchard there are 136 apple trees, each tree yielding 17 bushels of apples: how many bushels did the whole orchard yield, and what would they be worth at 42 cents a bushel?

73. A flour merchant bought 1845 barrels of flour at 7 dollars per barrel. He sold at one time 528 barrels, at 9 dollars a barrel; at another time 856 barrels at 8 dollars a barrel; how many barrels had he left, and at what cost?

74. What are 25 hogsheads of sugar worth, each weighing 872 pounds, at $6\frac{1}{2}$ cents a pound?

75. It is estimated that the whole amount of land appropriated by the General Government for educational purposes, to the 1st of January, 1854, was 52770231 acres. What would be the value of this land at the Government price of one dollar and twenty-five cents an acre?

76. If 30 men can do a piece of work in 25 days, how long will it take one man to do it?

77. A man desired that his property should be equally divided among his 5 children, giving each twenty-seven hundred dollars: what was the amount of his property?

78. Bought 9 chests of tea, each containing 72 pounds, at $37\frac{1}{2}$ cents a pound: what was the cost of the whole?

79. A farm consisting of 127 acres, was sold at auction for $37,565 an acre: what sum of money did it bring?

80. A drover bought 127 head of beef cattle at an average of 39 dollars per head; he sold 86 of them for 43 dollars per head; for how much per head must he sell the remainder, to clear on the first cost 1246 dollars?

81. What will 75 firkins of butter cost, each firkin weighing 56 pounds, at 16 cents a pound?

82. A merchant bought a box of goods containing 37 pieces, each piece containing 46 yards, worth 7 dollars a yard: what did the box of goods cost?

83. A bond was given April 20th, 1850, and was paid Sept. 4th, 1856: what will be the product, if the time which elapsed from the date of the bond to the time it was paid be multiplied by 45?

84. What distance will a wheel 16 feet 8 inches in circumference measure on the ground, if rolled over 84 times?

85. What is the difference between twice eight and fifty, and twice fifty-eight?

86. How much wood in 4 piles, each containing 5 cords, 6 cord feet and 32 cubic feet?

87. A man bought 56 acres of land for $25 an acre, and 94 acres for $32 an acre; if he sells the whole at $30 an acre, will he gain or lose, and how much?

88. If 12 men can build a wall in 16 days, how many men will be required to build a wall nine times as long in half the time?

89. A farmer sold 4 cows for $25,50 each; 12 sheep for $2,12½ each; and 3 calves for $7,25 each; what was the amount of the sale?

90. If it requires 116 tons of iron to construct one mile of railroad, how much would it require to construct a railroad from Albany to Buffalo, it being 326 miles?

91. A merchant bought 960 pounds of cheese at 9 cents a pound; 148 pounds of butter at 12½ cents a pound. He gave in payment, 12 yards of cloth, at $4,75 a yard; 186 pounds of

sugar at 7 cents a pound, and the remainder in cash: how much cash did he pay?

92. If a family consume 12*gal.* 2*qt.* 1*pt.* of ale in a week, how much will they consume in 14 weeks?

93. How much brandy will supply an army of 25,000 men for one month, if each man requires 1*gal.* 2*qt.* 1*pt.* 2*gi.*

94. It is estimated that the French, during the years 1854 and 1855, transported to the Crimea 80000 horses, and that 70000 of them were lost in the same time. Supposing the first cost of each horse to be $100, and the cost of transportation $95 per head, what was the value of the horses lost?

95. A man purchased a piece of woodland containing 27 acres, at 39 dollars per acre; each acre produced on an average 70 cords of wood, which, being sold, yielded a nett profit of 45 cents a cord: how much did the profit on the wood fall short of paying for the land?

BILLS OF PARCELS.

96. CHICAGO, *June* 10, 1856.

Mr. John C. Smith, *Bought of David Toombs.*

14 pounds of tea,	at 75 cents,	- -	$
9 " " coffee,	14 "	- - -	
42 " " sugar,	11 "	- - -	
3 " " pepper,	12½ "	- - -	
5 " " chocolate,	56 "	- - -	
12 " " candles,	16 "	- - -	
Received payment,			$

DAVID TOOMBS.

97 NEW YORK, *March 20th*, 1857.

Mr. Jacob Johns, *Bought of George Bliss & Co*

48 pounds of sugar at 9½ cents a pound, - - -	$
6 hogs. of molasses, each containing 63 gallons, at 27 cents a gallon, - - - - - - - - -	
8 casks of rice, 285 lbs. each, at 5 cts. a pound,	
9 chests of tea, 86 lbs. each, at 87½ cts. a pound,	
4 bags of coffee, each 67 lbs., at 11 cts. a pound,	
Received payment,	$

 GEO. BLISS & Co.

98. HARTFORD, *November 21st*, 1856.

Gideon Jones, *Bought of Jacob Thrifty.*

78 chests of tea, at $55,65 per chest,	- -	$
251 bags of coffee, 100 pounds each, at 12½ cts. per pound, - -	- - -	
317 boxes of raisins, at $2,75 per box,	- -	
1049 barrels of shad, at $7,50 per barrel,	- - -	
76 barrels of oil, 32 gallons each, at $1,08 per gal.,		
	Amount,	$

Received the above in full, JACOB THRIFTY.

99. BALTIMORE, *Jan. 1st*, 1855.

Mr. Abel Wirt, *Bought of Timothy Stout.*

10 yards of broadcloth, at $4,37½,	- - -	$
75 " " sheeting, " ,09	- - -	
42 " " plaid prints " ,45	- - -	
5 barrels of Genesee flour, at $7,87½,	- -	
7 pairs of boots, at $1,60 per pair,	- -	
18 bushels of corn, at 72 cts. per bushel,	-	
		$

100. MONTREAL, *Oct. 16th*, 1855.

Mr. Chas. Snow, *Bought of Vose, Duncan & Co.*

45 yards of broadcloth	at 9*s.* 6*d.*	- -	£ *s.* *d.*
56 " "	" 12*s.* 9¼*d.*	- - -	
16 " vestings,	" 6*s.* 8½*d.*	- - -	
24 lbs. colored thread,	" 5*s.* 4*d.*	- - -	
72 pairs silk hose,	" 7*s.* 5¾*d.*	- - -	
108 yards carpeting,	" 14*s.* 10*d.*	- - -	
Received payment,			£

VOSE, DUNCAN & CO.

DIVISION.

57. DIVISION *is the operation of finding from two numbers a third, which multiplied by the first, will produce the second.*

The first number, or number by which we divide, is called the *divisor.*

The second number, or number to be divided, is called the *dividend.*

The third number, or result, is called the *quotient.*

The quotient shows how many times the dividend contains the divisor.

When the quotient is expressed by an integral number, the division is said to be exact. When it cannot be so expressed, the part of the dividend that is undivided, is called the *remainder.*

58. There are always three parts in every division, and sometimes four: 1st. the dividend; 2d. the divisor; 3d. the quotient; and 4th. the remainder.

There are three signs used to denote division; they are the following:

$12 \div 3$ expresses that 12 is to be divided by 3.

$\frac{12}{3}$ expresses that 12 is to be divided by 3.

3)12 expresses that 12 is to be divided by 3.

When the last sign is used, if the divisor does not exceed 12, we draw a line beneath the dividend and set the quotient under it. If the divisor exceeds 12, we draw a curved line on the right of the dividend, and set the quotient at the right.

59. SHORT DIVISION is the operation of dividing when the work is performed mentally, and the results only written down. It is limited to the cases in which the divisors do not exceed 12.

---

57. What is division? What is the number to be divided called? What is the number called by which we divide? What is the answer called? What is the number called which is left?

58. How many parts are there in division? Name them. How many signs are there in division? Make and name them.

59. What is short division? How is it generally performed? Where is the quotient written? To what cases is it limited?

1. Divide 456 by 4.

ANALYSIS.—The number 456 is made up of 4 hundreds, 5 tens, and 6 units, each of which is to be divided by 4. Dividing 4 hundreds by 4, we have the quotient, 1 hundred: 5 tens divided by 4, gives 1 ten and 1 ten over: reducing this to units and adding in the 6, we have 16 units, which contains 4, 4 times: hence, the quotient is 114: that is, the dividend contains the divisor 114 times.

OPERATION.

```
4)456
  114
```

2. Divide £11 8*s.* 7*d.* 3*far.* by 5.

ANALYSIS.—Dividing £11 by 5, the quotient is £2 and £1 remainder. Reducing this to shillings and adding in the 8, we have 28*s.*, which divided by 5, gives 5*s.* and 3*s.* over. This being reduced to pence and 7*d.* added, gives 43*d.* Dividing by 5, we have 8*d.* and 3*d.* remainder. Reducing 3*d.* to farthings, adding 3 farthings, and again dividing by 5, gives the last quotient figure 3*far.*

OPERATION.

```
   £   s.  d. far.
5)11   8   7   3
   2   5   8   3
```

3. Divide £6 8*s.* 8*d.* by 8.

Here we have to pass to shillings before making the first division.

OPERATION.

```
  £  s.  d.
8)6  8   8
    16   1
```

4. Divide 11772 by 327.

ANALYSIS.—Having set down the divisor on the left of the dividend, it is seen that 327 is not contained in the first three figures on the left, which are 117 hundreds. But by observing that 3 is contained in 11, 3 times and something over, we conclude that the divisor is contained at least 3 times in the first *four* figures, 1177 tens, which is a *partial dividend.* Set down the quotient figure 3, and multiply the divisor by it: we thus get 981 tens, which being less than 1177, the quotient figure is not too great: we subtract the 981 tens from the first four figures of the dividend, and find a remainder 196 tens, which being less than the divisor, the quotient figure is not too small. Reduce this remainder to units and add in the 2, and we have 1962.

OPERATION.

```
327)11772(36
    981
    1962
    1962
```

As 3 is contained in 19, 6 times, we conclude that the divisor is contained in 1962 as many as 6 times. Setting down 6 in the quo-

tient and multiplying the divisor by it, we find the product to be 1962. Therefore the entire quotient is 36, or the divisor is contained 36 times in the dividend.

60. From the above analysis, we have the following rule for the division of numbers.

I. *Begin with the highest order of units of the dividend, and pass on to the lower orders until the fewest number of figures be found that will contain the divisor: divide these figures by it for the first figure of the quotient: the unit of this figure will be the same as that of the lowest order in the partial dividend.*

II. *Multiply the divisor by the quotient figure so found, and subtract the product from the partial dividend.*

III. *Reduce the remainder to units of the next lower order, and add in the units of that order found in the dividend: this gives a new partial dividend. Proceed in a similar manner until units of every order shall have been divided.*

DIRECTIONS FOR THE OPERATIONS.

NOTES.—There are five operations in Long Division. 1st. To write down the numbers: 2d. To divide, or find how many times: 3d. To multiply: 4th. To subtract: 5th. To bring down, to form the partial dividends.

2. The product of a quotient figure by the divisor must never be larger than the corresponding partial dividend: if it is, the quotient figure is too large and must be diminished.

3. When any one of the remainders is greater than the divisor, the quotient figure is too small and must be increased.

4. The unit of any quotient figure is the same as that of the partial dividend from which it is obtained. The pupil should always name the unit of every quotient figure.

---

60. Give the rule for the division of numbers.

NOTES.—1. How many operations are there in division? Name them.

2. If a partial product is greater than the partial dividend, what does it indicate? What then do you do?

3. What do you do when any one of the remainders is greater than the divisor?

4. What is the unit of any figure of the quotient? When the divisor is contained in simple units, what will be the unit of the quotient figure?

5. If any partial dividend is less than the divisor, the corresponding quotient figure is 0.

6. When there is a remainder, after division, write it at the right of the quotient, and place the divisor under it.

PRINCIPLES RESULTING FROM DIVISION.

1. When the divisor is equal to the dividend, the quotient will be 1.

2. When the divisor is less than the dividend, the quotient will be greater than 1. The quotient will be as many times greater than 1, as the dividend is times greater than the divisor.

3. When the divisor is greater than the dividend, the quotient will be less than 1. The quotient will be such a part of 1, as the dividend is of the divisor.

4. When the divisor is 1, the quotient will be equal to the dividend.

PROOF.

61. There are two methods of proof for division: 1st. By multiplication; 2d. By the excess of 9's.

FIRST METHOD.

By the definition of division, the quotient is such a number as multiplied by the divisor will produce the dividend (Art. 57).

In example 4, each product of the divisor by a figure of the quotient is a *partial product*, and the sum of these products is the product of the divisor and quotient (page 57, Note). Each product is taken,

---

When it is contained in tens, what will be the unit of the quotient figure? When it is contained in hundreds? In thousands?

5. If any partial dividend is less than the divisor what is the corresponding figure of the quotient?

6. When there is a remainder after division, what do you do with it?

NOTE.—1. When the divisor is equal to the dividend, what will the quotient be?

2. When the divisor is less than the dividend, how will the quotient compare with 1? How many times will it be greater than 1?

3. When the divisor is greater than the dividend, how will the quotient compare with 1? What part will the quotient be of 1?

4. When the divisor is 1, what will the dividend be?

61. How many methods of proof are there for division? What are they? What is the proof by multiplication? What is the proof by the 9's?

separately, from the dividend, and nothing remains. But, taking each product away, in succession, leaves the same remainder as would be left if their sum were taken away at once. Hence, the number 36, when multiplied by the divisor 327, gives a product equal to the dividend 11772; therefore, 36 is the quotient (Art. 57): hence, to prove division,

OPERATION

```
327)11772(36
    1981
    ----
    1962
    1962
    ----
```

Multiply the divisor by the quotient and add in the remainder, if any. If the work is right, the result will be the same as the dividend.

NOTE.—Divide 325 by 19. The quotient is 17, and 16 remainder: the *true quotient* is $17\frac{16}{19}$; for, this being multiplied by the divisor 19, will give the dividend. If the pupil knew how to dispose of the fractional part, we should simply say, "*Multiply the divisor by the quotient*," which is exactly what we do under the rule.

PROOF BY 9's.

Since the dividend is equal to the product of the divisor and quotient, it follows that if the excess of 9's in the divisor be multiplied by the excess of the 9's in the quotient, the excess of 9's in the product will be equal to the excess of 9's in the dividend (Art. 56). Hence,

Find the excess of 9's in the divisor and in the quotient: multiply them together, and note the excess of 9's in the product: if this is equal to the excess of 9's in the dividend, the work may be regarded as right.

$$\begin{array}{lll} \text{Divisor,} & 327, \text{ excess of 9's - - } & 3 \\ \text{Quotient,} & 36, \text{ " - - - } & 0 \\ \text{Dividend,} & 11772, \text{ " - - - } & 0 \end{array} \left.\begin{array}{l} \\ \\ \end{array}\right\} \text{Product, 0.}$$

EXAMPLES.

(1.)	(2.)	(3.)	(4.)
3)19737	4)147368	5)1346840	6)1650930

(5.)	(6.)	(7.)
6)47689872	9)10324683	7)506321494

(8.)	(9.)	(10.)
£ *s.* *d.*	*A.* *R.* *P.*	*yd.* *qr.* *na.*
3)47 19 3	9)37 3 17	5)47 3 1
15 " 19 " 9	4 " 0 " 33	

(11.)	(12.)	(13.)
$ *cts.* *m.*	$ *cts.* *m.*	$ *cts.*
8)634 75 6	7)1468 0 96	12)802346 16

14. Divide 734947644 by 48.
15. Divide 8536752 by 36.
16. Divide 3367598 by 19.
17. Divide 49300 by 725.
18. Divide 6477150 by 145.
19. Divide 770 by 28.
20. Divide $87,256 by 5.
21. Divide $495,704 by 129.
22. Divide $29,25 by 26.
23. Divide $10,125 by 27.
24. Divide $347,49 by 429.
25. Divide $751,50 by 150.
26. Divide $5711,04 by 108.
27. Divide $315 by $35.
28. Divide $50065 by $527.
29. Divide $432 by 54.
30. Divide 334422198 by 438.
31. Divide 714394756 by 1754.
32. Divide 47159407184 by 3574.
33. Divide 5719487194715 by 45705.
34. Divide 4715714937149387 by 17493.
35. Divide 671493471549375 by 47143.
36. Divide 571943007145 by 37149.
37. Divide 1714347149347 by 57143.
38. Divide 49371547149375 by 374567.
39. Divide 171493715947143 by 571007.
40. Divide 6754371495671594 by 678957.
41. Divide 7149371478 by 121.
42. Divide 71900715708 by 57149.
43. Divide 14714937148475 by 123456.
44. Divide 729*A.* 2*R.* 7*P.* by 41.
45. Divide 365*da.* 6*hr.* by 240.
46. Divide 1298*mi.* 2*fur.* 33*rd.* by 37.
47. Divide 95*hhd.* 6*gal.* by 120.
48. Divide 232*bush.* 3*pk.* 7*qt.* by 105.
49. Divide $18306,25 by 725.
50. **Bought 7 yds. of cloth for 16*s.* 4*d.*: what did it cost per yd.?**

51. A man travelled 265*mi.* 6*fur.* 16*rd.* in 12 days: how far did he travel in one day?

52. If 569*A.* 2*R.* 23*P.* be equally divided between 9 persons, how much will 5 of them have?

53. The annual income of a gentleman is $10000: how much is that per day, counting 365 days to the year?

54. What number multiplied by 9999 will produce 987551235?

55. A gentleman owning an estate of $75000, gave one-fourth of it to his wife, and the remainder was divided equally among his five children: how much did each receive?

56. The expenditure of the United States for 1853 was $54026818: how much would that be per day, allowing 365 days to the year?

57. If 28 yards of cloth cost $133, what will one yard cost?

58. If I pay $637,50 for 51 yards of cloth, what is the price per yard?

59. The city of New York, in 1850, had 104 periodical publications, with an aggregate circulation of 78747600 copies: what would be the average circulation of each?

60. Bought 19 bushels of wheat for $30,875: what was the cost of one bushel?

61. How long will 9125 loaves of bread last 5 families, if each family consume 5 loaves a day?

62. The product of two numbers is 7207272072, and the multiplier 9009: what is the multiplicand?

63. How many rings, each weighing 4*dwt.* 12*gr.*, can be made from 10*oz.* 11*dwt.* 12*gr.* of gold?

64. If iron is worth 2 cents a pound, how much can be bought for $67,50?

65. If 14 sticks of hewn timber measure 12*T.* 38*ft.* 118*in.*, how much does each stick contain?

66. In 1850, Pennsylvania manufactured 285702 tons of pig iron, and employed 9285 hands: what was the average product of each hand?

67. The number of college libraries in the United States in 1850, was 213, containing 942321 volumes: what would be the average *number of volumes* in each?

CONTRACTIONS AND APPLICATIONS.

CONTRACTIONS IN MULTIPLICATION.

62. Contractions in Multiplication are short methods of finding products when the multipliers are particular numbers.

63. *To multiply by 25.*

1. Multiply 356 by 25.

ANALYSIS.—If we annex two ciphers to the multiplicand, we multiply it by 100 (Art. 55): this product is 4 times too great; for the multiplier is but *one-fourth* of 100; hence, to multiply by 25,

OPERATION.

$$4\overline{)35600}$$
$$8900$$

Annex two ciphers to the multiplicand and divide the result by 4.

EXAMPLES.

2. Multiply 287 by 25.
3. Multiply 184 by 25.
4. Multiply 6741 by 25.
5. Multiply 3074 by 25.

64. *When the multiplier contains a fraction.*

What is the product of 15 multiplied by $3\frac{1}{5}$?

ANALYSIS.—The multiplicand is to be taken 3 and one-fifth times: taking *it one-fifth times,* gives 3, which we write in the units place: then, taking it 3 times, gives 45, and the sum 48 is the product; hence

OPERATION

$$\begin{array}{r} 15 \\ \underline{3\frac{1}{5}} \\ 3 \\ \underline{45} \\ 48 \text{ Ans.} \end{array}$$

RULE.—*Take such a part of the multiplicand as the fraction is of* 1; *then multiply by the integral number, and the sum of the products will be the required product.*

2. Multiply 327 by $8\frac{1}{3}$.
3. Multiply 23474 by $16\frac{1}{2}$.
4. Multiply 34700 by $127\frac{1}{5}$.
5. Multiply 1272 by $12\frac{1}{8}$.
6. Multiply 9824 by $272\frac{1}{4}$.
7. Multiply 3828 by $73\frac{1}{6}$.

---

62. What are contractions in multiplication?
63. How do you multiply by 25?
64. How do you multiply when the multiplier contains a fraction?

65. *To multiply by* $12\frac{1}{2}$.

1. Multiply 286 by $12\frac{1}{2}$.

ANALYSIS.—Since $12\frac{1}{2}$ is *one-eighth* of 100, *Annex two ciphers to the multiplicand and divide the result by* 8.

OPERATION.
$$8)\underline{28600}$$
$$3575$$

EXAMPLES.

1. Multiply 384 by $12\frac{1}{2}$.
2. Multiply 476 by $12\frac{1}{2}$.
3. Multiply 14800 by $12\frac{1}{2}$.
4. Multiply 670418 by $12\frac{1}{2}$.

66. *To multiply by* $33\frac{1}{3}$.

1. Multiply 975 by $33\frac{1}{3}$.

ANALYSIS.—Annexing two ciphers to the multiplicand, multiplies it by 100: but the multiplier is *one-third* of 100: hence,

Annex two ciphers and divide the result by 3.

OPERATION.
$$3)\underline{97500}$$
$$32500$$

EXAMPLES.

1. Multiply 1679252 by $33\frac{1}{3}$.
2. Multiply 1480724 by $33\frac{1}{3}$.
3. Multiply 10675512 by $33\frac{1}{3}$.
4. Multiply 4442172 by $33\frac{1}{3}$.

67. *To multiply by* 125.

1. Multiply 1125 by 125.

ANALYSIS.—Annexing three ciphers to the multiplicand, multiplies it by 1000: but 125 is but *one-eighth* of one thousand: hence,

Annex three ciphers and divide the result by 8.

OPERATION.
$$8)\underline{1125000}$$
$$140625$$

EXAMPLES.

1. Multiply 59264 by 125.
2. Multiply 17593408 by 125.
3. Multiply 1940812 by 125.
4. Multiply 140588 by 125.

CONTRACTIONS IN DIVISION.

68. Contractions in Division are short methods of finding the quotient, when the divisors are particular numbers.

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65. How do you multiply by $12\frac{1}{2}$?
66. How do you multiply by $33\frac{1}{3}$?
67. *How do you multiply* by 125?
68. *What are Contractions* in Division?

69. By reversing the last four processes, we have the four following rules:

1. To divide any number by 25:
Multiply the number by 4, *and divide the product by* 100.

2. To divide any number by $12\frac{1}{2}$:
Multiply the number by 8, *and divide the product by* 100.

3. To divide any number by $33\frac{1}{3}$:
Multiply the number by 3, *and divide the product by* 100.

4. To divide any number by 125:
Multiply by 8, *and divide the product by* 1000.

EXAMPLES.

1. Divide 6350 by 25.	9. Divide 1760600 by $12\frac{1}{2}$.
2. Divide 21345 by 25.	10. Divide 67500 by $33\frac{1}{3}$.
3. Divide 656280 by 25.	11. Divide 1308400 by $33\frac{1}{3}$.
4. Divide 7278675 by 25.	12. Divide 15851400 by $33\frac{1}{3}$.
5. Divide 5287215 by 25.	13. Divide 8072400 by $33\frac{1}{3}$.
6. Divide 12225 by $12\frac{1}{2}$.	14. Divide 281250 by 125.
7. Divide 10650 by $12\frac{1}{2}$.	15. Divide 6015750 by 125.
8. Divide 11925 by $12\frac{1}{2}$.	16. Divide 2026875 by 125.

70. *When the divisor is a composite number.*

1. How many feet and yards are there in 288 inches?

ANALYSIS.—Since there are 12 inches in 1 foot, there will be as many feet in 288 inches as 12 is contained times in 288; viz., 24 feet, *in which the unit is* 1 *foot.* Since 3 feet make 1 yard, there will be as many yards in 24 feet as 3 is contained times in 24; viz., 8 yards: *in which the unit is* 1 *yard.* We have thus passed, by division, from the unit 1 inch to the unit 1 foot, and then to the unit 1 yard; that is, in each

OPERATION.

```
12)288
  3)24
     8
```

---

69. What rules do you get by reversing the four previous rules? Give them.

70. What is a composite number? Under how many points of view may division be regarded? What are they? What is the rule for division when the divisor is a composite number? When there are remainders after division, how do you find the remainder in units of the dividend?

operation, we have *increased the unit as many times as there are units in the divisor.*

Let us now use the same numbers, in an entirely different question.

2. If 288 dollars be equally divided among 36 men, what will be the share of each?

ANALYSIS.—Since 288 dollars is to be equally divided among 36 men, each will have as many dollars as 36 is contained times in 288. Dividing 288 into 12 equal parts, we find that each part is 24 dollars. If *each of these parts* be now divided into 3 equal parts, there will then be 36 parts in all, each equal to 8 dollars: here, *the unit of the result is the same as that of the dividend.* Hence, we may regard division under two points of view:

$36 = 12 \times 3$

OPERATION.

$$\begin{array}{r} 12)\underline{288} \\ 3)\underline{24} \\ 8 \end{array}$$

1. *As a process of reduction, in which the unit of each succeeding dividend is increased as many times as there are units in the divisor:*

2. *As a process of separating a number into equal parts; in which case the unit of a part will be the same as that of the dividend.*

Hence, the following rule when the divisor is a composite number:

RULE.—*Divide the dividend by one of the factors of the divisor; then divide the quotient, thus arising, by a second factor, and so on, till every factor has been used as a divisor: the last quotient will be the answer.*

EXAMPLES.

Divide the following numbers by the factors:

1. 2322 by $6 = 2 \times 3$.	5. 1145592 by $72 = 8 \times 9$.
2. 37152 by $24 = 4 \times 6$.	6. 185760 by $96 = 8 \times 12$.
3. 19152 by $36 = 6 \times 6$.	7. 115776 by $64 = 8 \times 8$.
4. 38592 by $48 = 4 \times 12$.	8. 463104 by $144 = 12 \times 12$.

NOTE. When there are remainders, after division, the operation is to be treated as one of Reduction.

9. Divide the number 3671 by $30 = 2 \times 3 \times 5$.

ANALYSIS.—Dividing 3671 by 2, we have a quotient 1835, and a remainder, 1. After the second division, we have a quotient 611, and a remainder, 2; and after the third division, the quotient 122, and the remainder, 1. Now, it is plain, from the first analysis, that,

OPERATION.

```
2)3671
3)1835  . . 1.
 5)611  . . 2.
   122  . . 1.
```

$1 \times 3 + 2 = 3 + 2 = 5$;

$5 \times 2 + 1 = 10 + 1 = 11$ rem.

Ans. $122\frac{11}{30}$.

1. The unit of the first quotient is as many times greater than the unit of the dividend, as the divisor is times greater than 1; and similarly for all the following quotients.

2. The unit of the first remainder is the same as the unit of the dividend; and the unit of any remainder is the same as that of the corresponding dividend.

3. The unit of any dividend is reduced to that of the preceding dividend, by multiplying it by the preceding divisor.

Hence, to find the remainder in *units of the given dividend*, is simply a case of reduction in which the divisors denote the units of the scale: therefore,

I. *Multiply the last remainder by the last divisor but one, and add in the preceding remainder.*

II. *Multiply this result by the next preceding divisor, and add in the remainder, and so on, till you reach the unit of the dividend.*

Divide the following numbers by the factors, and find the remainders:

1. 416705 by $315 = 7 \times 9 \times 5$.
2. 804106 by $462 = 3 \times 2 \times 7 \times 11$.
3. 756807 by $3456 = 4 \times 8 \times 9 \times 12$.
4. 8741659 by $105 = 3 \times 5 \times 7$.
5. 947043 by $385 = 5 \times 7 \times 11$.
6. 4704967 by $1155 = 11 \times 7 \times 5 \times 3$.
7. 71874607 by $7560 = 8 \times 7 \times 9 \times 5 \times 3$.

71. *When the divisor is* 10, 100, 1000, *&c.*

1. Divide 3278 by 1000 = 10 × 10 × 10.

ANALYSIS.—We divide 3278 by 10, by simply cutting off 8, giving 327 tens and 8 units remainder. We again divide by 10, by cutting off the 7, giving 32 hundreds and 7 tens remainder. We again divide by 10 by cutting off the 2, giving a quotient of 3 thousands and 2 hundreds remainder. The quotient then is 3, and a remainder of 2 hundreds 7 tens and 8 units, or 278.

OPERATION.

```
10)327|8
10)32|7  .  8 rem.
10)3|2 . .  7 rem.
   3   . .  2 rem
```

$3\frac{278}{1000}$ *Ans.*

RULE.—*Cut off from the right of the dividend as many figures as there are ciphers in the divisor, considering the figures at the left the quotient, and those at the right the remainder.*

72. *When any divisor contains significant figures with one or more ciphers at the right hand.*

2. Divide 875896 by 32000.

ANALYSIS.—The divisor 32000 = 32 × 1000. Dividing by 1000 gives a quotient 875, and 896 remainder. Then dividing by 32 gives a quotient 27, and 11 remainder, which gives the result $27\frac{11896}{32000}$. Hence,

OPERATION.

```
32|000)875|896(27
       64
       235
       224
        11896 rem.
```

Ans. $27\frac{11896}{32000}$.

RULE.—*Cut off, by a line, the ciphers from the right of the divisor, and an equal number of figures from the right of the dividend: divide the remaining figures of the dividend by the remaining figures of the divisor, and the remainder, if any, with the figures cut off from the dividend annexed, will form the true remainder.*

EXAMPLES.

Divide the following numbers:

1. 1972654 by 420000.	4. 11428729800 by 72000.
2. 1752000 by 12000.	5. 36981400 by 146000.
3. 73199006 by 801400.	6. 141614398 by 63000.

71. How do you divide when the divisor is 1, with ciphers annexed?

72. How do you divide when the divisor contains significant figures, with ciphers annexed? How do you find the true remainder?

73. *When the divisor contains a fraction.*

1. Divide 856 by $4\frac{1}{5}$

	OPERATION.
$21=3\times7$	3)4280
	7)1426 − 2
	203 − 5
	Ans. $203\frac{11}{21}$.

ANALYSIS.—There are 5 fifths in 1; hence, in 4 there are 20 fifths; therefore, $4\frac{1}{5}=21$ fifths. In the dividend 856, there are 5 times as many fifths as units 1; that is, 4280 fifths; therefore, the quotient is 4280 divided by 21, equal $203\frac{11}{21}$. Hence, when the divisor contains a fractional part,

Reduce the divisor and dividend to the fractional part in the divisor, and then divide as in integral numbers.

Find the quotients in the following examples:

1. $3245 \div 16\frac{1}{2}$.	5. $87317 \div 9\frac{3}{5}$.
2. $47804 \div 15\frac{1}{3}$.	6. $87906 \div 12\frac{4}{7}$.
3. $870631 \div 14\frac{1}{4}$.	7. $95675 \div 15\frac{5}{9}$.
4. $37214 \div 51\frac{1}{8}$.	8. $71096 \div 17\frac{3}{7}$.

APPLICATIONS IN MULTIPLICATION.

74. The analysis of a practical question, in Multiplication, requires that the multiplier be an abstract number; and then the unit of the product will be *the same as the unit of the multiplicand.*

75. *To find the cost of several things, when we know the cost of unity and the number of things:*

1. What will six yards of cloth cost at 8 dollars a yard?

ANALYSIS.—Six yards of cloth will cost 6 times as much as 1 yard. Since 1 yard of cloth cost 8 dollars, 6 yards will cost 6 times 8 dollars, which are 48 dollars; therefore, 6 yards of cloth at 8 dollars a yard, will cost 48 dollars: hence,

The cost of any number of things is equal to the price of a single thing multiplied by the number of things.

But we have seen that the product of two numbers will be the same, (that is, will contain the same number of units) which-

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73. How do you divide when the divisor contains a fraction?

74. What does the analysis of a practical question require?

75. How do you find the cost of a single thing? How is it done in *practice?*

ever be taken for the multiplicand (Art. 50). Hence, in practice, we may multiply the two factors together, taking either for the multiplier, and *then assign the proper unit to the product.* We generally take the *less* number for the multiplier.

76. *To find the cost when the price is an aliquot part of a dollar:*

1. Find the cost of 45 bushels of apples, at 25 cents a bushel.

OPERATION.

$$\begin{array}{r} 4)45,00 \\ \hline \$11,25 \end{array}$$

ANALYSIS.—If the price were 1 dollar a bushel, the cost would be as many dollars as there are bushels. But the price is 25 cents = $\frac{1}{4}$ of a dollar; hence, the cost will be one-fourth as many dollars as there are bushels; that is, as many dollars as 4 is contained times in 45 = 11 dollars and 1 dollar over. This is reduced to cents by adding two ciphers; then dividing again by 4, we have the entire cost: hence,

Take such a part of the number which denotes the commodity, as the price is of 1 *dollar.*

EXAMPLES.

1. What would be the cost of 284 bushels of potatoes, at 50 cents a bushel?

2. At $33\frac{1}{3}$ cents a gallon, what will 51 gallons of molasses cost?

3. What cost 112 yards of calico, at $12\frac{1}{2}$ cents a yard?

4. If a pound of butter cost 20 cts., what will 175 pounds cost?

5. What will 576 bushels of wheat cost, at $1,50 a bushel?

2)$576	cost at	1 dollar	a bushel.
288	"	50 cents	"
$864	"	$1,50	"

6. What will it cost to dig a ditch 129 rods long, at $\$1,33\frac{1}{3}$ a rod?

7. At $1,25 a barrel, what will 96 barrels of apples cost?

8. What will 3 pieces of cloth cost, each piece containing 25 yards, at $1,20 a yard?

77. *To find the cost of articles sold by the* 100 *or* 1000.

1. What will 544 feet of lumber cost at 2 dollars per 100?

---

76. How do you find the cost when the price is an aliquot part of a dollar?

77. How do you find the cost of articles sold by the hundred or thousand?

ANALYSIS.—At 2 dollars a foot, the cost would be 544 × 2 = 1088 dollars; but as 2 dollars is the price of 100 feet, it follows that 1088 dollars is 100 times the cost of the lumber; therefore, if we divide 1088 dollars by 100 (which is done by cutting off two of the right hand figures, Art. 71), we obtain the cost.

NOTE.—Had the price been so much per 1000, we should have divided by 1000: hence,

Multiply the quantity by the number denoting the price; if the price be by the 100, *cut off* two *figures on the right hand of the product; if by the* 1000, *cut off* three, *and the remaining figures will be the answer in the same denomination as the price, which, if cents or mills, may be reduced to dollars.*

EXAMPLES.

1. What will be the cost of 3742 feet of timber at $3,25 per 100?

2. At $12,50 per 1000, what will 5400 feet of boards cost?

3. *Richard Ames,* *Bought of John Maple.*

1275	feet	of boards	at	$9,00	per 1000
3720	"	"	"	15,25	"
715	"	scantling	"	8,75	"
1200	"	timber	"	12,06	"
2550	"	lathing	"	75	100
965	"	plank	"	1,12½	"

Received payment, *John Maple.*

76. *To find the cost of articles sold by the ton.*

What is the cost of 640 pounds of hay at $11,50 per ton?

ANALYSIS.—Since there are 2000 pounds in a ton, the cost of 1000 pounds will be half as much as of 1 ton: viz., $5,75. Multiply this by the number of pounds (640), and cut off three places from the right (Art. 71), in addition to the two places cut off for cents; hence,

OPERATION.

```
2)$11,50
   5,75  price of 1000 lbs.
    640
  23000
  3450
$3,68000
```

76. How do you find the cost of articles sold by the ton?

Multiply one-half the price of a ton by the number of pounds, and cut off three figures from the right hand of the product. The remaining figures will be the answer in the same denomination as the price of a ton.

EXAMPLES.

1. What will be the cost of 1575 pounds of plaster at $3,84 per ton?

2. At $7,37½ a ton, what will 3496 pounds of coal cost?

3. What will 1260 pounds of hay cost at $9,40 per ton? at $10,25? at $14,60?

4. What will be the cost of transportation of 5482 pounds of iron from Pittsburgh to New York at $6,65 per ton?

APPLICATIONS IN DIVISION.

77. Abstractly, the object of division is to find from two given numbers a third, which, multiplied by the first, will produce the second. Practically, it has three objects:

1. Given the number of things and their cost, to find the price of unity.

2. Given the cost of a number of things and the price of unity, to find the number of things.

3. To divide any number of things into a given number of equal parts.

ANALYSIS.—Consider the number denoting cost or price as abstract; then make the division and assign the proper unit to the quotient. Hence, we have the following

RULES.

I. *Divide the number denoting the cost by the number of things: the quotient will be the price of unity.*

II. *Divide the number denoting the cost by the price of unity: the quotient will be the number of things.*

III. *Divide the whole number of things by the number of parts into which they are to be divided: the quotient will be the number in each part.*

---

77. What is the object of Division abstractly? How many objects has it practically? Name the objects. Give the rules for the three cases.

LONGITUDE AND TIME.

78. The equator of the earth, like that of other circles, is divided into 360°, which are called *degrees of longitude.*

79. The sun apparently goes round the earth once in 24 hours. This time is called a *day.*

Hence, in 24 hours, the sun apparently passes over 360° of longitude; and in 1 hour over $360° \div 24 = 15°$.

80. Since the sun, in passing over 15° of longitude, requires 1 hour or 60 minutes of time, 1° will require 60 minutes $\div 15 = 4$ minutes of time; and 1′ of longitude will require one-sixtieth of 4 minutes, which is 4 seconds of time: hence,

15° of longitude require 1 hour.
1° of longitude requires 4 minutes.
1′ of longitude requires 4 seconds.

Hence, we see that,

1. *If the degrees of longitude be multiplied by 4, the product will be the corresponding time in minutes.*

2. *If the minutes in longitude be multiplied by 4, the product will be the corresponding time in seconds.*

1. What is the time in hours, minutes and seconds in 56° 47′?

ANALYSIS.—First reduce the degrees to hours and minutes; then reduce the minutes to minutes and seconds, and take the sum.

OPERATION.

	m.	hr.	m.	sec.
$56° \times 4 =$	224	= 3	44	
$47' \times 4 =$	188 sec. =		3	8
		3	47	8

81. When the sun is on the meridian of any place, it is 12 o'clock, or noon, at that place.

---

78. How is the equator of the earth supposed to be divided?

79. How does the sun appear to move? What is a day? How far does the sun appear to move in 1 hour?

80. How do you reduce degrees of longitude to time? How do you reduce minutes of longitude to time?

81. What is the hour when the sun is on the meridian? When the sun is on the meridian of any place, how will the time be for all places east? How for all places west? If you have the difference of time, how do you find the time?

Now, as the sun apparently goes from east to west, at the instant of noon, at one place, it will be *past* noon for all places at the east, and *before* noon for all places at the west.

If then, we find the difference of time between two places and know the exact time at one of them, the corresponding time at the other will be found by *adding* their difference, if that other be *east*, or by subtracting it, if *west*.

82. *To reduce time to degrees and minutes of longitude.*

1. The difference of time between Boston and New Orleans is 1 hour 11 minutes and 48 seconds: what is the difference of longitude?

ANALYSIS.—Since 1 hour corresponds to 15° of longitude, there will be as many times 15° as there are hours: Since 1° corresponds to 4 minutes of time, there will be as many degrees as 4 is contained times in the minutes: Since 1′ corresponds to 4 seconds of time, there will be as many minutes as 4 is contained times in the seconds: hence,

OPERATION.

$$\begin{array}{rl} 15^\circ \times 1 = & 15^\circ \\ 11 \div 4 = & 2^\circ\ 45' \\ 48 \div 4 = & 12' \\ \hline \text{Diff.} & 17^\circ\ 57' \end{array}$$

1. *Multiply* 15° *by the number of hours, and the product will be degrees of longitude:*

2. *Divide the minutes by* 4, *and the quotient will be degrees and minutes of longitude:*

3. *Divide the seconds by* 4, *and the quotient will be minutes and seconds of longitude. The sum of these results will be the difference of longitude.*

EXAMPLES.

1. The longitude of Albany is 73° 42′ west, and that of Buffalo 78° 55′ west: what is the difference of longitude and what the difference of time?

2. The longitude of New York is 74° 1′ west, and that of Springfield, Illinois, 89° 33′ west: what would be the time at New York when it is 12 M. at Springfield?

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82. How do you reduce time to degrees and minutes of longitude?

3. When it is 12 M. at New York, it is 11 o'clock 6 minutes and 28 seconds at Cincinnati: what is their difference of longitude?

4. The longitude of Philadelphia is 75° 10′ west, and that of New York 74° 1′ west: what is the difference of time between these two places?

5. Washington is in longitude 77° 2′ west, New Orleans in 89° 2′ west. When it is 9 o'clock A. M. at Washington, what is the time at New Orleans?

6. If the difference of time between two places be 42*mi.* 16*sec.*, what is the difference of longitude?

7. What is the difference of longitude between two places if the difference of time is 2*h.* 20*mi.* 44*sec.*?

8. The longitude of St. Louis is 90° 15′ west; a person at that place observed an eclipse of the moon at 10*h.* 40*mi.* P. M.; another person, in a neighboring state, observed the same eclipse 22*mi.* 12*sec.* earlier: what was the longitude of the latter place, and the time of observation?

9. If the difference of time between London and Oregon City is 8 hours, what is the difference in longitude?

10. The difference of longitude between St. Louis and New York is 15° 35′. In travelling from New York to St. Louis will a watch, keeping accurate time, be fast or slow at St. Louis, and how much?

APPLICATIONS OF THE PRECEDING RULES.

1. What will it cost to build a wall 96 rods long, at \$1,33⅓ a rod?

2. A farmer wishes to put 1066*bush.* 2*pk.* of potatoes into 474 barrels, how much must he put into each barrel?

3. At \$4,32 a yard, what will 12¼ yards of cloth cost?

4. How many barrels of apples, each containing 2½ bushels, can I buy for \$36, at 45 cents a bushel?

5. The quotient arising from a certain division is 1236; the divisor is 375, and the remainder 184: what is the dividend?

6. The Croton Water Works of New York are capable of discharging 60000000 gallons of water every 24 hours: what would be the average amount per minute?

7. The population of the United States, in 1850, was 23191876. It has been estimated that 1 person in every 400 dies from intemperance: how many deaths then may be attributed to this cause, in the United States, during that year?

8. At the rate of 45 miles an hour, how long would it take a railroad car to pass around the globe, a distance of 25000 miles?

9. If a quantity of provisions will last 25 men 2*mo.* 3*wk.* 6*da.*, how long will it last 10 men?

10. If a man's salary is $1200 a year, and his expenses are $640 annually, how many years will it be before he will save $6720?

11. How long will it take to count 20 millions at the rate of 80 per minute?

12. If 3160 barrels of pork cost $47400, how many barrels can be bought for $11475?

13. What will be the cost of 6 firkins of butter, each containing 96 pounds, at $12\frac{1}{2}$ cents a pound?

14. What will 1000 quills cost, at $\frac{1}{2}$ cent a piece?

15. What will be the cost of $85\frac{1}{2}$ yards of cloth, at $$9\frac{1}{2}$ a yard?

16. What will be the cost of 1*hhd.* 2*gal.* 3*qt.* of brandy, at $56\frac{1}{4}$ cents a quart?

17. What will be the cost of 196 yards of cotton goods, at 1*s.* 6*d.* per yard?

18. At 2*s.* 8*d.* per bushel, what will 1246 bushels of oats cost?

19. If 112*lb.* of cheese cost £2 16*s.*, what is that per pound?

20. What will be the cost of 1426 pounds of hay, at $9,75 per ton?

21. How much must I pay for the transportation of 3840 pounds of iron, from Albany to Buffalo, at $4,50 per ton?

22. Bought 124 barrels of potatoes, each containing $2\frac{1}{4}$ bushels, at $33\frac{1}{3}$ cents a bushel: what is the whole cost?

23. If fifteen hundred tons of coal cost $11812,50, what will one ton cost?

24. If 789 pounds of leather cost $142,02, what is that per lb.?

25. There are three numbers, whose continued product is 16200; one of the numbers is 25; another 18: what is the third number?

26. If 1*dwt.* of gold is worth 92 cents, what would be the weight of $10059,28 in gold?

27. A man sold his house and lot for $4200, and took his pay in railroad stock, at 84 dollars a share; how many shares did he receive?

28. A person bought 640 acres of land, at 15 dollars an acre. He afterwards sold 160 acres at 20 dollars an acre; 240 acres at 18 dollars, and for the remainder he received $4560. What was his entire gain, and what did he receive per acre on the last sale?

29. A piece of ground 60 feet long and 48 feet wide is enclosed by a wall 12 feet high and $2\frac{1}{2}$ feet thick: how many cubic feet in the wall?

30. What will be the cost of transportation, from Montreal to Boston, of 325640 feet of lumber at $2,37$\frac{1}{2}$ per thousand?

31. Bought 684 pounds of hay, at $12.40 a ton: what will it cost me?

32. At $2,12$\frac{1}{2}$ a hundred, what will 786 feet of lumber cost?

33. How many shingles will it require to cover the roof of a building 40 feet long and 26 feet wide, with rafters 16 feet long, allowing one shingle to cover 24 square inches?

34. If 14*lb.* 8*oz.* 12*dwt.* 3*gr.* of silver be made into 9 tea-pots of equal weight, what will be the weight of each?

35. A man bought 320 barrels of flour for $2688: at what rate must he sell it to gain $1,60 on each barrel?

36. A farmer has a granary containing 449*bush.* 1*pk.* 2*qt.* of wheat; he wishes to put it into 182 bags: how much must he put in each bag?

37. A trader bought 750 barrels of flour for which he paid $4875; he sold the same for $7,25 a barrel: what was his profit on each barrel?

38. How many sheep, at $1,62½ a head, can be bought for $169?

39. If a person save $6,37½ a day, how long will it take him to save $267,75?

40. How many canisters, each holding *3lb. 10oz.*, can be filled from a chest of tea containing *58lb.*

41. In 26 hogsheads the leakage has reduced the whole amount to *1358gal. 2qt.*; if the same quantity has leaked out of each hogshead, how much still remains in each?

42. The number of college libraries in the United States in 1850 was 213, containing 942312 volumes: what would be the average number of volumes to each?

43. A man bought a piece of land for $3475,25, and sold the same for $3801,65, by which transaction he made $3,40 an acre: how many acres were there?

44. The whole amount of gold produced in California in the year 1855, was as follows: $43313281, sent to the Atlantic States; $6500000, sent directly to England; and $8500000 retained in the country. In 1854, the total product of gold in California was $57715000: how much more was produced in 1855 than in 1854?

45. If the forward wheels of a carriage are 12 feet in circumference, and the hind wheels 16 feet 6 inches, how many more times will the forward wheels turn round than the hind wheels, in running a distance of 264 miles?

46. If a certain township is 9 miles long, 4⅓ miles wide, how many farms of 192 acres each does it contain?

47. The total number of land warrants issued during the year ending Sept. 30th, 1855, was 34337, embracing 4093850 acres of land: what was the average number of acres to each warrant?

48. The amount of foreign imports brought into the United States during the fiscal year of 1855 was $261382960; during the year 1854 it was $305780253: how much was the decrease?

49. The longitude of Philadelphia is 75° 10′, and that of New Orleans 89° 2′; when it is 12 M. at Philadelphia, what is the time at New Orleans?

50. The sun passes the meridian at 12 M., the moon at 8*hr.* 30*m.* P. M.: what is the difference in longitude between the sun and moon?

51. Two persons, A and B, observed an eclipse of the moon; A observed its commencement at 9*hr.* 42*mi.* P. M.; B was in longitude 73° 20′, and observed its commencement 23 minutes earlier than A: what was A's longitude, and B's time of observation?

52. If in 11 piles of wood there are 120 cords, 7 cord feet, 5 cubic feet, how much is there in each pile?

53. If 16*cwt.* 2*qr.* 11*lb.* 10*oz.* of flour be put into 9 barrels, how much will each barrel contain?

54. A miller bought a quantity of wheat for $625,40, which he floured and put into barrels at an expense of $110,12½: what profit did he make by selling it for $900?

55. America was discovered Oct. 11th, 1492: how long to the commencement of the Revolution, April 19th, 1775?

56. From a hogshead of wine a merchant draws 18 bottles, each containing 1*pt.* 3*gills*; he then fills three 6 gallon demijons, and 4 dozen bottles each containing 2*qt.* 1*pt.* 3*gills*: how much remained in the cask?

57. In 753689 yards, how many degrees and statute miles?

58. In 189*mi.* 3*fur.* 6*rd.* 1*ft.* how many feet?

59. If 24 men can build 768 rods of wall in 1 day, how many rods can 48 men build in 9 days?

60. A certain number increased by 1764, and the sum multiplied by 209, gives the product of 7913576: what is the number?

61. If a man travel 146*mi.* 7*fur.* 14*rd.* 14*ft.* in 5 days, how much is that for each day?

62. If 325 acres of land cost $17712,50, how many acres can be bought for $545?

63. A merchant having $324 wishes to purchase an equal number of yards of two kinds of cloth; one kind was worth 4 dollars a yard, the other was worth 5 dollars a yard: how many yards of each can he buy?

64. From one-fourth of a piece of cloth containing 68*yd.* 3*qr.* a tailor cut 5 suits of clothes: how much did each suit contain?

65. A manufacturer having £5 10*s.*, distributed it among his laborers, giving every man 18*d.*, every woman 12*d.*, and every boy 10*d.*; the number of men, women and boys, were equal: what was the number of each?

66. It is estimated that 1 out of every 1585 persons in Great Britain is deaf and dumb. The population, according to the census of 1851, was 20936468: how many deaf and dumb persons were there in the entire population?

67. A grocer in packing 6 dozen dozen eggs broke half a dozen dozen, and sold the remainder for $1\frac{1}{2}$ cents a piece: how much did he receive for the eggs?

68. How much time will a man save in 50 years by rising 45 minutes earlier each day?

69. Richard Roe was born at 6 o'clock, A. M., June 24th, 1832: what will be his age at 3 o'clock, P. M., on the 10th day of January, 1858?

70. During the year 1855, there were shipped to Great Britain, from the United States, 408434 barrels of flour; 2550092 bushels of wheat; 1048540 bushels of corn. Supposing the flour to have sold for $10,25 a barrel, the wheat for $2,12$\frac{1}{2}$ a bushel, and the corn for $0,94 a bushel, what was the value of the whole?

71. A man dying without making a will, left a widow and 4 children. The law provides, in such cases, that the widow shall receive one-third of the personal property, and that the remainder shall be equally divided among the children. The estate was valued as follows: a farm, at $5000; 5 horses, at $85 each; a yoke of oxen, for $110; 25 cows, at $22 each; 150 sheep, at $2 each; some lumber, at $45; farming utensils, at $174; household furniture, at $450; grain and hay, at $380: what was the share of the widow and each child?

72. The amount of gold coin in the United States in 1855 was estimated at about $241200000. Adopting the same ratio of increase as from 1850 to 1855, the population of the United

States in 1855 would be about 26800000. In an equal distribution of the gold, how much would each person receive?

73. How many shingles will it take to cover the two sides of the roof of a building, 55 feet long, with rafters $16\frac{1}{2}$ feet in length, allowing each shingle to be 15 inches long and 4 inches wide, and to lay one-third to the weather?

74. If the longitude of St. Petersburgh is 30° 45′ east, and that of Washington 77° 2′ west, what is the difference of longitude between the two places, and the difference of time?

75. When it is 6 o'clock, A. M., at Washington, what is the time at St. Petersburgh?

76. A vessel sails from New York to Liverpool. After a number of days the captain, by taking an observation of the sun, finds that his chronometer, which gives New York time, differs 1*hr.* 44*m.* from the time at the place of observation. If his chronometer shows the time to be 3*hr.* 12*mi.* P. M., what is the correct time, at the place of observation, and how far is he east from New York?

77. A cistern containing 960 gallons, has two pipes; 45 gallons run in every hour by one pipe, and 25 gallons run out by the other: how long a time will be required to fill the cistern?

78. A speculator sold 840 bushels of wheat for $2180, which was $500 more than he gave for it: what did it cost him a bushel?

79. The whole number of gallons of rum manufactured in the United States in 1850, was 6500500 gallons; if it be valued at 50 cents a gallon, how many schoolhouses could be built, worth $750 each, with the proceeds?

80. A farmer sold a grocer 30 bushels of potatoes at $37\frac{1}{2}$ cents a bushel, for which he received 6 gallons of molasses at 45 cents a gallon; 60 pounds of mackerel at $6\frac{1}{2}$ cents a pound, and the remainder in sugar at 10 cents a pound: how many pounds of sugar did he receive?

81. If a man travel 12*mi.* 3*fur.* 20*rd.* in one day, how long will it take him to travel 174*mi.* 1*fur.* at the same rate?

82. If a man sell 2*bar.* 12*gal.* 2*qt.* of beer in one week, how much will he sell in 12 weeks?

83. A liquor merchant had 550 pint bottles, 400 quart bottles, 350 two quart bottles, 375 three quart bottles, and 150 jugs, holding a gallon each: how many barrels of wine will fill them?

84. How many yards of carpeting, one yard wide, will it take to cover the floors of two parlors, each 18 feet long, and 16 feet wide, and what will it cost at $1,33$\frac{1}{3}$ a yard?

85. How many rolls of wall paper, each 10 yards long and 2 feet wide, will it take to cover the sides of a room 22 feet long and 16 feet wide and 9 feet high?

86. Two persons are 1*mi.* 4*fur.* 20*rd.* apart, and are travelling the same way. The hindmost gains upon the foremost 5 rods in travelling 25 rods: how far must he travel to overtake the foremost?

87. A man sold 500 bushels of wheat at $1,75 a bushel, and took his pay in sugar at 5 cents a pound. He afterwards sold one-half of it: what quantity of sugar had he left?

88. A man bought 7 barrels of sugar at $12,87$\frac{1}{2}$ a barrel; he kept two barrels for his own use, and sold the remainder for what the whole cost him: what did he receive per barrel?

89. A flour merchant bought a quantity of flour for $18750, and sold the same for $26250, by which he gained $3 a barrel: how many barrels were there?

90. Three men rented a farm and raised 964*bush.* 2*pk.* 4*qt.* of grain, which was to be divided in proportion to the rent paid by each. The first was to have one-half the whole; the second one-third the remainder; and the third had what was left: how much did each have?

91. A vessel in longitude 70° 25′ east, sails 105° 30′ 56″ west, then 46° 50′ east, then 10° 5′ 40″ west, then 39° 11′ 36″ east; in what longitude is she then, and how many days will it take her to sail to longitude 77° west, if she sail 3° 20′ each day?

92. A privateer took a prize worth $25000, which was divided into 125 shares, of which the captain took 12 shares;

2 lieutenants, each 5 shares; 6 midshipmen, each 3 shares; and the remainder was divided equally among 85 seamen: how much did each receive?

93. If the longitude of Boston is 71° 4′, and a gentleman in travelling from Boston to Chicago finds that his watch is 1*hr.* 5*m.* 44*sec.* too fast by the time of the latter place, what is the longitude of Chicago, provided his watch has kept time accurately?

94. What time would it be in Boston when it was 8*hr.* 27*mi.* 30*sec.*, A. M., in Chicago?

95. What time would it be at Chicago when it was 12 M. at Boston?

96. Two places lie exactly east and west of each other, and by observation it is found that the sun comes to the meridian of the latter place 1 hour and 16 minutes after the former: how far apart are they in degrees and minutes of longitude?

97. In 12 bales of cloth, each bale containing 16 pieces, and each piece containing 20 ell English, how many yards?

98. How many eagles can be made from 24*lb.* 4*oz.* 6*pwt.* 18*gr.* of gold, making no allowance for waste, if each eagle weighs 11*pwts.* 9*gr.*?

99. A man paid $3284,82 for some wheat. He sold 740 bushels at 2 dollars a bushel; the remainder stood him in $1,42 a bushel: how many bushels did he purchase?

100. A speculator gave $8968 for a certain number of barrels of flour, and sold a part of it for $2618, at $7 a barrel, and by so doing lost $2½ on each barrel; for how much must he sell the remainder to gain $1060 on the whole?

101. A man sold 105*A.* 2*R.* 20*P.* of land for as many dollars as there were perches of land, payable in instalments, at the rate of 1 dollar an hour. If the contract was closed at 12 o'clock, M., April 1st, 1856, what length of time will be allowed the purchaser to pay the debt, reckoning 365 days 6 hours to the year?

PROPERTIES OF NUMBERS.

PRIME AND COMPOSITE NUMBERS.

83. AN INTEGRAL NUMBER is the unit 1, or a collection of such units.

84. One number is said to be *divisible* by another when the quotient is an integral number. The division is then said to be *exact.*

85. A COMPOSITE NUMBER is one that may be produced by the multiplication of two or more numbers, called *factors;* thus, $30 = 2 \times 3 \times 5$, is a composite number, in which the factors are 2, 3 and 5.

NOTE 1.—A composite number is exactly divisible by any one of its factors, or by the product of two or more of them.

86. A PRIME NUMBER is a number that is divisible *only* by itself and 1; thus, 1, 2, 3, 5, 7, 11, 13, &c., are prime numbers.

87. Two numbers are said to be prime to each other when they have no *common factor;* thus, 4 and 9 are *prime to each other*, though both are composite numbers.

88. Any number (prime or composite), as 26, may be put under the form, 1×26; hence, every number is divisible by itself and 1, and therefore, these are not reckoned among the *factors* or *divisors* either of prime or composite numbers.

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83. What is an integral number?

84. When is one number said to be divisible by another? How is the division then said to be?

85. What is a composite number? By what is a composite number always divisible?

86. What is a prime number?

87. When are two numbers prime to each other?

88. What numbers are not reckoned among the divisors of prime or composite numbers?

89. Every factor of a composite number is a divisor, and is either prime or composite; and, since every composite factor may be again divided, it follows that

Every number is equal to the product of all its prime factors.

For example, $24 = 3 \times 8$; but 8 is a composite number of which the factors are 2 and 4; and 4 is a composite number of which the factors are 2 and 2; hence,

$$24 = 3 \times 8 = 3 \times 2 \times 4 = 3 \times 2 \times 2 \times 2; \text{ and}$$
$$60 = 5 \times 12 = 5 \times 3 \times 4 = 5 \times 3 \times 2 \times 2.$$

Hence, to find the prime factors of any number:

Divide the number by any prime number that will exactly divide it: then divide the quotient by any prime number that will exactly divide it, and so on, till a quotient is found which is prime; the several divisors and the last quotient will be the prime factors of the given number.

NOTE.—It is most convenient, in practice, to use *at each division* the least prime number that is a divisor.

1. What are the prime factors of 105?

ANALYSIS.—Three being the least divisor that is a prime number, we divide by it, giving the quotient 35; then 5 is the least prime divisor of this quotient; hence, 3, 5 and 7 are the prime factors of 105.

OPERATION.

```
3)105
 5)35
    7
```

EXAMPLES.

1. What are the prime factors of 9? 10? 12? 14? 16? 18? 24? 27? 28?

2. What are the prime factors of 30? 22? 32? 36? 38? 40? 45? 49?

3. What are the prime factors of 50? 56? 58? 60? 64 66? 68? 70? 72?

4. What are the prime factors of 76? 78? 80? 82? 84? 86? 88? 90?

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89. To what product is every number equal? How do you find the prime factors of any number?

5. What are the prime factors of 92? 94? 96? 98? 99? 100? 102? 104?

6. What are the prime factors of 105? 106? 108? 110? 115? 116? 120? 125?

7. What are the prime factors of 302? 305? 604? 875? 975? 655?

NOTE.—The prime factors, when the numbers are small, may generally be seen by inspection. The teacher can easily multiply the examples.

90. When there are several numbers, and it is required to find the prime factors common to all of them:

Find the prime factors of each, and then select those factors which are common to all the numbers.

8. What are the prime factors common to 150, 210 and 270?

9. What are the prime factors common to 42, 126, and 168?

10. What are the prime factors common to 105, 315 and 525?

11. What are the prime factors common to 84, 126 and 210?

12. What are the prime factors common to 168, 256, 410, and 820?

13. What are the prime factors common to 420, 630, 1050, and 2100?

91. DIVISIBILITY OF NUMBERS.

1. Two is the only even number which is prime.
2. Two divides every even number, and no odd number.
3. Three divides every number the sum of whose figures is divisible by 3.
4. Four divides every number when the two right hand figures are divisible by 4.
5. Five divides every number which ends in 0 or 5.
6. Six divides every even number that is divisible by 3.
7. Ten divides every number ending in 0.

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90. How do you find the prime factors common to several numbers?

91. 1. How many even numbers are prime? 2. What numbers will 2 divide? 3. What numbers will 3 divide? 4. What numbers will 4 divide? 5. What numbers will 5 divide? 6. What numbers will 6 divide? 7. What

8. *When the divisor is a composite number, and each factor will exactly divide the dividend, their product will exactly divide it.*

For, dividing by the factors separately, gives the same quotient as dividing by their product (Art. 70).

9. *Any number which will divide one factor of a product will divide the product.*

Thus, take any number, as $30 = 5 \times 6$; any number which will divide 5 or 6 will divide 30.

10. *Any number which will exactly divide each of two numbers will divide their sum: and any number which will divide their sum and one of the numbers, will divide the other.*

For, take *any two numbers*, as 9 and 12; then,

$$9 + 12 = 21.$$

Now, any divisor that will divide two of these numbers will divide the other; else, *we should have a whole number equal to a fraction, which is impossible.*

11. *Any number which will exactly divide each of two numbers will divide their difference: and any number which will divide their difference and one of the numbers, will divide the other.*

For, let 24 and 8 be any two numbers; then,

$$24 - 8 = 16.$$

Now, any divisor that will divide two of these numbers will divide the other; else, *we should have a whole number equal to a fraction, which is impossible.*

12. *If there is a remainder after division, any number which will exactly divide the dividend and divisor will also divide the remainder.*

---

numbers will 10 divide? 8. When will the divisor exactly divide the dividend? 9. When will any number divide a product? 10. When will a number divide the sum of two numbers? When will it divide either of them separately? 11. When will a number exactly divide the difference of two numbers? 12. If a number divides the dividend and divisor, what other number will it always divide?

For, we always have

$$\text{Dividend} = \text{Divisor} \times \text{Quotient} + \text{Rem.}$$

or $$\text{Dividend} - \text{Divisor} \times \text{Quotient} = \text{Rem.};$$

hence, by principle (11) any number which will divide the dividend and divisor will also divide the remainder, after division.

GREATEST COMMON DIVISOR.

92. A Common Divisor of two or more numbers is any number that will divide each of them without a remainder; hence, it is merely a *common factor* of the numbers.

93. The Greatest Common Divisor of two or more numbers is the *greatest* number that will divide each of them without a remainder; hence, it is their greatest *common factor.*

For example, 2 and 3 are common divisors of 12 and 18; but 6 is their *greatest* common divisors, since there is no number greater than 6 that will exactly divide both of them; hence, it is their greatest common factor.

Note.—Since 1 and the number itself will divide *every number*, they are not reckoned among the common divisors.

Hence, to find the greatest common divisor of two or more numbers,

I. *Resolve each number into its prime factors:*

II. *The product of all the factors common to each result will be the greatest common divisor.*

EXAMPLES.

1. What is the greatest common divisor of 12 and 20?

Analysis.—There are three prime factors in 12; viz., 2, 2 and 3: there are three prime factors in 20; viz., 2, 2 and 5: the factors 2 and 2 are common; hence, $2 \times 2 = 4$ is the greatest common divisor.

OPERATION.

$$12 = 2 \times 2 \times 3$$
$$20 = 2 \times 2 \times 5$$

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92. What is a common divisor of two or more numbers?

93. What is the greatest common divisor of two or more numbers? How do you find the greatest common divisor of two or more numbers?

2. What is the greatest common divisor of 18 and 36.
3. What is the greatest common divisor of 12, 24 and 60
4. What is the greatest common divisor of 15, 50 and 40?
5. What is the greatest common divisor of 24, 18 and 144?
6. What is the greatest common divisor of 50, 100 and 80?
7. What is the greatest common divisor of 56, 84 and 140?
8. What is the greatest common divisor of 84, 154 and 210?

SECOND METHOD.

94. When the numbers are large, another method is used for finding their greatest common divisor.

1. Let it be required to find the greatest common divisor of the numbers 216 and 408.

OPERATION.

```
216)408(1
    216
    192)216(1
        192
         24)192(8
            192
```

ANALYSIS.—The greatest common divisor cannot be greater than the least number 216. Now, as 216 will divide itself, let us see if it will divide 408; for if it will, it is the greatest common divisor sought. Making the division, we find a quotient 1 and a remainder 192; hence, 216 is not a common divisor.

The greatest common divisor of 216 and 408 will divide the remainder 192 (Art. 91–12); and if 192 will exactly divide 216, it will be the greatest common divisor. We find that 192 is contained in 216 once, and a remainder 24. The greatest common divisor of 192 and 216 will divide the remainder 24; and if 24 will exactly divide 192, it will also divide 216, and consequently 408; now 24 exactly divides 192, and hence is the greatest common divisor sought.

Hence, to find the greatest common divisor,

Divide the greater number by the less, and then divide the preceding divisor by the remainder, and so on, till nothing remains: the last divisor will be the greatest common divisor.

NOTES.—1. If the last remainder is 1, the numbers have no com mon divisor; that is, *they are prime with respect to each other* (Art. 87).

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94. What is the rule when the numbers are large?

2. If, in the course of the operation, any one of the remainders is a *prime number*, and will not exactly divide the *preceding divisor*, it is certain that no common divisor exists, and it is unnecessary to divide further.

EXAMPLES.

1. What is the greatest common divisor of 3328 and 4592?

2. What is the greatest common divisor of 2205 and 4501?

3. What is the greatest number that will divide 16082 and 25740?

4. What is the greatest number that will divide 620, 1116 and 1488?

5. What is the greatest common divisor of 5270, 5952, 5394 and 3038?

6. What is the greatest common divisor of 4617, 7695, 6642 and 8424?

7. A farmer has 315 bushels of corn, and 810 bushels of wheat; he wishes to draw the corn and wheat to market separately in the fewest number of equal loads: how many bushels must he draw at a load?

8. The Illinois Central Railroad Company have 15750 acres of land in one location, and 21725 acres in another. They wish to divide the whole into lots of equal extent, containing the greatest number of acres that will give an exact division: how many acres will there be in each lot?

9. A man has a corner lot of land 1044 feet long, and 744 feet wide. The adjacent sides are bounded by the highway and he wishes to build a board fence with the fewest panels of equal length: what must be the length of the panels?

10. A farmer has 231 bushels of barley, 369 bushels of oats, and 393 bushels of wheat, all of which he wishes to put into the smallest number of bags of equal size, without mixing: how many bushels must each bag contain?

11. Three persons, A, B, and C, each agree to purchase a lot of cows at the same price per head, provided each man can thus invest his whole money. A has $286, B $462, and C $638; how many cows could each man purchase?

LEAST COMMON MULTIPLE.

95. A MULTIPLE of a number is any product in which the number enters as a factor; hence, a multiple of any number is exactly divisible by the number.

96. A COMMON MULTIPLE of two or more numbers is any number which each will divide without a remainder.

97. THE LEAST COMMON MULTIPLE of two or more numbers is the *least* number which they will separately divide without a remainder.

NOTES.—1. Since the least common multiple is exactly divisible by a divisor, it can be resolved into two factors, one of which is the divisor and the other the quotient.

2. If the divisor be resolved into its prime factors, the equal factor of the least common multiple may be resolved into the same factors; hence, *the least common multiple will contain every prime factor of its divisor.*

3. The question of finding the least common multiple of several numbers, is therefore reduced to finding a number which shall contain all their prime factors and *none others.*

1. What is the least common multiple of 6, 12 and 18?

OPERATION.

$$\begin{array}{r|l} 2 & 6 \;..\; 12 \;..\; 18 \\ \hline 3 & 3 \;..\; 6 \;..\; 9 \\ \hline & 1 \;..\; 2 \;..\; 3 \end{array}$$

$$2 \times 3 \times 2 \times 3 = 36$$

ANALYSIS.—Having placed the given numbers in a line, if we divide by 2, we find the quotients 3, 6 and 9; hence, 2 is a prime factor of all the numbers. Dividing by 3, we find that 3 is a prime factor of the quotients 3, 6, and 9; and hence, the remainders 2 and 3 are prime factors of 12 and 18; hence, the prime factors of all the numbers are 2, 3, 2 and 3, and their product 36 is the *least common multiple.*

98. Therefore, to find the least common multiple of several numbers:

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95. What is a multiple of a number?
96. What is a common multiple of two or more numbers?
97. What is the least common multiple of two or more numbers?
98. How do you find the least common multiple of several numbers?

I. *Place the numbers on the same line, and divide by any prime number that will exactly divide two or more of them, and set down in a line below the quotients and the undivided numbers.*

II. *Then divide as before, until there is no prime number greater than* 1 *that will exactly divide any two of the numbers.*

III. *Then multiply together the divisors and the numbers of the lower line, and their product will be the least common multiple.*

NOTE.—If the numbers have no common prime factor, their product will be their least common *multiple.*

EXAMPLES.

1. What is the least common multiple of 4, 9, 10, 15, 18, 20, 21?

2. What is the least common multiple of 8, 9, 10, 12, 25, 32, 75, 80?

3. What is the least common multiple of 1, 2, 3, 4, 5, 6, 7, 9?

4. What is the least common multiple of 9, 16, 42, 63, 21, 14, 72?

5. What is the least common multiple of 7, 15, 21, 28, 35, 100, 125?

6. What is the least common multiple of 15, 16, 18, 20, 24, 25, 27, 30?

7. What is the least common multiple of 9, 18, 27, 36, 45, 54?

8. What is the least common multiple of 4, 10, 14, 15, 21?

9. What is the least common multiple of 7, 14, 16, 21, 24?

10. What is the least common multiple of 49, 14, 84, 168, 98?

11. A can dig 9 rods of ditch in a day; B 12 rods in a day; and C 16 rods in a day: what is the smallest number of rods that would afford exact days of labor to each, working alone? In what time would each do the whole work?

12. What is the smallest sum of money for which a blacksmith can hire a number of journeymen for 1 month, at $15,

$16, $21, and $24 each, and what will be the number of men employed at each price?

13. A farmer has a number of bags containing 2 bushels each; of barrels, containing 3 bushels each; of boxes, containing 7 bushels each; and of hogsheads, containing 15 bushels each: what is the smallest quantity of wheat that would fill each an exact number of times, and *how many times* would that quantity fill each?

14. Four persons start from the same point to travel round a circuit of 300 miles in circumference. A goes 15 miles a day, B 20 miles, C 25 miles, and D 30 miles a day. How many days must they travel before they will all come together again at the same point, and how many times will each have gone round?

NOTE.—First find the number of days that it will take each to travel round the circuit.

CANCELLATION.

99. CANCELLATION is a method of shortening Arithmetical operations by omitting or *cancelling* common factors.

1. Divide 36 by 18. First, $36 = 9 \times 4$; and $18 = 9 \times 2$

ANALYSIS.—Thirty-six divided by 18 is equal to 9×4 divided by 9×2: by *cancelling*, or striking out the 9's, we have 4 divided by 2, which is equal to 2.

OPERATION.

$$\frac{36}{18} = \frac{\cancel{9} \times 4}{\cancel{9} \times 2} = 2.$$

NOTE.—The figures cancelled are slightly crossed.

The operations, in cancellation, depend on two principles:

1. *The cancelling of a factor, in any number, is equivalent to dividing the number by that factor*

2. *If the dividend and divisor be both divided by the same number, the quotient will not be changed.*

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99. What is cancellation? On what principles do the operations of cancellation depend?

PRINCIPLES AND EXAMPLES.

1. Divide 56 by 32.

ANALYSIS.—Resolve the dividend and divisor into factors, and then cancel those which are common.

OPERATION.

$$\frac{56}{32} = \frac{\cancel{8} \times 7}{\cancel{8} \times 4} = \frac{7}{4}.$$

2. In 72 times 25, how many times 45?

ANALYSIS.—We see that 9 is a factor of 72 and 45. Divide by 9, and write the quotient 8 over 72, and the quotient 5 below 45. Again, 5 is a factor of 25 and 5. Divide 25 by 5, and write the quotient 5 over 25. Dividing 5 by 5, reduces the divisor to 1, which need not be set down: hence, the true quotient is 40.

OPERATION.

$$\frac{\overset{8}{\cancel{72}} \times \overset{5}{\cancel{25}}}{\underset{\cancel{5}}{\cancel{45}}} = 40.$$

NOTE.—The operation may be performed in another way, by writing the divisor on the left of a vertical line, and the dividend on the right; in which case, the quotients, in cancelling, are written above, and at the side of the numbers, as 5, 8 and 5. If we conceive the horizontal line, first used, to be turned up from left to right, the dividend, which was above the line, will fall at the right, and the divisor, which was below it, at the left.

OPERATION.

$$\begin{array}{r|l} \overset{5}{\cancel{45}} & \overset{8}{\cancel{72}} \\ & \overset{5}{\cancel{25}} \\ \hline \end{array}$$

Ans. 40.

100. Hence, to perform the operations of cancellation:

I. *Resolve the dividend and divisor into such factors as shall give all the factors common to both.*

II. *Cancel the common factors and then divide the product of the remaining factors of the dividend by the product of the remaining factors of the divisor.*

NOTES.—1. Since every factor is cancelled by *division*, the quotient 1 always takes the place of the cancelled factor, but is omitted when it is a multiplier of other factors.

2. If one of the numbers contains a factor equal to the product of two or more factors of the other, all the factors may be cancelled.

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100. How do you perform the operations of cancellation?

3. If the product of two or more factors of the dividend is equal to the product of two or more factors of the divisor, such factors may be cancelled.

4. It is generally more convenient to set the dividend on the right of a *vertical* line and the divisor on the left.

EXAMPLES.

1. What is the quotient of $2 \times 4 \times 8 \times 13 \times 7 \times 16$ divided by $26 \times 14 \times 8$?

2. What is the quotient of $42 \times 3 \times 25 \times 12$ divided by $28 \times 4 \times 15 \times 6$?

3. What is the quotient of $125 \times 60 \times 24 \times 42$ divided by $25 \times 120 \times 36 \times 5$?

4. How many times is $11 \times 39 \times 7 \times 2$ contained in $44 \times 18 \times 26 \times 14$?

5. What is the quotient of 8 times 240 multiplied by 5 times 114, divided by 24 times 57 multiplied by 6 times 15?

6. What is the value of $(22 + 8 + 16) \times (18 + 10 + 21)$ divided by $(9 + 5 + 7) \times (15 + 8)$?

7. Divide $(140 + 86 - 34) \times (107 - 19)$ by $(237 - 141) \times (17 + 20 - 15)$?

8. Divide $(12 \times 5) - (2 \times 9) \times (42 + 30)$ by $(5 \times 8) \times (2 \times 9) \times (10 + 17)$?

9. What is the quotient of $240 \times 441 \times 16$ divided by $175 \times 56 \times 27$?

10. What is the quotient of 64 times 840 multiplied by 9 times 124, divided by 32 times 560 multiplied by 4 times 31?

11. How many dozens of eggs, worth 14 cents a dozen, must be given for 18 pounds of sugar, worth 7 cents a pound?

12. A dairyman sold 5 cheeses, each weighing 40 pounds, at 9 cents a pound: how many pounds of tea, worth 50 cents a pound, must he receive for the cheeses?

13. Bought 12 yards of cloth at $1,84 a yard, and paid for it in potatoes at 48 cents a bushel: how many bushels of potatoes will pay for the cloth?

14. How many firkins of butter, each containing 56 pounds,

at 25 cents a pound, will pay for 4 barrels of sugar, each weighing 175 pounds, at 8 cents a pound?

15. A man bought 10 cords of wood, at 20 shillings a cord, and paid in labor at 12 shillings a day: how many days must he labor?

16. How many pieces of cloth, each containing 36 yards, at $3,50 a yard, must be given for 96 barrels of flour, at $10,50 a barrel?

17. A farmer exchanged 492 bushels of wheat, worth $1,84 a bushel, for an equal number of bushels of barley, at 87 cents a bushel, of corn at 60 cents a bushel, and of oats at 45 cents a bushel: how many bushels of each did he receive?

18. How many barrels of flour, worth $7 a barrel, must be given for 250 bushels of oats, at 42 cents a bushel?

19. If 48 acres of land produce 2484 bushels of corn, how many bushels will 120 acres produce?

20. A man works 12 days at 9 shillings a day, and receives in pay wheat at two dollars a bushel: how many bushels did he receive?

21. A grocer sold 6 hams, each weighing 14 pounds, at 10 cents a pound, and took his pay in apples at 48 cents a bushel: how many bushels of apples did he receive?

22. How long will it take a man, travelling 36 miles a day, to go the same distance that another man has travelled in 15 days at the rate of 27 miles a day?

23. A man took 4 loads of apples to market, each load containing 12 barrels, and each barrel 3 bushels. He sells them at 45 cents a bushel, and receives in payment, a number of boxes of tea, each box containing 20 pounds, worth 72 cents a pound: how many boxes of tea did he receive?

COMMON FRACTIONS.

101. The unit 1 denotes an entire thing, as 1 apple, 1 chair, 1 pound of tea.

If the unit 1 be divided into two equal parts, each part is called *one-half.*

If the unit 1 be divided into three equal parts, each part is called *one-third.*

If the unit 1 be divided into four equal parts, each part is called *one-fourth.*

If the unit 1 be divided into twelve equal parts, each part is called *one-twelfth;* and if it be divided into *any number* of equal parts, we have a like expression for each part.

The parts are thus written:

$\frac{1}{2}$	is read, one-half.	$\frac{1}{7}$	is read, one-seventh.
$\frac{1}{3}$	- - one-third.	$\frac{1}{8}$	- - one-eighth.
$\frac{1}{4}$	- - one-fourth.	$\frac{1}{10}$	- - one-tenth.
$\frac{1}{5}$	- - one-fifth.	$\frac{1}{15}$	- - one-fifteenth.
$\frac{1}{6}$	- - one-sixth.	$\frac{1}{50}$	- - one-fiftieth.

The $\frac{1}{2}$, is an *entire half;* the $\frac{1}{3}$, an *entire third;* the $\frac{1}{4}$, an *entire fourth;* and the same for each of the other equal parts; hence, *each equal part is an entire thing,* and is called a *fractional unit.*

The unit, or whole thing which is divided, is called the *unit of the fraction.*

NOTE.—In every fraction let the pupil distinguish carefully, between the *unit of the fraction* and the *fractional unit.* The first is the *whole thing* from which the fraction is derived; the second, *one of the equal parts* into which that thing is divided.

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101. What is a unit? What is each part called when the unit 1 is divided into two equal parts? When it is divided into 3? Into 4? Into 5? Into 12?

102. Each fractional unit may, like the unit 1, become the base of a collection: thus, suppose it were required to express 2 of each of the fractional units: we should then write

$\frac{2}{2}$	which is read	2 halves	$= \frac{1}{2} \times 2.$
$\frac{2}{3}$	- - - -	2 thirds	$= \frac{1}{3} \times 2.$
$\frac{2}{4}$	- - - -	2 fourths	$= \frac{1}{4} \times 2.$
$\frac{2}{5}$	- - - -	2 fifths	$= \frac{1}{5} \times 2.$
&c.,	&c.,	&c.,	&c.

If it were required to express 3 of each of the fractional units, we should write

$\frac{3}{2}$	which is read	3 halves	$= \frac{1}{2} \times 3.$
$\frac{3}{3}$	- - - -	3 thirds	$= \frac{1}{3} \times 3.$
$\frac{3}{4}$	- - - -	3 fourths	$= \frac{1}{4} \times 3.$
$\frac{3}{5}$	- - - -	3 fifths	$= \frac{1}{5} \times 3.$
&c.,	&c.,	&c.,	&c.; hence,

A FRACTION is one of the equal parts of a unit, or a collection of such equal parts.

Fractions are expressed by two numbers, one written above the other, with a line between them. The lower number is called the *denominator*, and the upper number the *numerator*.

The denominator denotes the number of equal parts into which the unit is divided; and hence, determines the *value of the fractional unit.* Thus, if the denominator is 2, the fractional unit is *one-half*; if it is 3, the fractional unit is *one-third;* if it is 4, the fractional unit is *one-fourth*, &c., &c.

The numerator denotes the *number* of fractional units taken. Thus, $\frac{3}{5}$ denotes that the fractional unit is $\frac{1}{5}$, and that 3 such units are taken; and similarly for other fractions.

How may the one-half be regarded? The one-third? The one fourth? What is each part called? What is the unit of a fraction? What is a fractional unit? How do you distinguish between the one and the other?

102. May a fractional unit become the base of a collection? What is a fraction? How are fractions expressed? What is the lower number called? What is the upper number called? What does the denominator denote? What does the numerator denote? In the fraction 3 fifths, what

In the fraction $\frac{3}{5}$, the base of the collection of fractional units is $\frac{1}{5}$, but this is not the *primary* base. For, $\frac{1}{5}$ is *one-fifth of the unit* 1; hence, *the primary base of every fraction is the unit* 1.

103. If we suppose a second unit to be divided into the same number of equal parts, such parts may be expressed in the same collection with the parts of the first: thus,

$\frac{3}{2}$	is read	3 halves.
$\frac{7}{4}$	- - -	7 fourths.
$\frac{16}{5}$	- - -	16 fifths.
$\frac{18}{6}$	- - -	18 sixths.
$\frac{25}{7}$	- - -	25 sevenths.

104. A whole number may be expressed fractionally by writing 1 below it for a denominator. Thus,

3	may be written	$\frac{3}{1}$	and is read,	3 ones.
5	- - - -	$\frac{5}{1}$	- - -	5 ones.
6	- - - -	$\frac{6}{1}$	- - -	6 ones.
8	- - - -	$\frac{8}{1}$	- - -	8 ones.

But 3 ones are equal to 3, 5 ones to 5, 6 ones to 6, and 8 ones to 8; hence, *the value of a number is not changed* by placing 1 under it for a denominator.

105. If the numerator of a fraction be divided by its denominator, the integral part of the quotient will express the number of entire units used in forming the fraction; and the remainder will show how many fractional units are over. Thus, $\frac{11}{3}$ are equal to 3 and 2 thirds, and is written $\frac{11}{3} = 3\frac{2}{3}$: hence,

A fraction has the same form as an unexecuted division.

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is the fractional base? What is the primary base? What is the primary base of every fraction?

103. If a second unit be divided into the same number of equal parts, may the parts be expressed with those of the first? How many units have been divided to obtain 6 thirds? To obtain 9 halves? 12 fourths?

104. How may a whole number be expressed fractionally? Does this change the value of the number?

105. If the numerator be divided by the denominator, what does the quotient show? What does the remainder show? What form has a fraction? What are the seven principles which follow?

From what has been said, we conclude that,

1st. *A fraction is one of the equal parts of a unit, or a collection of such equal parts:*

2d. *The denominator shows into how many equal parts the unit is divided, and hence indicates the value of the fractional unit:*

3d. *The numerator shows how many fractional units are taken:*

4th. *The value of every fraction is equal to the quotient arising from dividing the numerator by the denominator:*

5th. *When the numerator is less than the denominator, the value of the fraction is less than* 1:

6th. *When the numerator is equal to the denominator, the value of the fraction is equal to* 1:

7th. *When the numerator is greater than the denominator, the value of the fraction is greater than* 1.

EXAMPLES IN WRITING AND READING FRACTIONS.

1. Read the following fractions:

$$\frac{8}{9}, \frac{7}{12}, \frac{5}{3}, \frac{6}{15}, \frac{21}{9}, \frac{16}{7}, \frac{18}{104},$$

What is the unit of the fraction, and what the fractional unit in each example? How many fractional units are taken in each?

2. Write 15 of the 19 equal parts of 1. Also, 37 of the 49 equal parts of 1.

3. If the unit of the fraction is 1, and the fractional unit one-fortieth, express 27 fractional units. Also, 95. Also, 106. Also, 87. Also, 41.

4. If the unit of the fraction is 1, and the fractional unit one 68th, express 45 fractional units. Also, 56. Also, 85. Also, 95. Also, 37.

5. If the unit of the fraction is 1, and the fractional unit one 90th, express 9 fractional units. Also, 87. Also, 75. Also, 65. Also, 85. Also, 90. Also, 100.

DEFINITIONS.

106. A PROPER FRACTION is one whose numerator is less than the denominator.

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106. What is a proper fraction? Give examples.

The following are proper fractions:

$\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{3}{4}, \frac{3}{7}, \frac{5}{8}, \frac{9}{10}, \frac{8}{9}, \frac{5}{6}.$

107. An Improper Fraction is one whose numerator is equal to, or exceeds the denominator.

Note.—Such a fraction is called *improper* because its value equals or exceeds 1.

The following are improper fractions:

$\frac{3}{2}, \frac{5}{3}, \frac{6}{5}, \frac{8}{7}, \frac{9}{8}, \frac{12}{6}, \frac{14}{7}, \frac{19}{7}.$

108. A Simple Fraction is one whose numerator and denominator are both whole numbers.

Note.—A simple fraction may be either proper or improper.

The following are simple fractions:

$\frac{1}{4}, \frac{3}{2}, \frac{5}{6}, \frac{8}{7}, \frac{9}{2}, \frac{8}{3}, \frac{6}{3}, \frac{7}{5}.$

109. A Compound Fraction is a fraction of a fraction, or several fractions connected by the word *of*.

The following are compound fractions:

$\frac{1}{2}$ of $\frac{1}{4}$, $\frac{1}{3}$ of $\frac{1}{2}$ of $\frac{1}{3}$, $\frac{1}{6}$ of 3, $\frac{1}{7}$ of $\frac{1}{8}$ of 4.

110. A Mixed Number is made up of a whole number and a fraction.

The following are mixed numbers:

$3\frac{1}{2}$, $4\frac{1}{3}$, $6\frac{2}{8}$, $5\frac{3}{5}$, $6\frac{5}{8}$, $8\frac{1}{7}$.

111. A Complex Fraction is one whose numerator or denominator is fractional; or, in which both are fractional.

The following are complex fractions:

$$\frac{\frac{1}{7}}{5}, \quad \frac{2}{19\frac{1}{2}}, \quad \frac{\frac{2}{3}}{\frac{4}{5}}, \quad \frac{45\frac{1}{8}}{69\frac{1}{7}}.$$

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107. What is an improper fraction? Why improper? Give examples

108. What is a simple fraction? Give examples. May it be proper or improper?

109. What is a compound fraction? Give examples.

110. What is a mixed number? Give examples.

111. What is a complex fraction? Give examples.

112. The numerator and denominator of a fraction, taken together, are called the *terms* of the fraction: hence, every fraction has two terms.

FUNDAMENTAL PROPOSITIONS.

113. By multiplying the unit 1, we form all the whole numbers,

2, 3, 4, 5, 6, 7, 8, 9, 10, &c.;

and by dividing the unit 1 by these numbers we form all the fractional units,

$\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{8}$, $\frac{1}{9}$, $\frac{1}{10}$, &c.

Now, since in 1 unit there are 2 halves, 3 thirds, 4 fourths, 5 fifths, 6 sixths, &c., it follows that the *fractional* unit becomes *less* as the denominators are *increased;* hence,

Any fractional unit is such a part of 1, *as* 1 *is of the denominator.*

Thus, $\frac{1}{2}$ is one-half of 1, since 1 is one-half of the denominator 2; $\frac{1}{3}$ is one-third of 1, since 1 is one-third of 3; $\frac{1}{4}$ is one fourth of 1; $\frac{1}{5}$, one-fifth of 1, &c. &c.

114. Let it be required to multiply $\frac{3}{8}$ by 4.

OPERATION.

$$\frac{3}{8} \times 4 = \frac{3 \times 4}{8} = \frac{12}{8}.$$

ANALYSIS.—In $\frac{3}{8}$ there are 3 fractional units, each of which is $\frac{1}{8}$, and these are to be taken 4 times. But 3 things taken 4 times, gives 12 things of the *same kind;* that is, 12 eighths; hence, the product is 4 times as great as the multiplicand: therefore, we have

PROPOSITION I.—*If the numerator of a fraction be multiplied by any number, the fraction will be increased as many times as there are units in the multiplier.*

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112. How many terms has every fraction? What are they?

113. How may all the whole numbers be formed? How may the fractional units be formed? What part of one, is one-half? What part of 1 is any fractional unit?

114. What is proved in proposition I?

EXAMPLES.

1. Multiply $\frac{3}{8}$ by 6, by 7.
2. Multiply $\frac{7}{8}$ by 4, by 9.
3. Multiply $\frac{5}{31}$ by 11, by 12.
4. Multiply $\frac{7}{25}$ by 12, by 14.
5. Multiply $\frac{47}{15}$ by 3, by 4.
6. Multiply $\frac{14}{19}$ by 7, by 9.
7. Multiply $\frac{47}{28}$ by 5, by 10.
8. Multiply $\frac{27}{29}$ by 3, by 11.

115. Let it be required to multiply $\frac{5}{12}$ by 4.

ANALYSIS.—In $\frac{5}{12}$ there are 5 fractional units, each of which is $\frac{1}{12}$. If we divide the denominator by 4, the quotient is 3, and the fractional unit becomes $\frac{1}{3}$, which is 4 *times as great as* $\frac{1}{12}$, because, if $\frac{1}{3}$ be divided into 4 equal parts each part will be $\frac{1}{12}$. If we take this fractional unit 5 times, the result $\frac{5}{3}$ will be 4 times *as great as* $\frac{5}{12}$; therefore, we have

OPERATION.

$$\frac{5}{12} \times 4 = \frac{5}{12 \div 4} = \frac{5}{3}.$$

PROPOSITION II.—*If the denominator of a fraction be divided by any multiplier, the value of the fraction will be increased as many times as there are units in that multiplier.*

EXAMPLES.

1. Multiply $\frac{5}{8}$ by 2, by 4.
2. Multiply $\frac{17}{16}$ by 8, by 4, 2.
3. Multiply $\frac{9}{24}$ by 2, 3, 4, 6, 8.
4. Multiply $\frac{7}{30}$ by 6, by 5, 10, 15.
5. Multiply $\frac{17}{48}$ by 2, 3, 4, 6, 8, 12, 16, and 24.
6. Multiply $\frac{6}{40}$ by 2, 4, 5, 10, 20.
7. Multiply $\frac{7}{35}$ by 7, and by 5.
8. Multiply $\frac{6}{42}$ by 21, 6, 7, 3, and 2.
9. Multiply $\frac{19}{36}$ by 2, 3, 4, 6, 9, and 12.

116. Let it be required to divide $\frac{9}{11}$ by 3.

ANALYSIS.—In $\frac{9}{11}$, there are 9 fractional units, each of which is $\frac{1}{11}$, and these are to be divided by 3. But 9 things, divided by 3, gives 3 *things of the same kind* for a quotient; hence, the quotient is 3 elevenths, a number which is one-third of $\frac{9}{11}$; hence, we have

OPERATION.

$$\frac{9}{11} \div 3 = \frac{9 \div 3}{11} = \frac{3}{11}.$$

PROPOSITION III.—*If the numerator of a fraction be divided by any number, the fraction will be diminished as many times as there are units in the divisor.*

11[5] oved in proposition II?
1 ed in proposition III?

EXAMPLES.

1. Divide $\frac{16}{19}$ by 2, 4, 8, 16.
2. Divide $\frac{14}{11}$ by 2, 7 and 14.
3. Divide $\frac{20}{19}$ by 2, 5, 4 and 10.
4. Divide $\frac{60}{26}$ by 5, 6, 10, 15 and 20.
5. Divide $\frac{18}{19}$ by 2, 3, 6, and 9.
6. Divide $\frac{24}{25}$ by 3, 6, 8, and 12.
7. Divide $\frac{27}{29}$ by 3, 9 and 27
8. Divide $\frac{54}{59}$ by 6, 9, 27 and 54.

117. Let it be required to divide $\frac{9}{11}$ by 3.

OPERATION.

$$\frac{9}{11} \div 3 = \frac{9}{11 \times 3} = \frac{9}{33}.$$

ANALYSIS.—In $\frac{9}{11}$, there are 9 fractional units, each of which is $\frac{1}{11}$. Now, if we multiply the denominator by 3, it becomes 33, and the fractional unit becomes $\frac{1}{33}$, which is one-third part of $\frac{1}{11}$. If, then, we take this fraciional unit 9 times, the result $\frac{9}{33}$ is just one-third part of $\frac{9}{11}$: hence, we have divided the fraction $\frac{9}{11}$ by 3: therefore, we have,

PROPOSITION IV.—*If the denominator of a fraction be multiplied by any divisor, the fraction will be diminished as many times as there are units in that divisor.*

EXAMPLES.

1. Divide $\frac{3}{4}$ by 6, 7 and 8.
2. Divide $\frac{4}{9}$ by 5, 4 and 9.
3. Divide $\frac{14}{17}$ by 3, 4 and 12.
4. Divide $\frac{30}{47}$ by 6, 8 and 11.
5. Divide $\frac{15}{17}$ by 7, 5 and 3.
6. Divide $\frac{14}{27}$ by 7, 8 and 6.
7. Divide $\frac{25}{19}$ by 3, 7 and 11.
8. Divide $\frac{11}{15}$ by 8, 4 and 10.

118. Let it be required to multiply both terms of the fraction $\frac{3}{5}$ by 4.

OPERATION.

$$\frac{3 \times 4}{5 \times 4} = \frac{12}{20}$$

ANALYSIS.—In $\frac{3}{5}$, the fractional unit is $\frac{1}{5}$, and it is taken 3 times. By multiplying the denominator by 4, the fractional unit becomes $\frac{1}{20}$, the value of which is is one-fourth of $\frac{1}{5}$. By multiplying the numerator by 4, we increase the *number* of fractional units taken, 4 times; that

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117. If the denominator of a fraction be multiplied by any number, how will the value of the fraction be effected?

118. If both terms of a fraction be multiplied by any number, how will the value of the fraction be effected?

is, we *increase the number just as many times* as we *decrease the value;* hence, the value of the fraction is not changed: therefore, we have

PROPOSITION V.—*If both terms of a fraction be multiplied by the same number, the value of the fraction will not be changed.*

EXAMPLES.

1. Multiply both terms of the fraction $\frac{7}{8}$ by 4, by 6, and by 5.
2. Multiply both terms of $\frac{8}{11}$ by 5, by 8, by 9, and 11.
3. Multiply both terms of $\frac{16}{19}$ by 7, by 8, and 9.
4. Multiply both terms $\frac{14}{29}$ by 5, 8, 6, and 12.
5. Multiply both terms of $\frac{23}{25}$ by 2, 3, 4, and 5.

119. Let it be required to divide the numerator and denominator of $\frac{6}{15}$ by 3.

OPERATION.

$$\frac{6\div3}{15\div3}=\frac{2}{5}.$$

ANALYSIS.—In $\frac{6}{15}$, the fractional unit is $\frac{1}{15}$, and it is taken 6 times. By dividing the denominator by 3, the fractional unit becomes $\frac{1}{5}$, the value of which is 3 *times as great as* $\frac{1}{15}$. By dividing the numerator by 3, we diminish the number of fractional units taken 3 times; that is, we *diminish the number just as many times as we increase the value:* hence, the value of the fraction is not changed; therefore, we have

PROPOSITION VI.—*If both terms of a fraction be divided by the same number, the value of the fraction will not be changed.*

EXAMPLES.

1. Divide both terms of $\frac{4}{8}$ by 2 and by 4.
2. Divide both terms of $\frac{3}{6}$ by 3.
3. Divide both terms of $\frac{24}{36}$ by 2, 3, 4, 6, and 12.
4. Divide both terms of $\frac{48}{64}$ by 2, 4, 8, and 16.
5. Divide both terms of $\frac{72}{96}$ by 2, 3, 4, 6, and 12.
6. Divide both terms of $\frac{36}{144}$ by 2, 3, 4, 6, and 36.

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119. If both terms of a fraction be divided by any number, how will the value of the fraction be effected?

REDUCTION OF FRACTIONS.

120. Reduction of Fractions is the operation of changing the fractional unit without altering the value of the fraction.

A fraction is in its *lowest terms,* when the numerator and denominator have no common factor.

CASE I.

121. *To reduce a fraction to its lowest terms.*

1. Reduce $\frac{70}{175}$ to its lowest terms.

Analysis.—By inspection, it is seen that 5 is a common factor of the numerator and denominator. Dividing by it, we have $\frac{14}{35}$. We then see that 7 is a common factor of 14 and 35: dividing by it, we have $\frac{2}{5}$. Now, there is no factor common to 2 and 5: therefore, $\frac{2}{5}$ is in its *lowest terms.*

1st. operation.

$5)\frac{70}{175} = \frac{14}{35}$.

$7)\frac{14}{35} = \frac{2}{5}$.

2d. The greatest common divisor of 70 and 175 is 35. (Art. 93); if we divide both terms of the fraction by it, we obtain, $\frac{2}{5}$. The value of the fraction is not changed in either operation, since the numerator and denominator are both divided by the same number (Art. 119): hence, the following

2d operation.

$35)\frac{70}{175} = \frac{2}{5}$.

Rule.—*Divide the numerator and denominator by their common factors, until they become prime with respect to each other.*

Or: 2d. *Divide the numerator and denominator by their greatest common divisor.*

EXAMPLES.

Reduce the following fractions to their lowest terms:

1. Reduce $\frac{7}{49}$.
2. Reduce $\frac{84}{420}$.
3. Reduce $\frac{104}{312}$.
4. Reduce $\frac{1049}{8392}$.
5. Reduce $\frac{275}{440}$.
6. Reduce $\frac{351}{795}$.
7. Reduce $\frac{172}{1118}$.
8. Reduce $\frac{63}{81}$ by 2d. method.
9. Reduce $\frac{315}{405}$ " "
10. Reduce $\frac{1157}{623}$ " "

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120. What is reduction of fractions? When is a fraction in its lowest terms?

121. How do you reduce a fraction to its lowest terms?

11. Reduce $\frac{792}{1386}$ by 2d. meth.
12. Reduce $\frac{374}{1030}$.
13. Reduce $\frac{410}{510}$.
14. Reduce $\frac{345}{1745}$.
15. Reduce $\frac{8343}{9747}$.
16. Reduce $\frac{549}{7143}$.
17. Reduce $\frac{2160}{2340}$.
18. Reduce $\frac{315}{1512}$.
19. Reduce $\frac{10560}{35520}$.
20. Reduce $\frac{6048}{38592}$.
21. Reduce $\frac{864}{21600}$.
22. Reduce $\frac{1080}{66420}$.

CASE II.

122. *To reduce an improper fraction to an equivalent whole or mixed number.*

1. In $\frac{278}{5}$ how many entire units?

ANALYSIS.—Since there are 5 fifths in 1 unit, there will be in 278 fifths as many units 1 as 5 is contained times in 278, viz., 55 and $\frac{3}{5}$ times. Hence, the following

OPERATION.

$$5)\underline{278}$$
$$55\tfrac{3}{5}.$$

RULE.—*Divide the numerator by the denominator, and the quotient will be the equivalent whole or mixed number.*

EXAMPLES.

Reduce the following fractions to whole, or mixed numbers

1. Reduce $\frac{108}{63}$.
2. Reduce $\frac{576}{48}$.
3. Reduce $\frac{1764}{324}$.
4. Reduce $\frac{19900}{800}$.
5. Reduce $\frac{135}{15}$ pounds.
6. Reduce $\frac{2358}{42}$ days.
7. Reduce $\frac{6284}{56}$ yards.
8. Reduce $\frac{4976}{224}$.
9. Reduce $\frac{102409}{160}$ acres.
10. Reduce $\frac{4478}{841}$.
11. Reduce $\frac{17959}{1256}$.
12. Reduce $\frac{526950}{2342}$.
13. Reduce $\frac{4790}{25}$.
14. Reduce $\frac{1512}{108}$.
15. Reduce $\frac{375941}{999}$.
16. Reduce $\frac{3745174}{349}$.

CASE III.

123. *To reduce a mixed number to an equivalent improper fraction.*

1. Reduce $12\frac{5}{7}$ to its equivalent improper fraction.

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122. What is an improper fraction? How do you reduce an improper fraction to its equivalent whole, or mixed number?

123. What is a mixed number? How do you reduce a mixed number to an improper fraction? How do you reduce a whole number to a fraction having a given denominator?

ANALYSIS.—Since in any number there are 7 times as many 7ths as units 1, there will be 84 sevenths in 12: To these add 5 sevenths, and the equivalent fraction becomes 89 sevenths. Hence, the following

OPERATION.

$12 \times 7 = 84$ sevenths.
add 5 sevenths.
gives $12\frac{5}{7} = 89$ sevenths.
Ans. $= \frac{89}{7}$.

RULE.—*Multiply the whole number by the denominator: to the product add the numerator, and place the sum over the given denominator.*

EXAMPLES.

1. Reduce $39\frac{7}{8}$ to its equivalent improper fraction.
2. Reduce $112\frac{9}{10}$ to its equivalent improper fraction.
3. Reduce $427\frac{11}{24}$ to its equivalent improper fraction.
4. Reduce $676\frac{37}{51}$ to an improper fraction.
5. Reduce $367\frac{9}{104}$ to an improper fraction.
6. Reduce $847\ \frac{36}{175}$ to an improper fraction.
7. Reduce $67426\ \frac{368}{879}$ to an improper fraction.
8. How many 200ths in $675\frac{187}{200}$?
9. How many 151ths in $187\frac{41}{151}$?
10. Reduce $149\frac{5}{9}$ to an improper fraction.
11. Reduce $375\frac{94}{99}$ to an improper fraction.
12. Reduce $17494\frac{543}{99999}$ to an improper fraction.
13. Reduce $4834\frac{57}{95}$ to an improper fraction.
14. Reduce $1789\frac{5}{9}$ to an improper fraction.
15. In $125\frac{6}{7}$ yards, how many sevenths of a yard?
16. In $375\frac{3}{4}$ feet, how many fourths of a foot?
17. In $464\frac{19}{63}$ hogsheads, how many sixty-thirds of a hogshead?
18. In $96\frac{11}{640}$ acres, how many 640ths of an acre?
19. In $984\frac{41}{112}$ pounds, how many 112ths of a pound?
20. In $35\frac{72}{366}$ years, how many 366ths of a year?
21. How many one hundred and thirty-fifths are there in the mixed number $87\frac{41}{135}$?
22. Place 4 sevens in such a manner that they shall express the number 78.
23. By means of 5 threes write a number that is equal to 334.

CASE IV.

123.* *To reduce a whole number to a fraction having a given denominator:*

1. Reduce 17 to a fraction of which the denominator shall be 5.

OPERATION.

$17 \times 5 = 85$

$17 = \frac{85}{5}$

ANALYSIS.—There are 17 times as many fifths in 17 as there are in 1. In 1, there are 5 fifths; therefore, in 17 there are 17 times 5 fifths or 85 fifths; hence,

RULE.—*Multiply the whole number by the denominator, and write the product over the required denominator.*

EXAMPLES.

1. Change 18 to a fraction whose denominator shall be 7.
2. Change 25 to a fraction whose denominator shall be 12.
3. Change 19 to a fraction whose denominator shall be 8.
4. Change 29 to a fraction whose denominator shall be 14.
5. Change 65 to a fraction whose denominator shall be 37.
6. Reduce 145 to a fraction having 9 for its denominator.
7. Reduce 450 to a fraction having 12 for its denominator.
8. Reduce 327 to a fraction having 36 for its denominator.
9. Reduce 97 to a fraction having 128 for its denominator.
10. Reduce 167 to a fraction whose denominator shall be 89.
11. Reduce 325 to a fraction whose denominator shall be 75.

CASE V.

124. *To reduce a compound fraction to a simple fraction.*

1. What is the equivalent fraction of $\frac{3}{5}$ of $\frac{4}{7}$?

OPERATION.

$\frac{3 \times 4}{5 \times 7} = \frac{12}{35}$

ANALYSIS.—Three-fifths of $\frac{4}{7}$ is three times $\frac{1}{5}$ of $\frac{4}{7}$: 1 fifth of $\frac{4}{7}$ is $\frac{4}{35}$ (Art. 117); and 3 times $\frac{4}{35}$ is $\frac{12}{35}$ (Art. 114); hence, $\frac{3}{5}$ of $\frac{4}{7} = \frac{12}{35}$; therefore,

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123.* How do you reduce a whole number to a fraction having a given denominator?

124. What is a compound fraction? How do you reduce a compound fraction to a simple fraction?

RULE.—*Multiply the numerators together for a new numerator, and the denominators together for a new denominator.*

NOTE.—1. If there are mixed numbers, reduce them to their quivalent improper fractions.

2. Cancel every factor common to the numerator and denominator before multiplying.

EXAMPLES.

1. Reduce $\frac{3}{4}$ of $\frac{5}{6}$ of $\frac{2}{3}$ to a simple fraction.
2. Reduce $\frac{2}{5}$ of $\frac{7}{9}$ of $\frac{3}{4}$ to a simple fraction.
3. Reduce $\frac{2}{3}$ of $\frac{3}{7}$ of $2\frac{1}{4}$ to a simple fraction.
4. Change $\frac{2}{9}$ of $\frac{3}{5}$ of $\frac{5}{8}$ of $3\frac{1}{3}$ to a simple fraction.
5. Change $\frac{9}{10}$ of $\frac{2}{3}$ of $\frac{7}{8}$ of $\frac{5}{14}$ to a simple fraction.
6. What is the value of $\frac{1}{4}$ of $\frac{1}{2}$ of $\frac{3}{4}$ of $12\frac{1}{2}$?
7. What is the value of $\frac{2}{7}$ of $\frac{5}{6}$ of $4\frac{1}{5}$?
8. What is the value of $\frac{9}{10}$ of $7\frac{1}{3}$ of $5\frac{5}{12}$?
9. Reduce $\frac{7}{8}$ of $9\frac{1}{3}$ of $6\frac{3}{7}$ of $2\frac{4}{5}$ to a whole or mixed number.
10. Reduce $\frac{9}{14}$ of $\frac{7}{12}$ of $21\frac{7}{9}$ to a whole or mixed number.
11. Reduce $\frac{3}{4}$ of $\frac{5}{6}$ of $\frac{5}{9}$ of $\frac{27}{101}$ of $\frac{5}{13}$ to a simple fraction.
12. Reduce $\frac{41}{110}$ of $\frac{3}{19}$ of $\frac{41}{108}$ of $\frac{3}{7}$ to a simple fraction.
13. Reduce $3\frac{5}{8}$ of $\frac{5}{9}$ of $\frac{27}{301}$ of 49 to a simple fraction.

CASE VI.

125. *To reduce fractions of different denominators to equivalent fractions that shall have a common denominator.*

1. Reduce $\frac{2}{3}$, $\frac{4}{5}$ and $\frac{3}{4}$ to a common denominator.

OPERATION.

$2 \times 5 \times 4 = 40$ 1st num.
$4 \times 3 \times 4 = 48$ 2d num.
$3 \times 5 \times 3 = 45$ 3d num.
$3 \times 5 \times 4 = 60$ denom.

ANALYSIS.—Multiplying both terms of the first fraction by 20, the product of the other denominators, gives $\frac{40}{60}$. Multiplying both terms of the second fraction by 12, the product of the other denominators, gives $\frac{48}{60}$. Multiplying both terms of the third by 15, the product of the other denominators, gives $\frac{45}{60}$. In each case both terms

125. How do you reduce fractions of different denominators to equivalent fractions having a common denominator? NOTE 1.—What reductions are first made? 2. When the numbers are small, how may the work be done? 3. How may the work often be shortened?

of the fraction have been multiplied by the same number; therefore, the value is not changed (Art. 118): hence, the following

RULE.—*Multiply the numerator of each fraction by all the denominators except its own, for the new numerators, and all the denominators together for a common denominator.*

NOTE.—1. Before multiplying, reduce to simple fractions when necessary.

2. When the numbers are small, the work may be performed mentally; thus,

$\frac{1}{3}, \frac{1}{4}, \frac{2}{5} = \frac{20}{60}, \frac{15}{60}, \frac{24}{60}$; and $\frac{2}{5}, \frac{1}{2}, \frac{3}{4} = \frac{16}{40}, \frac{20}{40}, \frac{30}{40}$.

EXAMPLES.

Reduce the following fractions to common denominators:

1. Reduce $\frac{3}{4}$, $5\frac{1}{3}$ and $\frac{6}{7}$.
2. Reduce $\frac{3}{5}$, $\frac{2}{3}$, $\frac{1}{7}$ and $\frac{1}{2}$ of 5.
3. Reduce $9\frac{1}{2}$, $4\frac{1}{3}$, $2\frac{3}{4}$ and $\frac{4}{5}$.
4. Reduce $\frac{2}{3}$, $\frac{7}{8}$, $\frac{5}{6}$, $\frac{1}{2}$ and $2\frac{1}{4}$.
5. Reduce $2\frac{1}{2}$ of 3, $\frac{6}{7}$, $\frac{4}{9}$ and $\frac{3}{5}$.
6. Reduce $2\frac{1}{2}$ of $3\frac{1}{7}$ of $\frac{2}{3}$, and $6\frac{1}{3}$ of $\frac{3}{2}$.
7. Reduce $\frac{3}{7}$ of $\frac{2}{3}$ of $\frac{5}{8}$ and $\frac{3}{4}$ of $\frac{5}{7}$ of $\frac{3}{5}$.
8. Reduce $4\frac{8}{9}$, $2\frac{1}{3}$, $5\frac{1}{2}$ and 6.
9. Reduce $5\frac{1}{5}$, $\frac{6}{5}$, $3\frac{1}{2}$ and $3\frac{2}{3}$.
10. Reduce $\frac{3}{4}$ of $5\frac{1}{3}$, $\frac{1}{2}$ of $3\frac{1}{7}$ and $\frac{7}{12}$ of $8\frac{1}{2}$.
11. Reduce $6\frac{1}{3}$ of 2, $\frac{3}{7}$, $5\frac{8}{7}$ and $\frac{1}{3}$.

NOTE.—3. We may often shorten the work by multiplying the numerator and denominator of each fraction by such a number as will make the denominators the same in all.

Reduce the following fractions to common denominators by this method:

1. Reduce $\frac{3}{4}$, $\frac{7}{12}$, $\frac{1}{2}$ and $\frac{5}{6}$ to a common denominator.
2. Reduce $\frac{6}{7}$, $\frac{8}{21}$ and $\frac{2}{3}$ to a common denominator.
3. Reduce $4\frac{1}{5}$, $\frac{9}{10}$ and $7\frac{1}{4}$ to a common denominator.
4. Reduce $10\frac{5}{9}$, $\frac{5}{6}$ and $7\frac{1}{3}$ to a common denominator.
5. Reduce $6\frac{1}{5}$, $\frac{5}{6}$ and $7\frac{1}{3}$ to a common denominator.
6. Reduce $\frac{4}{5}$, $\frac{7}{8}$, $14\frac{1}{2}$ and $3\frac{3}{4}$ to a common denominator.
7. Reduce $\frac{7}{12}$, $\frac{8}{9}$, $2\frac{5}{6}$ and $1\frac{3}{8}$ to a common denominator.
8. Reduce $\frac{6}{7}$, $\frac{1}{6}$, $\frac{16}{21}$ and $\frac{2}{3}$ to a common denominator.
9. Reduce $\frac{9}{11}$, $\frac{3}{4}$, $\frac{19}{22}$ and $\frac{1}{2}$ to a common denominator.
10. Reduce $2\frac{1}{2}$, $5\frac{1}{6}$, $\frac{9}{10}$ and $4\frac{5}{12}$ to a common denominator.

CASE VII.

125*. *To reduce fractions to their least common denominator.*

The least common denominator of two or more fractions is the number which contains only the prime factors of their denominators. Hence, before beginning the operation, reduce every fraction to a simple fraction and to its *lowest terms.*

1. Reduce $\frac{3}{4}$, $\frac{5}{6}$ and $\frac{4}{9}$ to their least common denominator.

OPERATION.

$(36 \div 4) \times 3 = 27$ 1st numerator.

$(36 \div 6) \times 5 = 30$ 2d numerator.

$(36 \div 9) \times 4 = 16$ 3d numerator.

$$\begin{array}{r|l} 2 & 4 \;.\; 6 \;.\; 9 \\ \hline 3 & 2 \;.\; 3 \;.\; 9 \\ \hline & 2 \;.\; 1 \;.\; 3 \end{array}$$

$2 \times 3 \times 2 \times 3 = 36$, least common denominator:

therefore, the fractions, reduced to their least common denominator, are

$$\tfrac{27}{36},\ \tfrac{30}{36},\ \text{and}\ \tfrac{16}{36}.$$

Hence, the following

RULE.—I. *Find the least common multiple of the denominators* (Art. 98): *this will be the least common denominator of the fractions.*

II. *Divide the least common denominator by the denominator of each fraction, separately; multiply the quotient by the numerator and place the product over the least common denominator: the results will be the new and equivalent fractions.*

EXAMPLES.

1. Reduce $\frac{3}{8}$, $\frac{4}{7}$ and $\frac{5}{12}$ to their least common denominator.
2. Reduce $\frac{5}{14}$, $\frac{3}{7}$ and $\frac{6}{21}$ to their least common denominator.
3. Reduce $2\frac{3}{4}$, $\frac{5}{16}$ and $\frac{9}{32}$ to their least common denominator.
4. Reduce $5\frac{3}{8}$, $4\frac{5}{12}$ and $\frac{7}{24}$ to their least common denominator.
5. Reduce $8\frac{7}{15}$, $\frac{2}{5}$ and $\frac{7}{30}$ to their least common denominator.
6. Reduce $9\frac{8}{11}$, $\frac{3}{22}$ and $\frac{5}{33}$ to their least common denominator.

125*. What is the least common denominator of two or more fractions? How do you find the least common denominator of two or more fractions?

7. Reduce $2\frac{1}{2}$, $3\frac{5}{21}$ and $\frac{1}{14}$ to their least common denominator.

8. Reduce $3\frac{5}{12}$, $\frac{7}{6}$, $\frac{3}{8}$ and $\frac{9}{16}$ to their least common denominator.

9. Reduce $\frac{8}{9}$, $\frac{5}{27}$ and $\frac{7}{36}$ to their least common denominator.

10. Reduce $4\frac{6}{13}$, $7\frac{3}{26}$ and $\frac{5}{39}$ to their least common denominator.

11. Reduce $3\frac{4}{9}$, $6\frac{5}{18}$ and $1\frac{1}{36}$ to their least common denominator.

12. Reduce $6\frac{2}{5}$, $8\frac{7}{10}$ and $2\frac{9}{20}$ to their least common denominator.

13. Reduce $5\frac{4}{11}$, $6\frac{3}{22}$ and $\frac{2}{33}$ to their least common denominator.

14. Reduce $\frac{9}{17}$, $2\frac{3}{34}$ and $1\frac{5}{68}$ to their least common denominator.

15. Reduce $5\frac{7}{9}$, $6\frac{5}{18}$, $\frac{7}{36}$ and $\frac{1}{72}$ to their least common denominator.

ADDITION OF COMMON FRACTIONS.

126. THE SUM of two or more fractions is a number which contains the unit 1 as many times as it is contained in the fractions separately.

ADDITION OF FRACTIONS is the operation of finding the sum of two or more fractions. There are two cases:

1*st. When the fractions have the same unit.*

2*d. When they have different units.*

CASE I.

127. *When the fractions have the same unit.*

1. What is the sum of $\frac{1}{2}$, $\frac{3}{2}$, $\frac{6}{2}$ and $\frac{3}{2}$?

ANALYSIS.—In this example, the unit of the fraction is 1, and the fractional unit $\frac{1}{2}$. There is one $\frac{1}{2}$ in the first, 3 one halves in the second, 6 in the third, and 3 in the fourth: hence, there are 13 halves in all, equal to $6\frac{1}{2}$.

OPERATION.

$1 + 3 + 6 + 3 = 13$

hence, $\frac{13}{2} = 6\frac{1}{2}$ sum.

2. What is the sum of $\frac{1}{2}$£ and $\frac{2}{3}$£?

ANALYSIS.—*The unit of both fractions* is 1£. In the first, the fractional unit is $\frac{1}{2}$£, and in the second, $\frac{1}{3}$£. These fractional units, being different, cannot be expressed.

OPERATION.

$\frac{1}{2}£ = \frac{3}{6}£$

$\frac{2}{3}£ = \frac{4}{6}£$

$\frac{3}{6}£ + \frac{4}{6}£ = £\frac{7}{6} = £1\frac{1}{6}$.

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126. What is the sum of two or more fractions? What is addition of fractions? How many cases are there? What are they?

127. How do you add fractions which have the same unit?

in one collection. But $\frac{1}{2}£ = \frac{3}{6}£$ and $\frac{2}{3}£ = \frac{4}{6}£$, in each of which the *fractional unit* is $\frac{1}{6}£$: hence, their sum is $\frac{7}{6}£ = £1\frac{1}{6}$.

NOTE.—*Only units of the same value, whether fractional or integral, can be expressed in the same collection.*

From the above analysis, we have the following

RULE.—I. *When the fractions have the same denominator, add their numerators, and place the sum over the common denominator:*

II. *When they have not the same denominator, reduce them to a common denominator, and then add as before.*

NOTE.—1. After the addition is performed, reduce every result to its simplest form; that is, improper fractions to mixed numbers, and the fractional parts to their lowest terms.

2. It often abridges the operations in fractions to reduce them to their least common denominators, before adding (Art. 125*.)

EXAMPLES.

1. Add $\frac{3}{5}$, $\frac{9}{5}$, $\frac{8}{5}$ and $\frac{7}{5}$.
2. Add $\frac{5}{9}£$, $\frac{7}{9}£$, $\frac{3}{9}£$ and $\frac{11}{9}£$.
3. Add $\$\frac{1}{11}$, $\$\frac{6}{11}$, $\$\frac{14}{11}$ and $\$\frac{8}{11}$.
4. Add $\frac{19}{15}$, $\frac{17}{15}$, $\frac{18}{15}$, and $\frac{7}{15}$.
5. Add $\frac{3}{7}$, $\frac{9}{7}$, $\frac{5}{7}$, $\frac{19}{7}$ and $\frac{11}{7}$.
6. Add $\frac{5}{12}$, $\frac{4}{12}$, $\frac{3}{12}$, $\frac{1}{12}$ and $\frac{20}{12}$.
7. Add $\frac{1}{4}$, $\frac{2}{5}$ and $\frac{9}{10}$.
8. Add $\frac{2}{3}$, $\frac{4}{5}$, $\frac{5}{6}$ and $\frac{7}{15}$.
9. Add $\frac{3}{4}$, $\frac{2}{7}$, $\frac{5}{8}$ and $\frac{9}{14}$.
10. Add $\frac{5}{9}$, $\frac{7}{12}$, $\frac{5}{18}$ and $\frac{21}{17}$.
11. Add $\frac{7}{8}$, $\frac{7}{12}$, $\frac{13}{16}$, $\frac{11}{18}$ and $\frac{19}{24}$.
12. Add $\frac{3}{4}$, $\frac{5}{8}$, $\frac{9}{16}$, $\frac{5}{32}$ and $\frac{15}{64}$.
13. Add $\frac{1}{16}$, $\frac{3}{7}$, $\frac{2}{8}$ and $\frac{4}{9}$.
14. Add $\frac{1}{5}$, $4\frac{1}{3}$ and $\frac{2}{5}$.
15. Add $\frac{3}{11}$, $\frac{5}{12}$, $\frac{13}{4}$ and $\frac{2}{3}$.
16. Add $\frac{9}{17}$, $\frac{5}{12}$, $\frac{2}{5}$ and $\frac{7}{8}$.*

NOTE.—Reduce each fraction to its least common denominator before adding.

17. What is the sum of $\frac{1}{5}$ and $\frac{1}{7}$?

ANALYSIS.—Reducing to a common denominator, we find the fractions to be $\frac{7}{35}$ and $\frac{5}{35}$, and their sum to be $\frac{12}{35}$. That is,

OPERATION.

$$\frac{1}{5}+\frac{1}{7}=\frac{7}{35}+\frac{5}{35}=\frac{12}{35}.$$

128. *The sum of two fractions whose numerators are each* 1, *is equal to the sum of their denominators divided by their product.*

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128. What is the sum of two fractions equal to when each numerator is equal to 1?

18. What is the sum of $\frac{1}{2}$ and $\frac{1}{3}$? of $\frac{1}{2}$ and $\frac{1}{5}$? of $\frac{1}{7}$ and $\frac{1}{9}$? of $\frac{1}{9}$ and $\frac{1}{10}$?

19. What is the sum of $\frac{1}{12}$ and $\frac{1}{10}$? of $\frac{1}{15}$ and $\frac{1}{16}$? of $\frac{1}{6}$ and $\frac{1}{9}$? of $\frac{1}{8}$ and $\frac{1}{5}$?

20. What is the sum of $12\frac{3}{5}$, $11\frac{2}{3}$ and $15\frac{5}{7}$?

OPERATION.

Whole Numbers.	*Fractions.*
$12+11+15=38$	$\frac{3}{5}+\frac{2}{3}+\frac{5}{7}=\frac{63}{105}+\frac{70}{105}+\frac{75}{105}=\frac{208}{105}=1\frac{103}{105}$:
then,	$38+1\frac{103}{105}=39\frac{103}{105}$. *Ans.*

NOTE.—When there are mixed numbers, *add the whole numbers and the fractions separately, and then add their sums.*

Find the sums of the following fractions:

21. Add $1\frac{9}{4}$, $3\frac{1}{7}$ and $\frac{1}{2}$ of 7.
22. Add $3\frac{33}{25}$, $7\frac{4}{5}$, $\frac{17}{24}$ and $2\frac{11}{15}$.
23. Add $2\frac{3}{5}$, $4\frac{7}{8}$ and $\frac{3}{4}$ of $5\frac{3}{10}$.
24. Add $12\frac{3}{4}$, $9\frac{2}{3}$, $\frac{4}{7}$ of $6\frac{1}{2}$.
25. Add $\frac{9}{10}$ of $6\frac{7}{8}$ and $\frac{4}{7}$ of $7\frac{1}{2}$.
26. Add $\frac{1}{5}$ of $9\frac{3}{8}$ and $\frac{2}{3}$ of $4\frac{5}{8}$.
27. Add $\frac{3}{5}$, $\frac{9}{10}$ of $\frac{5}{11}$ of 8 and $2\frac{1}{2}$.
28. Add $4\frac{3}{8}$, $\frac{9}{11}$ of $\frac{1}{6}$ of $15\frac{1}{2}$.
29. Add $3\frac{5}{7}$, $4\frac{5}{8}$ and $16\frac{5}{11}$.
30. Add $3\frac{5}{7}$, $4\frac{5}{8}$ and $\frac{1}{3}$ of 16.
31. Add $6\frac{3}{4}$, $13\frac{3}{7}$, $18\frac{1}{8}$ and $132\frac{1}{6}$.
32. Add $12\frac{5}{7}$, $26\frac{8}{9}$, and $40\frac{13}{18}$.

33. Bought a cord of wood for $2\frac{5}{8}$ dollars; a barrel of flour for $\$9\frac{5}{6}$; and some pork for $\$5\frac{3}{4}$: what was the entire cost?

34. A person travelled in one day $35\frac{1}{3}$ miles; the next, $28\frac{4}{7}$ miles; and the next, $25\frac{7}{21}$ miles: how many miles did he travel in the three days?

35. A grocer bought 4 firkins of butter, weighing respectively $54\frac{3}{4}$, $55\frac{3}{8}$, $51\frac{7}{16}$ and $50\frac{21}{32}$ pounds: what was their entire weight?

36. I paid for groceries at one time $\frac{7}{12}$ of a dollar; at another, $3\frac{4}{9}$ dollars; at another, $7\frac{3}{4}$ dollars; and at another, $5\frac{1}{6}$ dollars: what was the whole amount paid?

37. A merchant had three pieces of Irish linen; the first piece contained $22\frac{5}{6}$ yards; the second $20\frac{7}{8}$ yards; and the third $21\frac{4}{5}$ yards: how many yards in the three pieces?

38. A man sold 5 loads of hay; the first weighed $18\frac{7}{12}$*cwt.*; the second $19\frac{11}{20}$*cwt.*; the third $19\frac{5}{9}$*cwt.*; the fourth $21\frac{11}{15}$*cwt.*; and the fifth $20\frac{13}{18}$*cwt.*: what was the weight of the whole?

39. A farmer has three fields; the first contains $17\frac{3}{5}$ acres; the second $25\frac{2}{9}$ acres; and the third $46\frac{8}{15}$ acres: how many acres in the three fields?

40. A man sold $112\frac{6}{7}$ bushels of wheat for $250\frac{4}{5}$ dollars; $9\frac{5}{12}$ bushels of corn for $62\frac{3}{8}$ dollars; $225\frac{9}{14}$ bushels of oats for $104\frac{7}{9}$ dollars: how many bushels of grain did he sell, and how much did he receive for the whole?

CASE II.

129. *When the fractions have different units.*

1. What is the sum of $\frac{4}{5}lb.$ and $\frac{3}{4}oz.$?

ANALYSIS.—In $\frac{4}{5}lb.$ there are $\frac{64}{5}oz.$ (Art. 41). Then, the units of the fractions being the same, viz., $1oz.$, we reduce to a common denominator and add, and obtain $13\frac{11}{20}oz.$

OPERATIONS.

$$\frac{4}{5}lb. = \frac{4}{5} \times 16oz. = \frac{64}{5}oz.$$

$$\frac{64}{5}oz. + \frac{3}{4}oz. = \frac{256}{20}oz. + \frac{15}{20}oz. = \frac{271}{20}oz. = 13\frac{11}{20}oz$$

SECOND METHOD.—Three-fourths of an ounce is equal to $\frac{3}{64}lb.$ (Art. 41). Then, by adding, we find the sum to be

$$\frac{3}{4}oz. = \frac{3}{4} \times \frac{1}{16}lb. = \frac{3}{64}lb.$$

$$\frac{4}{5}lb. + \frac{3}{64}lb. = \frac{256}{320}lb. + \frac{15}{320}lb. = \frac{271}{320}lb.$$

$$\frac{271}{320}lb. = 13\frac{11}{20}oz. = 13oz.\ 8\frac{4}{5}dr.$$

THIRD METHOD.—Find the value of each fractional part in terms of integers of the lower denominations, and then add.

$$\frac{4}{5}lb. = \frac{4}{5} \times 16oz. = \frac{64}{5}oz. = 12oz.\ 12\frac{4}{5}dr.$$

$$\frac{3}{4}oz. = \frac{3}{4} \times 16dr. = \frac{48}{4}dr. = \quad 12$$

$$\text{Sum} \quad 13 \quad 8\frac{4}{5}$$

RULE.—*Reduce the given fractions to the same unit, and then add as in Case* 1.

Or: *Reduce the fractions separately to integers of lower denominations, and then add the denominate numbers.*

EXAMPLES.

1. Add $\frac{3}{8}$ of a yard to $\frac{5}{9}$ of an inch.
2. Add together $\frac{1}{3}$ of a week, $\frac{1}{4}$ of a day, and $\frac{1}{2}$ of an hour.
3. Add $\frac{3}{4}cwt.$, $\frac{42}{2}lb.$, $13oz.$, $\frac{1}{2}cwt.$ and $6lb.$ together.
4. Add $\frac{1}{5}$ of a pound troy to $\frac{1}{8}$ of an ounce.

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129. How do you add fractions when they have different units?

5. Add $\frac{4}{9}$ of a ton to $\frac{5}{12}$ of a hundred weight.

6. Add $\frac{5}{9}$ of a chaldron to $\frac{3}{7}$ of a bushel.

7. What is the sum of $\frac{3}{4}$ of a tun, and $\frac{3}{5}$ of a hogshead of wine?

8. Add $\frac{1}{5}$ of $\frac{3}{4}$ of a year, $\frac{3}{8}$ of $\frac{5}{9}$ of a day, and $\frac{7}{9}$ of $\frac{2}{3}$ of $\frac{3}{8}$ of $19\frac{1}{2}$ hours, together.

9. Add $\frac{5}{8}$ of an acre, $\frac{3}{5}$ of 19 square feet, and $\frac{3}{7}$ of a square inch, together.

10. What is the sum of $\frac{1}{7}$ of a yard, $\frac{1}{7}$ of a foot, and $\frac{1}{7}$ of an inch?

11. What is the sum of $\frac{2}{3}$ of a £, and $\frac{5}{9}$ of a shilling?

12. What is the sum of $\frac{1}{4}$ of a week, $\frac{1}{3}$ of a day, $\frac{1}{2}$ of an hour, and $\frac{3}{4}$ of a minute?

13. Add together $\frac{7}{8}$ of a mile, $\frac{2}{3}$ of a yard, and $\frac{3}{4}$ of a foot.

14. What is the sum of $\frac{3}{5}$ of a year, $\frac{1}{3}$ of a week, and $\frac{1}{8}$ of a day?

15. Add $\frac{4}{7}$ of a ton to $\frac{5}{6}$ of a hundred weight.

16. Add $\frac{3}{5}$*lb.* troy, $\frac{1}{6}$*oz.* and $\frac{5}{8}$*pwt.*

17. Add together $\frac{3}{19}$ of a circle, $3\frac{5}{8}$ signs, $\frac{2}{3}$ of a degree, and $\frac{2}{9}$ of $5\frac{1}{7}$ minutes.

18. What is the sum of $\frac{7}{8}$*yd.*, $\frac{3}{5}$ of $\frac{5}{8}$*qr.* and $3\frac{1}{3}$*na.*?

19. Add $\frac{3}{16}$ of a cord, $\frac{5}{9}$ cubic feet, and $\frac{2}{9}$ of $\frac{1}{2}$ of $24\frac{3}{7}$ cubic feet.

20. What is the sum of $\frac{3}{4}$ of $\frac{1}{2}$ of 4 cords, $\frac{5}{6}$ of $\frac{9}{16}$ of 15 cord feet, and $\frac{5}{9}$ of $31\frac{1}{2}$ cubic feet?

21. Add $\frac{5}{6}$ of 3 ell English to $\frac{5}{12}$ of a yard.

22. Add together $\frac{4}{5}$ of 3*A.* 1*R.* 20*P.*, $\frac{3}{8}$ of an acre, and $\frac{3}{4}$ of 3*R.* 15*P.*

23. What is the sum of $\frac{7}{12}$ of a ton, $\frac{3}{10}$ of a *cwt.*, and $\frac{5}{12}$ of an ounce?

24. What is the sum of $\frac{1}{2}$ of $\frac{3}{5}$ of a mile, $\frac{3}{5}$ of a furlong, $\frac{4}{33}$ of a rod and $\frac{1}{2}$ of a foot?

25. What is the sum of $\frac{1}{25}$ of a year, $\frac{5}{12}$ of a week, $\frac{7}{9}$ of a day and $\frac{3}{4}$ of an hour?

SUBTRACTION.

130. The difference between two fractions is such a number as added to the less will give the greater.

SUBTRACTION of Common Fractions is the operation of finding the difference between two fractional numbers. There are two cases:

1. *When the fractions have the same unit:* 2d. *When the fractions have different units.*

CASE I.

131. *When the fractions have the same unit.*

1. What is the difference between $\frac{3}{4}$ and $\frac{1}{4}$?

ANALYSIS.—The unit of both fractions is the same, being the abstract unit 1. The *fractional unit* is also the same, being $\frac{1}{4}$ in each; hence, the difference of the fractions is equal to the difference of the fractional units, which is $\frac{2}{4}$.

OPERATION.

$\frac{3}{4} - \frac{1}{4} = \frac{2}{4}$.

2. What is the difference between $\frac{1}{2}$*lb.* and $\frac{2}{3}$ of a pound?

ANALYSIS.—The unit in both fractions is 1*lb.* The fractional unit of the first is $\frac{1}{2}$*lb.* and of the second $\frac{1}{3}$*lb.* Reducing to the same fractional unit, we have $\frac{12}{18}$*lb.* and $\frac{12}{18}$*lb.*, the difference of which is $\frac{3}{18}$*lb.*; hence,

OPERATION.

$\frac{4}{6}-\frac{3}{6}=\frac{12}{18}-\frac{12}{18}=\frac{3}{18}$*lb.*

RULE I.—*If the fractional unit is the same in both, subtract the less numerator from the greater, and place the difference over the common denominator.*

II. *When the fractional units are different, reduce to a common denominator: then subtract the less numerator from the greater, and place the difference over the common denominator.*

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130. What is the difference between two fractions? What is Subtraction of Common Fractions? How many cases are there? What are they?

131. How do you make the subtraction when the fractions have the same unit?

NOTE.—Reduce each fraction to a simple form and to its lowest terms before reducing to a common denominator.

EXAMPLES.

1. From $\frac{3}{7}$ take $\frac{1}{7}$.
2. From $\frac{14}{13}$ take $\frac{11}{13}$.
3. From $\frac{16}{25}$ take $\frac{12}{25}$.
4. From $\frac{234}{325}$ take $\frac{104}{305}$.
5. From $\frac{6}{7}$ take $\frac{4}{5}$.
6. From $\frac{11}{12}$ take $\frac{13}{16}$.
7. From $\frac{14}{15}$ take $\frac{12}{13}$.
8. From $37\frac{11}{15}$ take $\frac{1}{3}$ of $5\frac{5}{6}$.
9. From $\frac{3}{4}$ take $\frac{5}{9}$.
10. From $\frac{7}{8}$ take $\frac{5}{18}$.
11. From 25 take $\frac{11}{15}$.
12. From $\frac{6}{15}$ of 3 take $\frac{1}{3}$ of $\frac{4}{9}$.
13. From $\frac{1}{7}$ of $\frac{3}{6}$ of 7 take $\frac{3}{8}$.
14. From $3\frac{5}{8}$ take $\frac{2}{3}$ of $\frac{7}{8}$.
15. From $\frac{2}{3}$ of 15 take $\frac{4}{5}$ of 3.
16. From $7\frac{1}{3}$ of 2 take $\frac{1}{5}$ of $\frac{2}{6}$.

17. To what fraction must I add $\frac{3}{5}$ that the sum may be $\frac{5}{8}$?

18. What number added to $1\frac{7}{9}$ will make 5?

19. What number is that to which if $7\frac{3}{4}$ be added the sum will be $17\frac{3}{5}$?

20. From the sum of $3\frac{5}{8}$ and $10\frac{4}{5}$ take the difference of $25\frac{1}{4}$ and $17\frac{11}{20}$.

21. What number is that from which if you subtract $\frac{1}{2}$ of $\frac{1}{8}$ of a unit, and to the remainder add $\frac{3}{5}$ of $\frac{7}{8}$ of a unit, the sum will be 9?

22. If I buy $\frac{2}{3}$ of $\frac{4}{5}$ of a vessel, and sell $\frac{1}{2}$ of $\frac{5}{6}$ of my share, how much of the whole vessel have I left?

23. A man bought a horse for $\frac{1}{3}$ of $\frac{4}{5}$ of $\frac{9}{10}$ of \$500, and sold him again for $\frac{6}{7}$ of $\frac{1}{5}$ of $\frac{2}{3}$ of \$1680: what did he gain by the bargain?

24. Bought wheat at $1\frac{7}{8}$ dollars a bushel, and sold it for $2\frac{1}{4}$ dollars a bushel: what did I gain on a bushel?

25. From a barrel of cider containing $31\frac{1}{2}$ gallons, $12\frac{4}{7}$ gallons were drawn: how much was there left?

26. Bought $10\frac{2}{3}$ cords of wood at one time, and $24\frac{5}{6}$ cords at another; after using $16\frac{7}{9}$ cords, how much remained?

27. A merchant bought two firkins of butter, one containing $54\frac{9}{10}$ pounds, and the other $56\frac{11}{12}$ pounds; he sold $43\frac{13}{15}$ pounds at one time, and $34\frac{4}{5}$ pounds at another: how much had he left?

28. A man having $\$50\frac{1}{2}$, expended $\$15\frac{7}{16}$ for dry goods, and $\$12\frac{7}{8}$ for groceries: how much had he left?

29. A boy having $\frac{3}{4}$ of a dollar, gave $\frac{1}{5}$ of it for an inkstand, and $\frac{1}{4}$ of it for a slate: how much had he left?.

30. Bought two pieces of cloth, one containing $27\frac{4}{9}$ yards, the other $32\frac{1}{8}$ yards, from which I sold $40\frac{17}{18}$ yards: how much had I left?

132. 1. What is the difference between $\frac{1}{6}$ and $\frac{1}{8}$?

ANALYSIS.—Reducing both fractions to a common denominator and subtracting, we find the difference to be $\frac{2}{48}$; that is,

OPERATION.

$\frac{1}{6} - \frac{1}{8} = \frac{8}{48} - \frac{6}{48} = \frac{2}{48}$.

The difference between two fractions, each of whose numerators is 1, *is equal to the difference of the denominators divided by their product.*

2. From $\frac{1}{9}$ take $\frac{1}{15}$.
3. From $\frac{1}{14}$ take $\frac{1}{15}$.
4. From $\frac{1}{19}$ take $\frac{1}{20}$.
5. From $\frac{1}{27}$ take $\frac{1}{30}$.

133. 1. What is the difference between $16\frac{1}{5}$ and $3\frac{1}{3}$?

ANALYSIS.—Since we cannot take $\frac{5}{15}$ from $\frac{3}{15}$, we borrow $1 = \frac{15}{15}$ from the whole number of the minuend, which added to $\frac{3}{15}$, gives $\frac{18}{15}$: then $\frac{5}{15}$ from $\frac{18}{15}$ leaves $\frac{13}{15}$. We must now carry 1 to the next figure of the subtrahend, and say 4 from 16 leaves 12. Hence, to subtract one mixed number from another,

OPERATION.

$$\begin{aligned} 16\tfrac{1}{5} &= 16\tfrac{3}{15} \\ 3\tfrac{1}{3} &= \underline{\;3\tfrac{5}{15}} \\ & \quad 12\tfrac{13}{15} \end{aligned}$$

Subtract the fractional part from the fractional part, and the integral part from the integral part.

1. What is the difference between $14\frac{4}{7}$ and $12\frac{6}{19}$?
2. What is the difference between $115\frac{3}{8}$ and $39\frac{7}{8}$?
3. What is the difference between $78\frac{3}{16}$ and $4\frac{7}{32}$?
4. What is the difference between $48\frac{5}{19}$ and $41\frac{15}{38}$?
5. What is the difference between $287\frac{5}{25}$ and $104\frac{37}{100}$.

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132. What is the difference between two fractions whose numerators are each 1?

133. How do you subtract one mixed number from another?

CASE II.

134. *When the fractions have different units.*

1. What is the difference between $\frac{1}{2}$ of a £ and $\frac{1}{3}$ of a shilling?

OPERATION.

ANALYSIS.—Reducing to the common unit 1*s.*, we find the difference to be $\frac{56}{6}s. = 9s.\ 8d.$

$\frac{1}{2}£ = \frac{1}{2} \times 20s. = \frac{20}{2}s.$
$\frac{20}{2}s. - \frac{1}{3}s. = \frac{60}{6} - \frac{4}{6}s. = \frac{56}{6}s.$
$= 9s.\ 8d.$

SECOND METHOD.—Reducing to the common unit 1£, we find the difference to be $\frac{29}{60}£ = 9s.\ 8d.$

$\frac{1}{3}s. = \frac{1}{3} \times \frac{1}{20}£ = £\frac{1}{60}.$
$\frac{1}{2}£ - \frac{1}{60}£ = \frac{30}{60}£ - \frac{1}{60}£.$
$= \frac{29}{60}£ = 9s.\ 8d.$

THIRD METHOD.—Reduce the fractions to integral units, and then subtract as in denominate numbers.

$\frac{1}{2}£ = 10s.$
$\frac{1}{3}s. = \quad 4d.$
$\quad 9s.\ 8d.$

RULE.—*Reduce the fractions to the same unit, and then subtract as in Case 1.*

Or: *Find the value of each fraction in units of lower denominations, and then subtract as in denominate numbers.*

EXAMPLES.

1. From $\frac{5}{6}$ of a pound troy take $\frac{5}{8}$ of an ounce.
2. From $\frac{3}{8}$ of a ton take $\frac{2}{3}$ of $\frac{3}{4}$ of a pound.
3. From $\frac{2}{3}$ of $\frac{5}{7}$ of a hogshead of wine take $\frac{3}{4}$ of $\frac{1}{2}$ of a quart.
4. From $\frac{3}{5}$ of a league take $\frac{5}{8}$ of a mile.
5. What is the difference between $1\frac{2}{3}s.$ and $\frac{2}{3}$ of $7\frac{1}{2}d.$?
6. What is the difference between $\frac{21}{28}$ of a degree and $\frac{3}{4}$ of $\frac{1}{7}$ of a degree?
7. From $\frac{5}{16}$ of a square mile take $36\frac{7}{9}$ acres.
8. From $\frac{6}{7}$ of a ton take $\frac{5}{9}$ of 12*cwt.*
9. From $1\frac{3}{4}lb.$ troy take $\frac{1}{6}$ of an ounce.
10. From $2\frac{3}{8}$ cords take $\frac{3}{4}$ of a cord foot.
11. From $\frac{1}{6}$ of a yard take $\frac{2}{3}$ of an inch.

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134. How do you subtract when the fractions have different units?

12. From $\frac{1}{3}$ of $\frac{3}{4}$ of a pound take $\frac{4}{5}$ of $\frac{1}{3}$ of a dram, apothecaries' weight.

13. A pound avoirdupois is equal to 14*oz.* 11*pwt.* 16*gr.* troy; what is the difference, in troy weight, between the ounce avoirdupois and the ounce troy?

MULTIPLICATION OF FRACTIONS.

135. Multiplication of Fractions is the operation of taking one number as many times as there are units in another, when one of the numbers is fractional, or when they are both fractional.

1. If 1 pound of tea cost $\frac{5}{8}$ of a dollar, what will $\frac{3}{7}$ of a pound cost?

Analysis.—The cost will be equal to the price of unity taken as many times as there are units in the quantity (Art. 75).

OPERATION.

$\$\frac{5}{8} \times \frac{3}{7} = \frac{5 \times 3}{8 \times 7} = \$\frac{15}{56}$.

One-seventh of a pound of tea will cost *one-seventh* as much as 1*lb.* Since 1*lb.* cost $\$\frac{5}{8}$, $\frac{1}{7}$ of 1*lb.* will cost $\frac{1}{7}$ of $\$\frac{5}{8} = \$\frac{5}{56}$. (Art. 117). But 3 sevenths of 1*lb.* will cost 3 times as much as $\frac{1}{7}$; that is, $\$\frac{5}{56} \times 3 = \$\frac{15}{56}$ (Art. 114). Hence, to multiply one fraction by another,

Rule.—*Multiply the numerators together for a new numerator, and the denominators together for a new denominator.*

Notes.—1. When the multiplier is less than 1, we do not take the whole of the multiplicand, but only such a part of it as the multiplier is of 1.

2. When the multiplier is a proper fraction, multiplication does not imply *increase*, as in the multiplication of whole numbers. The product is the same part of the multiplicand which the multiplier is of 1.

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135. What is multiplication of fractions? How do you multiply one fraction by another? When the multiplier is less than 1, what part of the multiplicand is taken? If the fraction is proper, does multiplication imply increase? What part is the product of the multiplicand? What do you do when either factor is a whole number?

3. If either of the factors is a whole number, write 1 under it for a denominator.

4. When either of the factors is a mixed number, it may be reduced to an improper fraction, or we may multiply the parts separately and take their sum.

1. Multiply $\frac{3}{7}$ by 8.
2. Multiply $\frac{8}{75}$ by 12.
3. Multiply $\frac{32}{40}$ by 9.
4. Multiply $\frac{14}{16}$ by 15.
5. Multiply $\frac{75}{96}$ by 12.
6. Multiply $\frac{4}{5}$ of $\frac{4}{7}$ by 35.
7. Multiply $3\frac{1}{2}$ of $\frac{2}{3}$ by 14.
8. Multiply $1\frac{3}{4}$ of $2\frac{1}{2}$ by 16.
9. Multiply $2\frac{1}{5}$ of $\frac{2}{7}$ by 70.
10. Multiply $4\frac{9}{8}$ of 8 by 36.

11. Multiply 36 by $4\frac{1}{9}$.

ANALYSIS.—The number 36 is to be taken $4\frac{1}{9}$ times; that is, 4 times and $\frac{1}{9}$ times. One-ninth of 36 is 4, which is written in the units place: then, 4 times 36 is 144; and the sum 148 is the product.

OPERATION.

$$\begin{array}{r} 36 \\ \underline{4\frac{1}{9}} \\ 4 \\ \underline{144} \\ 148\ \textit{Ans.} \end{array}$$

12. Multiply 67 by $9\frac{1}{12}$.
13. Multiply 842 by $7\frac{1}{9}$.
14. Multiply 360 by $12\frac{3}{5}$.
15. Multiply 460 by $11\frac{3}{4}$.
16. Multiply 620 by $10\frac{5}{4}$.
17. Multiply 1340 by $8\frac{3}{4}$.

EXAMPLES.

1. Multiply $\frac{4}{9}$ by 8.
2. Multiply 15 by $\frac{6}{7}$.
3. Multiply 11 by $\frac{8}{15}$.
4. Multiply $7\frac{7}{8}$ by 8.
5. Multiply $9\frac{1}{2}$ by $18\frac{3}{4}$.
6. Multiply $3\frac{2}{7}$ by $4\frac{14}{33}$.
7. Multiply $\frac{175}{81}$ by 9.
8. Multiply $\frac{3}{4}$ by $\frac{3}{5}$.
9. Multiply $\frac{7}{8}$ by $\frac{3}{5}$.
10. Multiply $\frac{1}{4}$ of $\frac{3}{8}$ by $\frac{5}{9}$.
11. Multiply $\frac{5}{12}$ by 9 of $\frac{6}{17}$.
12. Multiply $\frac{1}{2}$ of $\frac{7}{8}$ by $\frac{4}{7}$ of $\frac{6}{16}$.
13. Multiply $\frac{7}{8}$ by 16.
14. Multiply 28 by $\frac{9}{14}$.
15. Multiply $\frac{21}{25}$ by 18.
16. Multiply $8\frac{7}{10}$ by 15.
17. Multiply $\frac{6}{11}$ of $\frac{2}{3}$ by $\frac{10}{24}$.
18. Multiply $5\frac{1}{4}$ by $\frac{4}{5}$ of $3\frac{1}{3}$.
19. Multiply $842\frac{1}{4}$ by $7\frac{1}{2}$.
20. Multiply $\frac{5}{9}$ by $\frac{6}{7}$.
21. Multiply $\frac{9}{10}$ by $7\frac{7}{11}$.
22. Multiply $\frac{16}{21}$ by $\frac{3}{4}$ of $\frac{7}{18}$.

23. Multiply $\frac{7}{11}$, $\frac{22}{23}$ and $\frac{46}{49}$ together.

24. Multiply $\frac{14}{27}$, $\frac{9}{28}$, $\frac{6}{13}$ and $\frac{26}{30}$ together.

25. What is the product of $\frac{12}{17}$ by $\frac{2}{9}$ of 17 ?

26. What is the product of 6 by $\frac{2}{3}$ of 5 ?

27. What is the product of $\frac{1}{8}$ of $\frac{1}{6}$ of 3 by $15\frac{1}{7}$?

28. Require the product of $\frac{2}{9}$ of $\frac{3}{5}$ by $\frac{5}{8}$ of $3\frac{2}{7}$.

29. Require the product of 5, $\frac{2}{3}$, $\frac{2}{7}$ of $\frac{3}{5}$, and $4\frac{1}{6}$.

30. Require the product of 14, $\frac{5}{6}$, $\frac{4}{5}$ of 9, and $6\frac{3}{7}$.

31. What will 7 yards of cloth cost at $\$\frac{3}{4}$ a yard ?

32. What will $12\frac{3}{4}$ bushels of apples cost at $\$\frac{8}{7}$ a bushel ?

33. If one bushel of wheat cost $\$1\frac{7}{9}$, what will $\frac{3}{8}$ of a bushel cost ?

34. If one horse eat $\frac{5}{8}$ of a ton of hay in one month, how much will 18 horses eat in the same time ?

35. If a man earn $\$\frac{15}{16}$ in one day, how much can he earn in 24 days ?

36. What will $3\frac{1}{2}$ yards of cloth cost at $\frac{7}{8}$ of a dollar a yard ?

37. At $16 a ton, what will $\frac{1}{12}$ of a ton of hay cost ?

38. If one pound of tea cost $\$1\frac{1}{4}$, what will $6\frac{1}{8}$ pounds cost ?

39. What will $3\frac{3}{8}$ boxes of raisins cost at $\$2\frac{1}{2}$ a box ?

40. At 75 cents a bushel, what will $\frac{11}{15}$ of a bushel of corn cost ?

41. If a lot of land be worth $\$75\frac{8}{15}$, what will $\frac{5}{11}$ of it be worth ?

42. If a man earn $56 in one month, how much can he earn in $\frac{9}{14}$ of a month ?

43. What will $17\frac{1}{2}$ yards of cambric cost at $2\frac{1}{2}$ shillings a yard ?

44. Bought $15\frac{5}{8}$ barrels of sugar at $\$20\frac{4}{5}$ a barrel, what did the whole cost ?

45. If one bushel of corn is worth $\frac{5}{6}$ of a dollar, what is $\frac{2}{3}$ of a bushel worth ?

46. If I own $\frac{7}{15}$ of a farm and sell $\frac{9}{14}$ of my share, what part of the whole farm do I sell ?

47. Bought a book for $\frac{9}{10}$ of a dollar and a knife for $\frac{5}{12}$ as much ; how much did I pay for the knife ?

48. At $\frac{2}{3}$ of $\frac{11}{12}$ of a dollar a pound, what will $\frac{4}{5}$ of $\frac{15}{16}$ of a pound of tea cost?

49. If hay is worth $\$9\frac{3}{4}$ a ton what is $\frac{2}{3}$ of $3\frac{1}{5}$ *cwt.* worth.

50. If a man can dig a cellar in $22\frac{1}{2}$ days, how many days will it take him to dig $\frac{5}{9}$ of it?

51. If a railroad train run 1 mile in $\frac{1}{20}$ of an hour, how long will it be in running $106\frac{2}{3}$ miles?

52. What will be the cost of $20\frac{7}{8}$ cords of wood at $\$3\frac{1}{5}$ a cord?

53. If a man walk $3\frac{1}{2}$ miles an hour, how far will he walk in $9\frac{5}{7}$ hours?

54. What will $14\frac{3}{5}$ bushels of potatoes cost at $31\frac{1}{4}$ cents a bushel?

55. What will $12\frac{1}{6}$ dozens of eggs bring at $18\frac{3}{4}$ cents a dozen?

56. At $\frac{5}{8}$ of a dollar a bushel, what will $102\frac{1}{3}$ bushels of rye cost?

57. What will $\frac{3}{5}$ of a firkin of butter cost at $\$18\frac{1}{16}$ a firkin?

58. A man, at his death, left his wife $15000; she at her death left $\frac{4}{5}$ of her share to her daughter: what part of the father's estate did the daughter receive?

59. A person owning $\frac{3}{7}$ of a cotton factory sold $\frac{3}{5}$ of his part to A, and the rest to B: what part of the whole did each buy?

60. A owned $\frac{7}{8}$ of a farm and sold $\frac{4}{5}$ of his share to B, who sold $\frac{5}{9}$ of what he bought to C, who sold $\frac{6}{7}$ of what he bought to D: what part of the whole did D have?

61. A owned $\frac{3}{5}$ of 200 acres of land, and sold $\frac{2}{3}$ of his share to B, who sold $\frac{1}{4}$ of what he bought to C: how many acres had each?

DIVISION.

136. DIVISION OF FRACTIONS is the operation of finding a number which multiplied by the divisor will produce the dividend, when one of the parts is fractional, or when both are fractional.

1. What is the quotient of $\frac{7}{8}$ divided by $\frac{14}{5}$?

ANALYSIS.—If the divisor and dividend be reduced to the same fractional unit, the quotient will be equal to the number of units of the dividend, divided by the number of units of the divisor.

The number of units of the dividend, after reduction, is equal to the product of its numerator multiplied by the denominatior of the divisor; and the number of units in the divisor is equal to the product of its numerator by the denominator of the dividend; hence,

OPERATION.

$$\frac{7}{8} \div \frac{14}{5} = \frac{35}{40} \div \frac{112}{40}$$

$$\text{quotient} = \frac{35}{112}.$$

$$\frac{7}{8} \times \frac{5}{14} = \frac{35}{112} = \frac{5}{16}.$$

$$\begin{array}{r|l} & \cancel{7} \\ 2 \quad 8 & \\ \cancel{14} & 5 \\ \hline 16 & 5 = \frac{5}{16} \quad \textit{Ans.} \end{array}$$

I. *Invert the terms of the divisor:*

II. *Multiply the numerators together for the numerator of the quotient, and the denominators together for the denominator of the quotient.*

NOTES.—1. If either the dividend or divisor is a whole number, make it fractional, by writing 1 under it for a denominator.

2. If the vertical line is used, the denominator of the dividend and the numerator of the divisor fall on the left, and the other terms on the right.

3. Cancel all common factors.

---

136. What is division of fractions? What is the rule for the division of fractions? What do you do when either the dividend or divisor is a whole number? Where do the parts fall when you use the vertical line? What do you do when either term of the fraction is a mixed number or a compound fraction? If the terms of the dividend are exactly divisible by the corresponding terms of the divisor, how do you find the quotient?

4. If the dividend and divisor have a common denominator, it will cancel, and the quotient of the numerators will be the answer.

5. When either term of the fraction is a mixed number, or a compound fraction, reduce to the form of a simple fraction before dividing.

6. If the numerator of the dividend is exactly divisible by the numerator of the divisor, and the denominator by the denominator, the division may be made without inverting the terms of the divisor.

EXAMPLES.

1. Divide $\frac{21}{15}$ by 7.
2. Divide $\frac{9}{14}$ by 6.
3. Divide $\frac{13}{15}$ by 9.
4. Divide $\frac{120}{319}$ by 40.
5. Divide $\frac{23}{64}$ by 13.
6. Divide 5 by $\frac{7}{10}$.
7. Divide 27 by $\frac{3}{4}$.
8. Divide $\frac{1}{8}$ by $\frac{1}{7}$.
9. Divide $\frac{9}{10}$ by $\frac{3}{8}$.
10. Divide $\frac{45}{80}$ by $\frac{5}{14}$.
11. Divide $\frac{2}{3}$ of $\frac{4}{5}$ by $\frac{6}{7}$ of $\frac{3}{4}$.
12. Divide $\frac{7}{8}$ of $\frac{6}{7}$ by $\frac{4}{5}$ of $\frac{8}{9}$.
13. Divide $\frac{3}{8}$ of $\frac{2}{3}$ by $\frac{3}{4}$ of $\frac{5}{6}$.
14. Divide 56 by $\frac{1}{12}$.
15. Divide 1000 by $\frac{80}{133}$.
16. Divide 725 by $\frac{25}{47}$.
17. Divide $4\frac{3}{8}$ by 5.
18. Divide $9\frac{5}{11}$ by 12.
19. Divide $\frac{1}{3}$ of $16\frac{1}{2}$ by $4\frac{1}{7}$.
20. Divide $9\frac{1}{6}$ by $\frac{1}{2}$ of 7.
21. Divide $\frac{5}{6}$ of 50 by $4\frac{1}{3}$.
22. Divide $300\frac{5}{28}$ by $6\frac{1}{4}$.
23. Divide $\frac{4}{7}$ of $3\frac{3}{4}$ by $\frac{19}{20}$ of $7\frac{1}{2}$.
24. Divide $9\frac{7}{8}$ by $8\frac{1}{3}$.
25. Divide $\frac{5}{9}$ of $\frac{7}{11}$ by $6\frac{1}{9}$.
26. Divide $\frac{12}{17}$ by 4.
27. Divide $\frac{20}{27}$ by 5.
28. Divide $\frac{60}{75}$ by 8.
29. Divide $\frac{432}{521}$ by 48.
30. Divide $\frac{42}{125}$ by 21.
31. Divide 36 by $\frac{9}{10}$.
32. Divide 420 by $\frac{3}{8}$.
33. Divide $\frac{9}{20}$ by $\frac{3}{8}$.
34. Divide $\frac{14}{25}$ by $\frac{7}{15}$.
35. Divide $\frac{2}{3}$ of $\frac{27}{50}$ by $\frac{20}{27}$.
36. Divide $\frac{7}{9}$ by $\frac{15}{16}$.
37. Divide $\frac{3}{5}$ of $\frac{8}{9}$ by $\frac{6}{7}$ of $\frac{3}{4}$.
38. Divide $\frac{1}{2}$ of $\frac{1}{4}$ of $\frac{2}{3}$ by $\frac{1}{8}$ of $\frac{4}{9}$.
39. Divide 650 by $\frac{100}{127}$.
40. Divide 1273 by $\frac{17}{56}$.
41. Divide 4324 by $\frac{128}{475}$.
42. Divide $6\frac{2}{9}$ by 8.
43. Divide $12\frac{4}{9}$ by 42.
44. Divide $3\frac{1}{6}$ by $9\frac{1}{2}$.
45. Divide 100 by $4\frac{3}{8}$.
46. Divide $44\frac{1}{33}$ by $\frac{213}{333}$.
47. Divide $111\frac{1}{9}$ by $33\frac{1}{3}$.
48. Divide $191\frac{1}{5}$ by $159\frac{1}{3}$.
49. Divide $5\frac{3}{8}$ by $\frac{3}{8}$ of $1\frac{1}{2}$.
50. Divide $5205\frac{1}{8}$ by $\frac{4}{5}$ of [illegible]

51. At $\frac{1}{8}$ of a dollar a pound, how much butter can be bought for $\frac{21}{32}$ of a dollar?

52. At $\frac{4}{5}$ of a dollar a yard, how much cloth can be bought for $\frac{7}{8}$ of a dollar?

53. If a bushel of potatoes cost $\frac{3}{8}$ of a dollar, how many can be bought for $\frac{9}{16}$ of a dollar?

54. If $\frac{1}{5}$ of a ton of hay will feed 1 horse one week, how many horses will $\frac{9}{10}$ of a ton feed, the same time?

55. If $\frac{6}{7}$ of a bushel of apples cost $\frac{3}{5}$ of a dollar, what will a bushel cost?

56. What will a barrel of flour cost, if $\frac{5}{18}$ of a barrel cost $\frac{6}{7}$ of a dollar?

57. If $\frac{3}{8}$ of a bushel of apples cost $\frac{2}{5}$ of a dollar, what will 1 bushel cost?

58. How much molasses at $\frac{2}{7}$ of a dollar a gallon, can be bought for $1\frac{5}{7}$ dollars?

59. A man sold $\frac{35}{54}$ of a mill, which was $\frac{7}{9}$ of his share: what part of the mill did he own?

60. What number multiplied by $\frac{3}{4}$, will give $15\frac{3}{4}$ for the product?

61. What number multiplied by $5\frac{1}{3}$, will give 146 for the product?

62. The dividend is $520\frac{1}{5}$, and the quotient $36\frac{9}{10}$: what is the divisor?

63. What number is that which if multiplied by $\frac{5}{8}$ of $\frac{3}{7}$ of $15\frac{1}{8}$, will produce $\frac{5}{8}$?

64. If 7*lb.* of sugar cost $\frac{42}{75}$ of a dollar, what will 1 pound cost?

65. If $10\frac{1}{2}$*lb.* of nails cost $\frac{3}{7}$ of a dollar, what is the price per pound?

66. If $\frac{4}{7}$ of a yard of cloth cost \$3, what is the cost of a yard?

67. A family consumes $165\frac{3}{9}$ pounds of butter in $8\frac{1}{2}$ weeks: how much do they consume in 1 week?

68. At $\$9\frac{3}{8}$ a barrel, how much flour can be bought for $\$138\frac{3}{4}$?

69. If a man divides $\$3\frac{5}{9}$ equally among 8 beggars, how much will he give them apiece?

70. If 8 pounds of tea cost $7\frac{5}{8}$ dollars, what is the price per pound?

71. If $\frac{4}{5}$ of a ton of hay sell for $\$10\frac{2}{3}$, what should 1 ton sell for?

72. If $\frac{7}{9}$ of an acre of ground produce $84\frac{7}{16}$ bushels of potatoes, how many bushels will 1 acre produce?

73. What quantity of cloth may be purchased for $\$5\frac{1}{16}$, at the rate of $\$6\frac{3}{4}$ a yard?

74. How long would a person be in travelling $125\frac{5}{7}$ miles, if he travelled $31\frac{6}{14}$ miles per day?

75. How many bottles, each holding $1\frac{2}{7}$ gallons, can be filled from a barrel of wine, containing $31\frac{1}{2}$ gallons?

76. How long will it take 11 men to do a piece of work that 1 man can do in $15\frac{8}{9}$ days?

77. If $\frac{4}{7}$ of a barrel of flour cost 6 dollars, what is the price per barrel?

78. Eighty-one is $\frac{9}{4}$ of how many times 8?

79. Five-eighths of 48 is $\frac{5}{9}$ of how many times 9?

80. How many times can a vessel, containing $\frac{4}{5}$ of a gallon, be filled from $\frac{1}{3}$ of a barrel of $31\frac{1}{2}$ gallons?

81. If $5\frac{1}{2}$*lb.* of tea cost $\$4\frac{2}{5}$, what is the price of 1 pound?

82. If $\frac{3}{4}$ of $\frac{5}{9}$ of a ship is worth \$2540, what is the whole vessel worth?

83. If $\frac{5}{6}$ of an acre of land cost $\$36\frac{1}{9}$, what will be the value of an acre?

84. If $\frac{5}{7}$ of $\frac{3}{4}$ of a barrel of flour will last a family 1 week, how long will $9\frac{5}{14}$ barrels last them?

REDUCTION OF COMPLEX FRACTIONS.

137. A **Complex Fraction** is only another form of expression for the division of fractions: thus, $\frac{\frac{7}{9}}{\frac{5}{6}}$, is the same as $\frac{7}{9}$ divided by $\frac{5}{6}$; and may be written, $\frac{7}{9} \div \frac{5}{6} = \frac{42}{45}$

138. *To reduce a complex fraction to a simple fraction:*

1. Reduce $\frac{6\frac{2}{3}}{1\frac{1}{7}}$ to a simple fraction.

Analysis.—Reducing the divisor and dividend each to a simple fraction, we have $\frac{20}{3}$ and $\frac{8}{7}$. Then $\frac{20}{3}$ divided by $\frac{8}{7}$ is equal to $\frac{20}{3} \times \frac{7}{8} = \frac{140}{24} = \frac{35}{6} = 5\frac{5}{6}$.

OPERATION.

$6\frac{2}{3} = \frac{20}{3}$ and $1\frac{1}{7} = \frac{8}{7}$.

$\frac{20}{3} \div \frac{8}{7} = \frac{20}{3} \times \frac{7}{8} = \frac{35}{6} = 5\frac{5}{6}$.

	5
3	$\cancel{20}$
2 $\cancel{8}$	7
6	35

Ans. $\frac{35}{6} = 5\frac{5}{6}$.

Rule.—*Divide the numerator of the complex fraction by its denominator.*

Or: *Multiply the numerator of the upper fraction into the denominator of the lower, for a new numerator; and the denominator of the upper fraction into the numerator of the lower, for a new denominator.*

Notes.—1. When either of the terms of a complex fraction is a mixed number, or a compound fraction, it must first be reduced to the form of a simple fraction.

2. When the vertical line is used, the numerator of the *upper* and the denominator of the *lower* numbers fall on the right of the vertical line, and the other terms on the left.

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137. What is a complex fraction?

138. How do you reduce a complex fraction to a simple fraction?

EXAMPLES.

Reduce the following to simple fractions:

1. Reduce $\dfrac{\frac{5}{6}}{\frac{4}{5}}$.

2. Reduce $\dfrac{\frac{8}{9}}{\frac{15}{16}}$.

3. Reduce $\dfrac{1\frac{5}{9}}{\frac{9}{16}}$.

4. Reduce $\dfrac{87\frac{1}{2}}{\frac{7}{8}}$.

5. Reduce $\dfrac{\frac{8}{9}}{4\frac{1}{2}}$.

6. Reduce $\dfrac{8\frac{4}{7}}{12}$.

7. Reduce $\dfrac{11\frac{3}{32}}{8\frac{7}{8}}$.

8. Reduce $\dfrac{20}{\frac{4}{7}}$.

9. Reduce $\dfrac{\frac{5}{9} \text{ of } 7\frac{3}{11}}{\frac{4}{11} \text{ of } 17\frac{3}{7}}$.

10. Reduce $\dfrac{.26\frac{8}{35}}{\frac{3}{5} \text{ of } 17}$.

11. Reduce $\dfrac{55\frac{4}{5}}{\frac{1}{8} \text{ of } 8\frac{2}{5}}$.

12. Reduce $\frac{5}{8}$ of $\frac{3}{10}$ of $\dfrac{9\frac{3}{4}}{13}$.

APPLICATIONS IN FRACTIONS.

1. What will $5\frac{1}{4}$ cords of wood cost at $\frac{1}{6}$ of $\frac{3}{7}$ of $\frac{4}{5}$ of \$50 a cord?

2. A farmer sold $\frac{3}{8}$ of a ton of hay for \$$6\frac{3}{5}$: what would be the price of a ton at the same rate?

3. A person walks $77\frac{2}{5}$ miles in $10\frac{1}{2}$ hours: at what rate is that per hour?

4. From the product of $\frac{2}{9}$ and $11\frac{1}{3}$, take $\frac{6}{13}$, and multiply the remainder by $20\frac{3}{4}$.

5. How much greater is $\frac{3}{4}$ of the sum of $\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{7}$ and $\frac{1}{9}$, than the sum of $\frac{1}{4}$, $\frac{1}{6}$ and $\frac{1}{8}$?

6. If $\frac{3}{5}$ of a ton of hay is worth \$$7\frac{1}{8}$, what is $2\frac{2}{9}$ tons worth?

7. If $\frac{2}{3}$ of a dollar will pay for $\frac{7}{8}$ of a yard of cloth, how many yards can be bought for \$$11\frac{3}{7}$?

8. What is the value of $3\frac{1}{2}$ cords of wood at \$$4\frac{2}{3}$ a cord?

9. What is the continued product of $14\frac{3}{7}$, $\dfrac{4}{5\frac{1}{3}}$, $\dfrac{\frac{3}{4}}{\frac{5}{7}}$, and $\dfrac{\frac{5}{7}}{9}$?

10. What is the sum and difference of $\dfrac{49\frac{5}{8}}{97}$ and $\dfrac{34\frac{3}{5}}{146\frac{3}{11}}$?

11. At $\frac{1}{5}$ of a dollar a peck, how many bushels of apples can be bought for \$$6\frac{3}{4}$?

12. What is the difference between $\frac{2}{3}$ of a league and $\frac{7}{10}$ of a mile?

13. Subtract $8\frac{5}{6}$*lb.* from $\frac{4}{7}$ of a *cwt.*

14. What is the sum of $4\frac{9}{10}$ miles, $\frac{2}{7}$ of a furlong, and $\frac{3}{8}$ of $1\frac{1}{2}$ yards?

15. At \$$1\frac{4}{5}$ per day, how many days labor can be obtained for \$$36\frac{3}{5}$?

16. Sold $7\frac{1}{4}$ bushels of apples for \$$3\frac{5}{8}$: what should I receive for $24\frac{2}{3}$ bushels?

17. A has 634 sheep, which are 124 more than $\frac{5}{6}$ of $2\frac{1}{4}$ times B's number: how many sheep had B?

18. At $\frac{9}{325}$ of a dollar a yard, how many yards of ribbon can be bought for $\frac{9}{25}$ of a dollar?

19. Paid \$$56\frac{2}{5}$ for 94 yards of muslin: how much was that per yard?

20. Bought $5\frac{1}{3}$ yards of cloth at \$$4\frac{1}{8}$ a yard, and paid for it in wheat at \$$1\frac{4}{7}$ a bushel: how many bushels were required?

21. What number must be taken from $27\frac{3}{4}$, and the remainder multiplied by $14\frac{2}{3}$, that the product shall be 100?

22. Three persons, A, B, and C, purchase a piece of property for \$6300; A pays $\frac{3}{7}$ of it, B, $\frac{4}{9}$, and C the remainder: what is the value of each one's share?

23. What number is that which being diminished by the difference between $\frac{3}{4}$ and $\frac{3}{5}$ of itself leaves a remainder equal to 34?

24. Add together $\frac{1}{3}$ of a week, $\frac{1}{4}$ of a day, and $\frac{1}{2}$ of an hour.

25. What is the sum of $\frac{2}{7}$ of £15, £$3\frac{3}{7}$, $\frac{1}{3}$ of $\frac{5}{7}$ of $\frac{3}{5}$ of £1, and $\frac{2}{3}$ of $\frac{3}{7}$ of a shilling?

26. If $\frac{1}{6}$ of John's marbles are équal to $\frac{1}{8}$ of James', and together they have 56: how many has each?

27. A person owning $\frac{3}{7}$ of 2000 acres of land, sold $\frac{2}{3}$ of his share: how many acres did he retain?

28. A boy having 240 marbles, divided them in the following manner: he gave to A, $\frac{1}{3}$, to B, $\frac{1}{10}$, to C, $\frac{1}{8}$, and to D, $\frac{1}{6}$, keeping the remainder himself: what number of marbles had each?

29. A man engaging in trade with \$3740, found at the end of 3 years that he had gained \$156$\frac{1}{3}$ more than $\frac{1}{3}$ of his capital: what was his average annual gain?

30. Two boys having bought a sled, one paying $\frac{3}{4}$ of a dollar, and the other $\frac{7}{8}$ of a dollar, sold it for $\frac{7}{16}$ of a dollar more than they gave for it: what did they sell it for, and what was each one's share of the gain?

31. A farmer having 126$\frac{6}{7}$ bushels of wheat, sold $\frac{5}{8}$ of it for \$2$\frac{1}{5}$ a bushel, and the remainder for \$1$\frac{3}{4}$ a bushel: how much did he receive for his wheat?

32. A man having \$19$\frac{1}{8}$, expended it for wheat and corn, of each an equal quantity; for the wheat he paid \$1$\frac{4}{5}$ a bushel, and for the corn \$$\frac{3}{4}$ a bushel: how much of each did he buy?

33. Two persons engage in trade: A furnished $\frac{7}{12}$ of the capital, and B, $\frac{5}{12}$; if B had furnished \$492$\frac{2}{3}$ more, their shares would have been equal: how much did each furnish?

34. A man being asked how many sheep he had, said he had them in 3 fields: in the first he had 63, which was $\frac{7}{8}$ of what he had in the second, and that $\frac{5}{3}$ of what he had in the second was just 4 times what he had in the third: how many sheep had he in all?

DUODECIMALS.

139. Duodecimals are a series of numbers which arise from dividing the unit 1 according to the uniform scale of 12; thus,

If the unit 1 foot be divided into 12 equal parts, each part is called an *inch* or *prime*, and marked ′. If an inch be divided into 12 equal parts, each part is called a *second*, and marked ″. If a second be divided, in like manner, into 12 equal parts, each part is called a *third*, and marked ‴; and so on for divisions still smaller.

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139. What are duodecimals? If the unit 1 foot be divided into 12 equal parts, what is each part called? If 1 inch be divided into 12 equal parts, what is each part called? If the second be divided in like manner, what is each part called? What are indices?

This division of the foot gives

$1'$	inch or prime - - - - - -	$= \frac{1}{12}$	of a foot.
$1''$	second is $\frac{1}{12}$ of $\frac{1}{12}$ - - - -	$= \frac{1}{144}$	of a foot.
$1'''$	third is $\frac{1}{12}$ of $\frac{1}{12}$ of $\frac{1}{12}$ - -	$= \frac{1}{1728}$	of a foot.

NOTE.—The marks $'$, $''$, $'''$, &c., which denote the *fractional units*, are called *indices*.

TABLE.

$12'''$	make	$1''$	second.
$12''$	"	$1'$	inch or prime.
$12'$	"	1	foot.

Hence: Duodecimals are denominate fractions, in which the primary unit is 1 *foot*, and the *scale* uniform, the units of it, at every point, being 12.

NOTE.—Duodecimals are chiefly used in measuring *surfaces* and *solids*.

ADDITION AND SUBTRACTION.

140. The units of duodecimals are reduced, added, subtracted, and multiplied like those of other denominate numbers. The *units of the scale* are 12, at every change of the unit.

EXAMPLES.

1. In $86'$ how many feet?
2. In $750''$ how many feet?
3. In $37000'''$ how many *ft.*?
4. In $67'$ how many feet?
5. In $470'''$ how many feet?
6. In $375''$ how many feet?
7. What is the sum of 8*ft.* $9'$ $7''$ and 6*ft.* $7'$ $3''$ $4'''$?
8. What is the difference between 32*ft.* $6'$ $6''$ and 29*ft.* $7'''$?
9. Add together 9*ft.* $6'$ $4''$ $3'''$, 12*ft.* $2'$ $9''$ $10'''$, 26*ft.* $0'$ $5''$, and 40*ft.* $1'$ $0''$ $3'''$.
10. What is the sum of 125*ft.* $0'$ $6''$, 45*ft.* $11'$ $0''$ $2'''$ and 12*ft.* $6'$?

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140. By what rules do you operate on duodecimal units? What are the units of the scale?

11. What is the sum of 84*ft.* 7′, 96*ft.* 0′ 11″, 42*ft.* 6′ 9″ 10‴ and 5′ 7″ 11‴?

12. From 127*ft.* 3′ 6″ 4‴ 11⁗ take 40*ft.* 0′ 10″ 7‴ 5⁗.

13. What is the difference between 425*ft.* 9′ 10″ and 107*ft.* 10′ 9″ 8‴?

14. What is the sum and difference of 325*ft.* 7′ 6″ 2‴ and 217*ft.* 10′ 9″?

15. What is the sum and difference of 1001*ft.* 0′ 0″ 10‴ and 720*ft.* 10′ 9″ 1‴?

MULTIPLICATION.

141. Multiplication of duodecimals is the operation of finding the superficial contents, or the contents of volume, when the linear dimensions are known.

To do this we begin with the *highest* unit of the multiplier and the *lowest* of the multiplicand, and recollect,

1st. That 1 linear foot × 1 linear foot = 1 square foot, (Art. 411), or, that a part of a foot × a part of a foot = some part of a square foot.

2d. That a square foot × by a foot in length = a cubic foot.

Note.—Observe that in the first multiplication the unit is changed, from a linear to a *superficial unit;* in the second multiplication, from a superficial unit to a unit of volume.

1. Multiply 6*ft.* 7′ 8″ by 2*ft.* 9′.

Analysis.—Since a prime is $\frac{1}{12}$ of a foot, and a second $\frac{1}{144}$, $2\times8'' = \frac{16}{144}$ of a square foot; which reduced to 12ths, is 1′ and 4″; that is, 1 twelfth, and 4 twelfths of twelfths of a square foot.

$2 \times 7' = 14$ twelfths $= 1$*ft.* 2′ - - -

$2 \times 6 = 12$ square feet - - - - -

$9' \times 8'' = \frac{72}{1728}$ of a square foot $= 6''$ -

$9' \times 7' = \frac{63}{144} = 5'\ 3''$ - - - - - -

$9 \times 6' = \frac{54}{12} = 4\ 6'$ - - - - - - -

OPERATION.

	ft.		
	6	7′	8″
	2	9′	
2 × 8″ =		1′	4″
2 × 7′ =	1	2′	
2 × 6 =	12		
9′ × 8″ =			6″
9′ × 7′ =		5′	3″
9′ × 6 =	4	6′	
Prod.	18	3′	1″

141. What is multiplication of duodecimals?

RULE.—I. *Write the multiplier under the multiplicand, so that units of the same order shall fall in the same column.*

II. *Begin with the highest unit of the multiplier and the lowest of the multiplicand, and make the index of each product equal to the sum of the indices of the factors.*

III. *Reduce each product of the first multiplication to square feet and 12ths of a square foot, and when there are three factors reduce the second products to units of volume.*

NOTE.—The index of the unit of any product is equal to the sum of the indices of the factors.

EXAMPLES.

1. How many cubic feet in a stick of timber 12 feet 6 inches long, 1 foot 5 inches broad, and 2 feet 4 inches thick?

ANALYSIS.—Beginning with the 1 foot, we say 1 time $4'$ is $4' = \frac{4}{12}$ of a square foot: then, 1 time 2 is 2 square feet. Next, 5 times $4'$ are $20'' = 1'$ and $8''$: then, 5 times 2 feet $= 10'$, and the $1'$ to carry, makes $11'\ 8''$. Then multiplying by the length 12 feet $6'$, we find the contents to be 41 $3'\ 10''$ cubic feet.

OPERATION.

ft.		
2	4′	
1	5′	
2	4′	
	11	8″
3	3′	8″
12	6′	
39	8′	0
1	7′	10″
41	3′	10″

2. Multiply 9*ft.* 6′ by 4*ft.* 7′.
3. Multiply 12*ft.* 5′ by 6*ft.* 8′.
4. Multiply 35*ft.* 4′ 6″ by 9*ft.* 10′.
5. What is the product of 45*ft.* 4′ 3″ by 12*ft.* 2′ 9″?
6. What is the product of 140*ft.* 0′ 2″ 4‴ by 20*ft.* 10′?
7. What is the product of 279*ft.* 10′ 6″ by 8′ 4″?
8. What are the contents of a board 14*ft.* 6′ 3″ long and 2*ft.* 9′ wide?
9. How many square feet in a floor 18*ft.* 9′ long, and 15*ft.* 10′ wide?
10. How many square yards in a ceiling 70*ft.* 9′ long, and 12*ft.* 8′ wide?

11. What will be the cost of paving a yard 64*ft.* 6′ square, at 5 cents a square foot?

12. What are the cubic contents of a block of marble, 6*ft.* 9′ long, 4*ft.* 8′ wide, and 2*ft.* 10′ thick?

13. There is a room 97*ft.* 4′ around it; it is 9*ft.* 6′ high: what will it cost to paint the walls, at 18 cents a square yard?

14. How many cubic feet of wood in a pile 36*ft.* 5′ long, 6*ft.* 8′ high, and 3*ft.* 6′ wide?

15. What will a pile of wood 26*ft.* 8′ long, 6*ft.* 6′ high, and 3*ft.* 3′ wide, cost, at $3,50 a cord?

16. How many cubic yards of earth were dug from a cellar which measured 38*ft.* 10′ long, 20*ft.* 6′ wide, and 9*ft.* 4′ deep?

17. At 16 cents a yard, what will it cost to plaster a room 22*ft.* 8′ long, 18*ft.* 9′ wide, and 11*ft.* 6′ high? There are to be deducted 8 windows, 6*ft.* 4′ high and 2*ft.* 9′ wide; 2 doors, 7*ft.* 6′ high and 3*ft.* 2′ wide, and the base moulding, which is 1 foot wide.

DIVISION OF DUODECIMALS.

142. Division of Duodecimals is the operation of finding from two duodecimal numbers a third, which multiplied by the first, will give the second.

1. A hall contains 103*sq. ft.* 4′ 5″ 8‴ 4^{iv}, and is 6*ft.* 11′ 8″ wide: what is its length?

OPERATION.

```
 ft.        sq. ft.                 ft.
6 11′ 8″) 103  4′  5″  8‴  4ⁱᵛ(14  9′  11″
           97  7′  4″
           ----------
            5  9′  1″  8‴
            5  2′  9″  0‴
            -------------
               6′  4″  8‴  4ⁱᵛ
               6′  4″  8‴  4ⁱᵛ
```

Analysis. — The units of the dividend are square feet and fractions of a square foot. The units of the divisor are linear feet and fractions of a linear foot.

First, consider how often the first two parts of the divisor are contained in the first part of the dividend. The first two parts of the divisor are nearly equal

142. What is Division of Duodecimals? How is it performed?

to 7 feet, and this is contained in 103*sq. ft.* 14 times and something over. Multiplying the divisor by this term of the quotient and subtracting, we find the remainder 5*ft.* 9′ 1″, to which bring down 8‴.

Next, consider how many times the first two parts of the divisor, (equal to 7 feet, nearly,) are contained in the first two parts of the remainder, reduced to the next lower unit; that is, 5*ft.* 9′ = 69′. Multiplying the divisor by the quotient figure 9′, and making the subtraction, we have, 6′ 4″ 8‴, to which bring down 4^{iv}.

Consider, again, how often, *nearly* 7 feet is contained in 6′ 4″=76″. Multiplying the divisor by the quotient 11″, we find a product equal to the last remainder. Hence, *the process of division is the same as that of other denominate numbers, except in the manner of selecting the quotient figure.*

NOTES.—1. If the integral unit of the dividend and divisor is the same, *the unit of the quotient will be abstract.*

2. If the unit of the dividend is a superficial unit, and the unit of the divisor a linear unit, *the unit of the quotient will be linear.*

3. If the unit of the dividend is a unit of volume, and the unit of the divisor linear, *the unit of the quotient will be superficial.*

4. If the unit of the dividend is a unit of volume, and the unit of the divisor superficial, *the unit of the quotient will be linear.*

EXAMPLES.

1. Divide 29*sq. ft.* 0′ 4″ by 6*ft.* 4′.

2. Divide 49*sq. ft.* 0′ 10″ 6‴ by 9*ft.* 6′.

3. What is the length of a floor whose area is 1176*sq. ft.* 1′ 6″, and breadth 24*ft.* 3′?

4. A load of wood, containing 119*cu. ft.* 2′ 6″ 8‴, is 3*ft.* 4′ high, and 4*ft.* 2′ wide: what is its length?

5. In a granite pillar there are 105*cu. ft.* 5′ 7″ 6‴; it is 3*ft.* 9′ wide, and 2*ft.* 3′ thick: what is its length?

6. There are 394*sq. ft.* 2′9″ in the floor of a hall that is 10*ft.* 7′ wide: what is its length?

7. A board 17*ft.* 6′ long, contains 27*sq. ft.* 8′ 6″: what is its width?

8. From a cellar 42*ft.* 10′ long, 12*ft.* 6′ wide, were thrown 158*cu. yds.* 17*cu. ft.* 4′ of earth: how deep was it?

DECIMAL FRACTIONS.

143. There are two kinds of Fractions: *Common Fractions* and *Decimal Fractions.*

A Common Fraction is one in which the unit is divided into any number of equal parts.

A Decimal Fraction is one in which the unit is divided according to the *scale of tens.*

144. If the unit 1 be divided into 10 equal parts, each part is called *one-tenth.*

If the unit 1 be divided into one hundred equal parts, each part is called *one-hundredth.*

If the unit 1 be divided into one thousand equal parts, the parts are called *thousandths,* and we have like expressions for the parts, when the unit is further divided according to the scale of tens.

These fractions may be written thus:

Three-tenths, - - - - - -	$\frac{3}{10}$.
Seventh-tenths, - - - - -	$\frac{7}{10}$.
Sixty-five hundredths, - - - -	$\frac{65}{100}$.
215 thousandths, - - - - -	$\frac{215}{1000}$.
1275 ten thousandths, - - - -	$\frac{1275}{10000}$.

From which we see, that in each case the denominator indicates the fractional unit; that is, determines whether the parts are tenths, hundredths, thousandths, &c.

---

143. How many kinds of fractions are there? What are they? What is a common fraction? What is a decimal fraction?

144. When the unit 1 is divided into 10 equal parts, what is each part called? What is each part called when it is divided into 100 equal parts? When into 1000? Into 10,000, &c.? How are decimal fractions formed? What gives denomination to the fraction?

145. The denominators of decimal fractions are seldom set down. The fractions are usually expressed by means of a period, placed at the left of the numerator.

Thus,	$\frac{3}{10}$	is written	.3
	$\frac{65}{100}$		.65
	$\frac{215}{1000}$		.215
	$\frac{1275}{10000}$		.1275

This method of writing decimal fractions is a mere language, and is used to avoid writing the denominators. The denominator, however, of every decimal fraction is always understood:

It is the unit 1 *with as many ciphers annexed as there are places of figures in the decimal.*

The place next to the decimal point, is called the place of *tenths*, and its unit is 1 tenth. The next place, at the right, is the place of *hundredths*, and its unit is 1 hundredth; the next is the place of *thousandths*, and its unit is 1 thousandth; and similarly for places still to the right.

DECIMAL NUMERATION TABLE.

Tenths. Hundredths. Thousandths. Ten thousandths. Hundred thousandths. Millionths. Ten millionths.

.4	is read 4 tenths.
.5 4	- - 54 hundredths.
.0 6 4	- - 64 thousandths.
.6 7 5 4	- - 6754 ten thousandths.
.0 1 2 3 4	- - 1234 hundred thousandths.
.0 0 7 6 5 4	- - 7654 millionths.
.0 0 4 3 6 0 4	- - 43604 ten millionths.

NOTE.—Decimal fractions are numerated from left to right; thus, *tenths, hundredths, thousandths, &c.*

146. Write and numerate the following decimals:

Six-tenths, - - - - -	.6
Six hundredths, - - -	.0 6
Six thousandths, - - -	.0 0 6
Six ten thousandths, - - -	.0 0 0 6
Six hundred thousandths, - -	.0 0 0 0 6
Six millionths, - - - -	.0 0 0 0 0 6
Six ten millionths, - - -	.0 0 0 0 0 0 6

Here we see that the same figure denotes different decimal units, according to the place which it occupies; therefore,

The value of the unit, in the different places, in passing from the left to the right, diminishes according to the scale of tens.

Hence, ten of the units in any place, are equal to one unit in the place *next to the left;* that is, ten thousandths make one hundredth, ten hundredths make one-tenth, and ten-tenths make the unit 1.

This scale of *increase*, from the *right* hand towards the *left*, is the same as that in whole numbers; therefore,

Whole numbers and decimal fractions may be united by placing the decimal point between them; thus,

Whole numbers.								Decimals.						
Tens of millions.	Millions.	Hundreds of thousands.	Tens of thousands.	Thousands.	Hundreds.	Tens.	UNITS.	Tenths.	Hundredths.	Thousandths.	Ten thousandths.	Hundred thousandths.	Millionths.	Ten millionths.
4	2	0	3	1	4	5	1.	2	0	4	3	0	7	8.

---

145. Are the denominators of decimal fractions generally set down? How are the fractions expressed? Is the denominator understood? What is it? What is the place next the decimal point called? What is its unit? What is the next place called? What is its unit? What is the third place called? What is its unit? Which way are decimals numerated?

A number composed partly of a whole number, and partly of a decimal, is called a *mixed number.*

RULE FOR WRITING DECIMALS.

Write the decimal as if it were a whole number, prefixing as many ciphers as are necessary to make it of the required denomination.

RULE FOR READING DECIMALS.

Read the decimal as though it were a whole number, adding the denomination indicated by the lowest decimal unit.

EXAMPLES.

Write the following numbers decimally:

(1.)	(2.)	(3.)	(4.)	(5.)
$\frac{6}{100}$	$\frac{17}{10}$	$\frac{5}{1000}$	$\frac{27}{100}$	$\frac{47}{1000}$.
(6.)	**(7.)**	**(8.)**	**(9.)**	**(10.)**
$6\frac{41}{100}$	$7\frac{8}{1000}$	$9\frac{5}{100}$	$10\frac{150}{100}$	$12\frac{327}{10}$.

Write the following numbers in figures, and numerate them:

1. Twenty-seven, and four-tenths.
2. Thirty-six, and fifteen thousandths.
3. Ninety-nine, and twenty-seven ten thousandths.
4. Three hundred and twenty thousandths.
5. Two hundred, and three hundred and twenty millionths.
6. Three thousand six hundred ten thousandths.
7. Five, and three millionths.
8. Forty, and nine ten millionths.

---

146. On what does the unit of a figure depend? How does the value change from the left towards the right? What do ten units of any one place make? How do the units of the places increase from the right towards the left? How may whole numbers be joined with decimals? What is such a number called? Give the rule for writing decimal fractions. Give the rule for reading decimal fractions.

9. Forty-nine hundred ten thousandths.
10. Fifty-nine and sixty-seven ten thousandths.
11. Four hundred and sixty-nine ten thousandths.
12. Seventy-nine, and four hundred and fifteen millionths.
13. Sixty-seven, and two hundred and twenty-seven ten thousandths.
14. One hundred and five, and ninety-five ten millionths.

UNITED STATES MONEY.

147. The denominations of United States Money correspond to the decimal division, if we regard 1 dollar as the unit.

For, *the dimes are tenths of the dollar, the cents are hundredths of the dollar, and the mills, being tenths of the cent, are thousandths of the dollar.*

EXAMPLES.

1. Express $37 and 26 cents and 5 mills, decimally.
2. Express $17 and 5 mills, decimally.
3. Express $215 and 8 cents, decimally.
4. Express $275 5 mills, decimally.
5. Express $9 8 mills, decimally.
6. Express $15 6 cents 9 mills, decimally.
7. Express $27 18 cents 2 mills, decimally.

ANNEXING AND PREFIXING CIPHERS.

148. Annexing a cipher is placing it on the right of a number.

If a cipher is annexed to a decimal it makes *one more decimal place*, and, therefore, a cipher must also be *added to the denominator* (Art. 145).

The numerator and denominator will therefore have been multiplied by the same number, and consequently the value of the fraction will not be changed (Art. 118): hence,

---

147. If the denominations of Federal Money be expressed decimally, what is the unit? What part of a dollar is 1 dime? What part of a dime is a cent? What part of a cent is a mill? What part of a dollar is 1 cent? 1 mill?

Annexing ciphers to a decimal fraction does not alter its value.

We may take as an example, $.5 = \frac{5}{10}$.

If we annex a cipher to the numerator, we must, at the same time, annex one to the denominator, which gives,

$.5 = \frac{50}{100} = .50$ by annexing one cipher.
$.5 = \frac{500}{1000} = .500$ by annexing two ciphers.
$.5 = \frac{5000}{10000} = .5000$ by annexing three ciphers.

Also, $.4 = \frac{4}{10} = .40 = \frac{40}{100} = .400 = \frac{400}{1000}$.

Also, $.7 = .70 = .700 = .7000 = .70000$.

149. Prefixing a cipher is placing it on the left of a number.

If ciphers are prefixed to the numerator of a decimal fraction, the same number of ciphers must be annexed to the denominator. Now, the numerator will remain unchanged while the denominator will be increased ten times for every cipher annexed; and hence, the value of the fraction will be *diminished* ten times for every cipher prefixed to the numerator (Art. 117).

Prefixing ciphers to a decimal fraction diminishes its value ten times for every cipher prefixed.

Take, for example, the fraction $.3 = \frac{3}{10}$.

$.3$ becomes $\frac{03}{100} = .03$ by prefixing one cipher.
$.3$ becomes $\frac{003}{1000} = .003$ by prefixing two ciphers.
$.3$ becomes $\frac{0003}{10000} = .0003$ by prefixing three ciphers:
in which the fraction is diminished ten times for every cipher prefixed.

---

148. When is a cipher annexed to a number? Does the annexing of ciphers to a decimal alter its value? Why not? What does five-tenths become by annexing a cipher? What by annexing two ciphers? Three ciphers? What does 7 tenths become by annexing a cipher? By annexing two ciphers? By annexing three ciphers?

149. When is a cipher prefixed to a number? When prefixed to a decimal, does it increase the numerator? Does it increase the denominator? What effect, then, has it on the value of the fraction?

ADDITION OF DECIMALS.

150. ADDITION of decimals is the operation of finding a single number which shall be equal in value to all the numbers added.

It must be remembered, that only units of the same value can be added together. Therefore, in setting down decimal numbers for addition, figures expressing the same unit must be placed in the same column.

The addition of decimals is then made in the same manner as that of whole numbers.

1. Find the sum of 87.06, 327.3 and .0567.

Place the decimal points in the same column: this brings units of the same value in the same column: then add as in whole numbers; hence,

OPERATION.

$$\begin{array}{r} 87.06 \\ 327.3 \\ .0567 \\ \hline 414.4167 \end{array}$$

RULE.—I. *Set down the numbers to be added so that figures of the same unit value shall stand in the same column.*

II. *Add as in simple numbers, and point off in the sum, from the right hand, as many places for decimals as are equal to the greatest number of places in any of the numbers added.*

PROOF.—The same as in simple numbers.

EXAMPLES.

1. Add 6.035, 763.196, 445.3741, and 91.5754 together.
2. Add 465.103113, .78012, 1.34976, .3549, and 61.11.
3. Add 57.406 + 97.004 + 4 + .6 + .06 + .3.
4. Add .0009 + 1.0436 + .4 + .05 + .047.

---

150. What is Addition? What parts of unity may be added together? How do you set down the numbers for addition? How will the decimal points fall? How do you then add? How many decimal places do you point off in the sum?

5. Add .0049 + 49.0426 + 37.0410 + 360.0039.

6. Add 5.714, 3.456, .543, 17.4957 together.

7. Add 3.754, 47.5, .00857, 37.5 together.

8. Add 54.34, .375, 14.795, 1.5 together.

9. Add 71.25, 1.749, 1759.5, 3.1 together.

10. Add 375.94, 5.732, 14.375, 1.5 together.

11. Add .005, .0057, 31.008, .00594 together.

12. Required the sum of 9 tens, 19 hundredths, 18 thousandths, 211 hundred-thousandths, and 19 millionths.

13. Find the sum of two, and twenty-five thousandths, five, and twenty-seven ten-thousandths, forty-seven, and one hundred twenty six-millionths, one hundred fifty, and seventeen ten-millionths.

14. Find the sum of three hundred twenty-seven thousandths, fifty-six ten-thousandths, four hundred, eighty-four millionths, and one thousand five hundred sixty hundred-millionths.

15. What is the sum of 5 hundredths, 27 thousandths, 476 hundred-thousandths, 190 ten-thousandths, and 1279 ten-millionths?

16. What is the sum of 25 dollars 12 cents 6 mills, 9 dollars 8 cents, 12 dollars 7 dimes 4 cents, 18 dollars 5 dimes 8 mills, and 20 dollars 9 mills?

17. What is the sum of 126 dollars 9 dimes, 420 dollars 75 cents 6 mills, 317 dollars 6 cents 1 mill, and 200 dollars 4 dimes 7 cents 3 mills?

18. A man bought 4 loads of hay, the first contained 1 ton 25 thousandths; the second, 997 thousandths of a ton; the third, 88 hundredths of a ton; and the fourth, 9876 ten-thousandths of a ton: what was the entire weight of the four loads?

19. Paid for a span of horses, $225,50; for a carriage, $127,055, and for harness and robes, $75,28: what was the entire cost?

20. Bought a barrel of flour for $9,375; a cord of wood for $2,12½; a barrel of apples for $1,62½; and a quarter of beef for $6,09: what was the amount of my bill?

21. A farmer sold grain, as follows: wheat, for $296.75;

corn, for $126,12½; oats, for $97,87½; rye, for $100,10; and barley, for $50,62½: what was the amount of his sale?

22. A person made the following bill at a store; 5 yards of cloth, for $16,408; 2 hats, for $4,87½; 4 pairs of shoes, for $6; 20 yards of calico, for $2, 378; and 12 skeins of silk, for $0,62½: what was the amount of his bill?

SUBTRACTION OF DECIMALS.

151. Subtraction of Decimal Fractions is the operation of finding the difference between two decimal numbers.

1. From 6.304 to take .0563.

Note.—In this example a cipher is annexed to the minuend to make the number of decimal places equal to the number in the subtrahend. This does not alter the value of the minuend (Art. 148): hence,

OPERATION.

6.3040
.0563
6.2477

Rule.—1. *Write the less number under the greater, so that figures of the same unit value shall fall in the same column.*

II. *Subtract as in simple numbers, and point off the decimal places in the remainder, as in addition.*

Proof.—Same as in simple numbers.

EXAMPLES.

1. From 3278 take .0879.
2. From 291.10001 take 41.496.
3. From 10.00001 take .111111.
4. Required the difference between 57.49 and 5.768.
5. What is the difference between .3054 and 3.075?
6. Required the difference between 1745.3 and 173.45.
7. What is the difference between seven-tenths and 54 ten thousandths?

---

151. What is subtraction of decimal fractions? How do you set down the numbers for subtraction? How do you then subtract? How many decimal places do you point off in the remainder?

8. What is the difference between .105 and 1.00075?

9. What is the difference between 150.43 and 754.355?

10. From 1754.754 take 375.49478.

11. Take 75.304 from 175.01.

12. Required the difference between 17.541 and 35.49.

13. Required the difference between 7 tenths and 7 millionths.

14. From 396 take 67 and 8 ten-thousandths.

15. From 1 take one-thousandth.

16. From 6374 take fifty-nine and one-tenth.

17. From 365.0075 take 5 millionths.

18. From 21.004 take 98 ten-thousandths.

19. From 260.3609 take 47 ten-millionths.

20. From 10.0302 take 19 millionths.

21. From 2.03 take 6 ten-thousandths.

22. From one thousand, take one-thousandth.

23. From twenty-five hundred, take twenty five hundredths.

24. From two hundred, and twenty seven thousandths, take ninety-seven, and one hundred twenty ten-thousandths.

25. A man owning a vessel, sold five thousand seven hundred sixty-eight ten thousandths of her: how much had he left?

26. A farmer bought at one time 127.25 acres of land, at another, 84.125 acres, at another, 116.7 acres. He wishes to make his farm amount to 500 acres: how much more must he purchase?

27. Bought a quantity of lumber for \$617.37½, and sold it for \$700: how much did I gain by the sale?

28. Having bought some cattle for \$325.50; some sheep for \$97.12½; and some hogs for \$60.87½; I sell the whole for \$510.10: what was my entire gain?

29. A dealer in coal bought 225.025 tons of coal; he sold to *A*, 1.05 tons, to *B*, 20.007 tons, to *C*, 40.1255 tons, and to *D*, 37.00056 tons: how much had he left?

30. A man owes \$2346.865, and has due him, from *A*, \$1240.06, and from *B*, \$1867.98½: how much will he have left after paying his debts?

31. Bought of each of two persons, 1284.05 pounds of wool, from which I sell to three persons, each 262.125 pounds: how much will I still have on hand?

MULTIPLICATION OF DECIMAL FRACTIONS.

152. MULTIPLICATION of decimal fractions is the operation of taking one number as many times as there are units in another, when one of the factors contains a decimal, or when they both contain decimals.

1. Multiply 8.03 by 6.104.

OPERATION.

$$\frac{803}{100} = 8.03$$

$$\frac{6104}{1000} = 6.104$$

```
    6.104
    1606
   803
  4818
 48.99906
```

ANALYSIS.—If we change both factors to common fractions, the product of the numerators will be the same as that of the decimal numbers, and *the number of decimal places will be equal to the number of ciphers in the two denominators*; hence,

RULE.—*Multiply as in simple numbers, and point off in the product, from the right hand, as many figures for decimals as there are decimal places in both factors; and if there be not so many in the product, supply the deficiency by prefixing ciphers.*

EXAMPLES.

1. Multiply 2.125 by 375 thousandths.
2. Multiply .4712 by 5 and 6 tenths.
3. Multiply .0125 by 4 thousandths.
4. Multiply 6.002 by 25 hundredths.
5. Multiply 473.54 by 57 thousandths.
6. Multiply 137.549 by 75 and 437 thousandths.
7. Multiply 3 and .7495 by 73487.

---

152. After multiplying, how many decimal places will you point off in the product? When there are not so many in the product, what do you do? Give the rule for the multiplication of decimals.

8. Multiply .04375 by 47134 hundred thousandths.

9. Multiply .371343 by seventy-five thousand 493.

10. Multiply 49.0754 by 3 and 5714 ten thousandths.

11. Multiply .573005 by 754 millionths.

12. Multiply .375494 by 574 and 375 hundredths.

13. Multiply two hundred and ninety-four millionths, by one millionth.

14. Multiply three hundred, and twenty-seven hundredths by 62.

15. Multiply 93.01401 by 10.03962.

16. Multiply 596.04 by 0.000012.

17. Multiply 38049.079 by 0.000016.

18. Multiply 1192.08 by 0.000024.

19. Multiply 76098.158 by 0.000032.

20. Multiply thirty-six thousand, by thirty-six thousandths.

21. Multiply 125 thousand, by 25 ten thousandths.

22. What is the product of 50 thousand, by 75 ten millionths?

23. What is the product of 48 hundredths, by 75 ten thousandths?

24. What are the contents of a lot of land 16.25 rods long, and 9.125 rods wide?

25. What are the contents of a board 12.07 feet long, and 1.005 feet wide?

26. What will 27.5 yards of cloth cost, at .875 dollars per yard?

27. At $25.125 an acre, what will 127.045 acres cost?

28. Bought 17.875 tons of hay, at $11.75 a ton: what was the cost of the whole?

29. A gentleman purchased a farm of 420.25 acres, for $35.08 an acre; he afterwards sold 196.175 acres to one man for $37.50 an acre, and the remainder to another person, for $36.125 an acre: what did he gain on the first cost?

30. A merchant bought two pieces of cloth, one containing 87.5 yards, at $2.75 a yard, and the other, containing 27.35 yards, at $3.125 a yard; he sold the whole at an average price of $2.94 a yard: did he gain or lose by the bargain, and how much?

CONTRACTIONS IN MULTIPLICATION.

153. CONTRACTION, in the multiplication of decimals, is a short method of finding the product of two decimal numbers in such a manner, that it shall contain but a given number of decimal places.

1. Let it be required to find the product of 2.38645 multiplied by 38.2175, in such a manner that it shall contain but four decimal places.

ANALYSIS.—It is proposed, in this example, to take the multiplicand 2.38645, 38 times, then 2 tenths times, then 1 hundredth times, then 7 thousandths times, then 5 ten-thousandths times, and the sum of these several products will be the product sought.

OPERATION.
2.38645
5712.83
715935
190916
4773
239
167
12
91.2042

Write the unit figure of the multiplier directly under that place of the multiplicand which is to be retained in the product, and the remaining places of integer figures, if any, at the right, and then write the decimal places at the left in their order, tenths, hundredths, &c.

When the numbers are so written, the product of any figure in the multiplier by the figure of the multiplicand directly over it, will be of the same order of value as the last figure to be retained in the product.

Therefore, the first figure of each product is always to be arranged directly under the *last retained figure of the multiplicand.* But it is the whole of the multiplicand which should be multiplied by each figure of the multiplier, and not a part of it only. Hence, to compensate for the part omitted, we begin

153. What is contraction in the multiplication of decimals? What is proposed in the example? How are the numbers written down for multiplication? When the numbers are so written, what will be the order of value of the product of any figure of the multiplier by the figure directly over it? Where then is the first figure of each product to be written? How do you compensate for the part omitted?

with the figure at the right of the one directly over any multiplier, and carry one when the product is greater than 5 and less than 15, 2 when it falls between 15 and 25, 3 when it falls between 25 and 35, and so on for the higher numbers.

For example, when we multiply by the 8, instead of saying 8 times 4 are 32, and writing down the 2, we say first, 8 times 5 are 40, and then carry 4 to the product 32, which gives 36. So, when we multiply by the last figure 5, we first say, 5 times 3 are 15, then 5 times 2 are 10 and 2 to carry make 12, which is written down.

EXAMPLES.

1. Multiply 36.74637 by 127.0463, retaining three decimal places in the product.

Contraction.	*Common way.*
36.74637	36.74637
3640.721	127.0463
3674637	11023911
734927	22047822
257225	14698548
1470	25722459
220	7349274
11	3674637
4668.490	4668.490346931

2. Multiply 54.7494367 by 4.714753, reserving five places of decimals in the product.

3. Multiply 475.710564 by .3416494, retaining three decimal places in the product.

4. Multiply 3754.4078 by .734576, retaining five decimal places in the product.

5. Multiply 4745.679 by 751.4549, and reserve only whole numbers in the product.

154. Note.—When a decimal number is to be multiplied by 10, 100, 1000, &c., the multiplication may be made by removing the decimal point as many places to the right hand as there are ciphers in the multiplier; and if there be not so many figures on the right of the decimal point, supply the deficiency by annexing ciphers.

$$\text{Thus, 4.27 multiplied by } \left.\begin{array}{l}10\\100\\1000\\10000\\100000\end{array}\right\} = \left\{\begin{array}{r}42.7\\427\\4270\\42700\\427000\end{array}\right.$$

$$\text{Also, 596.027 multiplied by } \left.\begin{array}{l}10\\100\\1000\\10000\\100000\end{array}\right\} = \left\{\begin{array}{r}5960.27\\59602.7\\596027\\5960270\\59602700\end{array}\right.$$

DIVISION OF DECIMAL FRACTIONS.

155. Division of Decimal Fractions is the operation of finding a number which multiplied by the divisor will produce the dividend, when one of the parts is a decimal fraction, or when both are fractional.

OPERATION.

```
2.043).71505(35
       6129
       -----
       10215
       10215
       -----
   Ans. 0.35.
```

Analysis.—Since the dividend must be equal to the product of the divisor and quotient, it must contain as many decimal places as both of them (Art. 152): therefore,

There must be as many decimal places in the quotient as the decimal places in the dividend exceed those in the divisor : hence,

Rule.—*Divide as in simple numbers, and point off in the*

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154. How do you multiply a decimal number by 10, 100. 1000, &c.? If there are not as many decimal figures as there are ciphers in the multiplier, what do you do?

155. If one decimal fraction be multiplied by another, how many decimal places will there be in the product? How does the number of decimal places in the dividend compare with those in the divisor and quotient? How do you determine the number of decimal places in the quotient? *Give the rule* for the division of decimals.

quotient, from the right hand, as many places for decimals as the decimal places in the dividend exceed those in the divisor; and if there are not so many, supply the deficiency by prefixing ciphers.

EXAMPLES.

1. Divide 4.6842 by 2.11.
2. Divide 12.82561 by 1.505.
3. Divide 33.66431 by 1.01.
4. Divide .010001 by .01.
5. Divide 24.8410 by .002.
6. Divide .0125 by 2.5.
7. Divide .051 by .012.
8. Divide .063 by 9.
9. Divide 1.05 by 14.
10. Divide 5.1435 by 4.05.
11. Divide .46575 by 31.05.
12. Divide 2.46616 by .145.

13. What is the quotient of 75.15204, divided by 3? By .3? By .03? By .003? By .0003?

14. What is the quotient of 389.27688, divided by 8? By .08? By .008? By .0008? By .00008?

15. What is the quotient of 374.598, divided by 9? By .9? By .09? By .009? By .0009? By .00009?

16. What is the quotient of 1528.4086488, divided by 6? By .06? By .006? By .0006? By .00006? By .000006?

17. Divide 17.543275 by 125.7.

18. Divide 1437.5435 by .7493.

19. Divide .000177089 by .0374.

20. Divide 1674.35520 by 960?

21. Divide 120463.2000 by 1728.

22. Divide 47.54936 by 34.75.

23. Divide 74.35716 by .00573.

24. Divide .37545987 by 75.714.

25. If 25 men remove 154.125 cubic yards of earth in a day, how much does each man remove?

26. If 167 dollars 8 dimes 7 cents and 5 mills be equally divided among 17 men, how much will each receive?

27. Bought 45.22 yards of cloth for $97.223: how much was it a yard?

28. If 375.25 bushels of salt cost $232.655, what is the price per bushel?

29. At $0.125 per pound, how much sugar can be bought for $2.25?

30. How many suits of clothes can be made from 34 yards of cloth, allowing 4.25 yards for each suit?

31. If a man travel 26.18 miles a day, how long will it take him to travel 366.52 miles?

32. A miller wishes to purchase an equal quantity of wheat, corn, and rye; he pays for the wheat, $2.225 a bushel; for the corn, $0.985 a bushel; and for the rye, $1.168 a bushel: how many bushels of each can he buy for $242.979?

33. A farmer purchased a farm containing 56 acres of woodland, for which he paid $46.347 per acre; 176 acres of meadow land, at the rate of $59.465 per acre; besides which there was a swamp on the farm that covered 37 acres, for which he was charged $13.836 per acre. What was the area of the land; what its cost; and what the average price per acre?

34. A person dying has $8345 in cash, and 6 houses, valued at $4379.837 each; he ordered his debts to be paid, amounting to $3976.480, and $120 to be expended at his funeral; the residue was to be divided among his five sons in the following manner: the eldest was to have a fourth part, and each of the other sons to have equal shares. What was the share of each son?

PARTICULAR CASES.

156. Note.—1. When any decimal number is to be divided by 10, 100, 1000, &c., the division is made by removing the decimal point as many places to the left as there are 0's in the divisor; and if there be not so many figures on the left of the decimal point, the deficiency must be supplied by prefixing ciphers.

---

156. How do you divide a decimal number by 10, 100, 1000, &c.? If there be not as many figures at the left of the decimal point as there ciphers in the divisor, what do you do?

$$49.87 \text{ divided by } \left\{\begin{array}{l}10\\100\\1000\\10000\end{array}\right\} = \left\{\begin{array}{l}4.987\\.4987\\.04987\\.004987\end{array}\right.$$

$$327.56 \text{ divided by } \left\{\begin{array}{l}10\\100\\1000\\10000\\100000\end{array}\right\} = \left\{\begin{array}{r}32.756\\3.2756\\.32756\\.032756\\.0032756\end{array}\right.$$

157. NOTE.—2. When there are more decimal places in the divisor than in the dividend, annex as many ciphers to the dividend as are necessary to make its decimal places equal to those of the divisor; *all the figures of the quotient will then be whole numbers.* Always bear in mind that *the quotient is as many times greater than* 1, *as the dividend is times greater than the divisor.*

EXAMPLES.

1. Divide 4397.4 by 3.49.

We annex one 0 to the dividend. Had it contained no decimal place, we should have annexed two.

OPERATION.

```
3.49)4397.40(1260
     349
     ---
     907
     698
     ----
     2094
     2094
     ----
   Ans. 1260
```

2. Divide 1097.01097 by .100001.
3. Divide 9811.0047 by .1629735.
4. Divide .1 by one ten-thousandths.
5. Divide 10 by one-tenth.
6. Divide 6 by .6. By .06. By .006. By .2. By .3. By .003. By .5. By .005. By .000012.

158. NOTE.—3. When it is necessary to continue the division farther than the figures of the dividend will allow, we may annex ciphers to it, and consider them as decimal places.

---

157. If there are more decimal places in the divisor than in the dividend, what do you do? What will the figures of the quotient then be?

158. How do you continue the division after you have brought down all the figures of the dividend? When the division does not terminate, what sign do you place after the quotient? What does it show?

EXAMPLES.

1. Divide 4.25 by 1.25.

In this example, after having exhausted the decimals of the dividend, we annex an 0, and then the decimal places used in the dividend will exceed those in the divisor by 1.

OPERATION.

```
1.25)4.25(3.4
     3.75
      500
      500
Ans.  3.4
```

2. Divide .2 by .06.

We see, in this example, that the division will never terminate. In such cases the division should be carried to the third or fourth place, which will give the answer true enough for all practical purposes, and the sign + should then be written, to show that the division may still be continued.

OPERATION.

```
.06).20(3.333 +
     18
      20
      18
       20
       18
        20
Ans. 3.333 +
```

3. Divide 37.4 by 4.5.
4. Divide 586.4 by 375.
5. Divide 94.0369 by 81.032.
6. Divide 36.2678 by 2.25.

159. Note.—4. If we regard 1 dollar as the unit of United States Currency, all the lower denominations, dimes, cents, and mills, are decimals of the dollar. Hence, all the operations upon United States Money are the same as the corresponding operations on decimal fractions.

CONTRACTIONS IN DIVISION.

160. Contractions in division of decimals, are short methods of finding a quotient which shall contain a given number of decimal places.

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159. What is the unit of the currency of the United States? What parts of this unit are the inferior denominations, dimes, cents, and mills?

160. What are the contractions in division? Explain the process of making the division?

EXAMPLES.

1. Divide 754.347385 by 61.34775, and find a quotient which shall contain three places of decimals.

Common Method.

61.34775)754.34738500(12.296

61347	75
14086	988
12269	550
1817	4385
1226	9550
590	48350
552	12975
38	353750
36	808650
1	545100

Contracted Method.

61.34775)754.347385(12.296
61348
14086
12269
1817
1227
590
552
38
37
1

In the operation, by the common method, the figures at the right of the vertical line, do not affect the quotient figures:

1. *Note the unit of the first quotient figure and then note the number of figures which the quotient is to contain.*

2. *Select as many figures of the divisor as you wish places of figures in the quotient, and multiply the figures so selected by the first quotient figure, observing to carry for the figures cast off as in the contraction of multiplication.*

3. *Use each remainder as a new dividend, and in each following division omit one figure at the right of the divisor.*

NOTE.—In the example above, the order of the first quotient figure was obviously tens; hence there were two places of whole numbers; and as there were three decimal places required in the quotient, *five figures* of the divisor must be used.

2. Divide 59 by .74571845, and let the quotient contain four places of decimals.

160. What figures may be omitted in the contracted method?

3. Divide 17493.407704962 by 495.783269, and let the quotient contain four places of decimals.

4. Divide 98.187437 by 8.4765618, and let the quotient contain seven places of decimals.

5. Divide 47194.379457 by 14.73495, and let the quotient contain as many decimal places as there will be integers in it.

REDUCTION OF COMMON AND DECIMAL FRACTIONS.

161. *To change a common to a decimal fraction.*

The value of a fraction is the quotient of the numerator divided by the denominator (Art. 105.)

1. Reduce $\frac{7}{8}$ to a decimal.

ANALYSIS.—If we place a decimal point after the 7, and then write any number of 0's, after it, the value of the numerator will not be changed (Art. 148). If then, we divide by the denominator, the quotient will be the decimal number: hence,

OPERATION.

$$\begin{array}{r} 8)\underline{.7000} \\ .875 \end{array}$$

RULE.—*Annex decimal ciphers to the numerator and then divide by the denominator, pointing off as in division of decimals.*

EXAMPLES.

Reduce the following common fractions to decimals:

1. Reduce $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$.
2. Reduce $\frac{4}{5}$, $\frac{7}{8}$, and $\frac{5}{16}$.
3. Reduce $\frac{3}{8}$ and $\frac{1}{25}$.
4. Reduce $\frac{3}{192}$ and $\frac{4}{15}$.
5. Reduce $\frac{1}{8}$ and $\frac{3}{1000}$.
6. Reduce $\frac{9}{35}$ and $\frac{15}{34}$.
7. Express $\frac{1412}{5907}$ decimally.
8. Express $\frac{275}{3842}$ decimally.
9. Reduce $\frac{7}{16}$ and $\frac{5}{64}$.
10. Express $\frac{14}{3125}$ decimally.
11. Reduce $\frac{67}{125}$ and $\frac{93}{250}$.
12. Reduce $\frac{3}{8}$ of $\frac{2}{5}$ of 6.
13. Reduce $\frac{4}{5}$ of $1\frac{1}{2}$.
14. Reduce $\frac{9}{16}$ of $\frac{43}{50}$.
15. Reduce $\frac{2}{3}$ of $\frac{2\frac{1}{2}}{3\frac{1}{4}}$.
16. Reduce $\frac{10\frac{1}{4}}{20}$ and $\frac{4\frac{1}{4}}{758}$.
17. What is the decimal value of $\frac{2}{3}$ of $\frac{3}{5}$ multiplied by $\frac{5}{12}$.

---

161. How do you change a common to a decimal fraction?

162. How do you change a decimal to the form of a common fraction?

18. What is the value, in decimals, of $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{7}{8}$ divided by $\frac{3}{8}$ of $\frac{2}{4}$?

19. A man owns $\frac{7}{8}$ of a ship; he sells $\frac{4}{22}$ of his share: what part is that of the whole, expressed in decimals?

20. Bought $\frac{11}{30}$ of $87\frac{3}{11}$ bushels of wheat for $\frac{9}{20}$ of 7 dollars a bushel: how much did it come to, expressed in decimals?

21. If a man receives $\frac{8}{5}$ of a dollar at one time, $7\frac{1}{2}$ at another, and $8\frac{3}{4}$ at a third: how much in all, expressed in decimals?

22. What mixed decimal is equal to $\frac{5}{4}$ of 18, and $\frac{8}{11}$ of $\frac{11}{12}$ of $7\frac{4}{9}$ added together?

23. What decimal is equal to $\frac{2}{3}$ of $3\frac{1}{2}$ taken from $\frac{3}{5}$ of $8\frac{3}{4}$?

24. What decimal is equal to $\frac{16}{21}$, $\frac{13}{7}$, $\frac{2}{9}$, added together?

162. *To change a decimal to the form of a common fraction.*

A decimal fraction may be changed to the form of a common fraction by simply writing its denominator (Art. 145).

EXAMPLES.

Express the following decimals in vulgar fractions.

1. Reduce .25 and .75.
2. Reduce .125 and .625.
3. Reduce .105 and .0025.
4. Reduce .8015 and .6042.
5. Reduce .68375.
6. Reduce .01875.
7. Reduce .22575.
8. Reduce .265625.

DENOMINATE DECIMALS.

163. A DENOMINATE DECIMAL is one in which the unit of the fraction is a denominate number. Thus, .3 of a dollar, .7 of a shilling, .8 of a yard, &c., are denominate decimals, in which the units are, 1 dollar, 1 shilling, 1 yard.

CASE I.

164. *To find the value of a denominate number in decimals of a higher unit.*

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163. What is a denominate decimal?

164. How do you find the value of a denominate number in decimals of a higher unit?

1. Reduce £1 4s. $9\frac{3}{4}d.$ to the decimal of a £.

ANALYSIS.—We first reduce 3 farthings to the decimal of a penny, by dividing by 4. We then annex the quotient .75*d.* to the 9 pence. We next divide by 12, giving .8125, which is the decimal of a shilling. This we annex to the shillings, and then divide by 20.

OPERATION.

$\frac{3}{4}d. = .75d.$; hence,

$9\frac{3}{4}d. = 9.75d.$

12)9.75*d.*

.8125*s.*, and

20)4.8125*s.*

£.240625; therefore,

£1 4*s.* $9\frac{3}{4}d.$ = £1.240625.

RULE.—I. *Divide the lowest denomination by the units of the scale which connect it with the next higher, annexing ciphers, if necessary.*

II. *Annex the quotient to the next higher denomination and divide by the units of the scale; and proceed in the same manner through all the denominations, to the required unit.*

NOTE.—When any denomination, between the highest and the lowest is wanting, the number to be prefixed to the corresponding quotient, is 0.

EXAMPLES.

1. Reduce 14 drams to the decimal of a *lb.*, Avoirdupois.
2. Reduce 78*d.* to the decimal of a £.
3. Reduce 63 pints to the decimal of a peck.
4. Reduce 9 hours to the decimal of a day.
5. Reduce 375678 feet to the decimal of a mile.
6. Reduce 7*oz.* 19*dwt.* of silver to the decimal of a pound
7. Reduce 3*cwt.* 7*lb.* 8*oz.* to the decimal of a ton.
8. Reduce 2.45 shillings to the decimal of a £.
9. Reduce 1.047 roods to the decimal of an acre.
10. Reduce 176.9 yards to the decimal of a mile.
11. Reduce 2*qr.* 14*lb.* to the decimal of a *cwt.*
12. Reduce 10*oz.* 18*dwt.* 16*gr.* to the decimal of a *lb.*
13. Reduce 3*qr.* 2*na.* to the decimal of a yard.
14. Reduce 1*gal.* to the decimal of a hogshead.
15. Reduce 17*h.* 6*m.* 43*sec.* to the decimal of a day.
16. Reduce 4*cwt.* $2\frac{3}{5}$*qr.* to the decimal of a ton.
17. Reduce 19*s.* 5*d.* 2*far.* to the decimal of a pound.

18. Reduce $1R.$ $37P.$ to the decimal of an acre.
19. Reduce $2qr.$ $3na.$ to the decimal of an *Eng. Ell.*
20. Reduce $2yd.$ $2ft.$ $6\frac{1}{2}in.$ to the decimal of a mile.
21. Reduce $15'$ $22\frac{1}{2}''$ to the decimal of a degree.
22. Reduce 290 cubic inches to the decimal of a ton of round timber.
23. Reduce $3bush.$ $3pk.$ to the decimal of a chaldron.
24. Reduce $17yd.$ $1ft.$ $6in.$ to the decimal of a mile.
25. What decimal part of a year is $9\frac{1}{2}$ months?
26. What decimal part of a *lb.* is $10oz.$ $18dwt.$ $16gr.$?
27. What decimal part of an acre is $1R.$ $14P.$?
28. What decimal part of a chaldron is $45pk.$?
29. What decimal part of a mile is 72 yards?
30. What part of a ream of paper is 9 sheets?
31. What part of a rod in length is 4.0125 inches?
32. Reduce $10wk.$ $2da.$ to the decimal of a year.
33. Reduce 4 ℥ 1 ʒ 1 ℈ $10gr.$ to the decimal of a ℔.
34. Reduce $3qt.$ $1.75pt.$ to the decimal of a *hhd.*
35. Reduce $24sq.\ yd.$ $1.8sq.\ ft.$ to the decimal of an acre.
36. Reduce $2qr.$ $1na.$ $0.36in.$ to the decimal of a yard.
37. Reduce $3ft.$ $4'$ $8''$ $3'''$ to feet and decimals of a foot.

CASE II.

165. *To find the value of a decimal in integers of less denominations.*

1. What is the value of .832296 of a £?

ANALYSIS.—First multiply the decimal by 20, which brings it to the denomination of shillings, and after cutting off from the right as many places for decimals as there are in the given number, we have $16s.$ and the decimal .645920 over. This is reduced to pence by multiplying by 12, and then to farthings by multiplying by 4.

OPERATION.

$$\begin{array}{r} .832296 \\ 20 \\ \hline 16.645920 \\ 12 \\ \hline 7.751040 \\ 4 \\ \hline 3.004160 \end{array}$$

Ans. $16s.$ $7d.$ $3far.$

---

165. How do you find the value of a decimal in integers of less denominations?

RULE.—I. *Multiply the decimal by the units of the scale which connect it with the next less denomination, pointing off as in the multiplication of decimals.*

II. *Multiply the decimal part of the product as before, and continue so to do until the decimal is reduced to the required denominations. The integers at the left form the answer.*

EXAMPLES.

1. What is the value of .6725 of a hundred weight?
2. What is the value of .61 of a pipe of wine?
3. What is the value of .83229 of a £?
4. Required the value of .0625 of a barrel of beer.
5. Required the value of .42857 of a month.
6. Required the value of .05 of an acre.
7. Required the value of .3375 of a ton.
8. Required the value of .875 of a pipe of wine.
9. What is the value of .375 of a hogshead of beer?
10. What is the value of .911111 of a pound troy?
11. What is the value of .675 of an English ell?
12. What is the value of .001136 of a mile in length?
13. What is the value of .000242 of a square mile?
14. Required the value of .4629 degrees.
15. Required the value of .875 of a yard.
16. Required the value of .3489 of a pound apothecaries.
17. Required the value of .759 of an acre.
18. Required the value of .01875 of a ream of paper.
19. Required the value of .0055 of a ton.
20. Required the value of .625 of a shilling.
21. Required the value of .3375 of an acre.
22. Required the value of .785 of a year, of 365¼ days.

CIRCULATING OR REPEATING DECIMALS.

166. In changing a common to a decimal fraction, there are two general cases:

1st. When the division terminates; and

2d. When it does not terminate.

In the first case, the quotient will contain a *limited number* of decimal places, and the value of the common fraction will be *exactly* expressed decimally.

In the second case, the quotient will contain an *infinite number* of decimal places, and the value of the common fraction cannot be *exactly* expressed decimally.

CASE I.

167. *When the division terminates:*

When a common fraction is reduced to its lowest terms (which we suppose to be done in all the cases that follow), there will be no factor common to its numerator and denominator (Art. 120).

1. Reduce $\frac{17}{50}$ to its equivalent decimal.

ANALYSIS.—Annexing one decimal 0 to the numerator multiplies it by 10, or by 2 and 5; hence, 2 and 5 become *prime factors* of the numerator every time that an 0 is annexed. But if the division be exact, these prime factors, and *none others*, must also be found in the denominator (Art. 91).

OPERATION.

$$\begin{array}{l} 50)17.00(.34 \\ \underline{\;\;15\,0\;\;} \\ \;\;\;\;2\,00 \\ \;\;\;\;2\,00 \end{array}$$

---

166. How many cases are there in changing a vulgar to a decimal fraction? What are they? What distinguishes one of these cases from the other?

167. How do you determine when a vulgar fraction can be exactly expressed decimally? How many decimal places will there be in the quotient?

2. Reduce $\frac{5}{36}$ to its equivalent decimal.

OPERATION.

$$\begin{array}{r} 36)\ 50\ (.1388+ \\ \underline{36} \\ 140 \\ \underline{108} \\ 320 \\ \underline{288} \\ 320 \\ 288 \end{array}$$

ANALYSIS. — $36 = 18 \times 2 = 9 \times 2 \times 2 = 3 \times 3 \times 2 \times 2$; in which we see that the denominator contains *other factors* than 2 and 5, and hence, *the fraction cannot be exactly expressed by decimals* (Art. 91). Hence, to determine whether a common fraction can be exactly expressed decimally:

I. *Decompose the denominator into its prime factors; and if there are no factors other than* 2 *and* 5, *the exact division can be made:*

II. *If there are other prime factors, the exact division cannot be made.*

NOTE.—Every decimal 0 annexed to the numerator, introduces the two factors 2 and 5; and these factors must be introduced until we have as *many of each* as there are in the denominator after it shall have been decomposed into its prime factors 2 and 5. But the quotient will contain as many decimal places as there are decimal 0's in the dividend (Art. 155); hence,

The number of decimal places in the quotient will be equal to the greatest number of equal factors 2 *or* 5, *in the divisor.*

EXAMPLES.

1. Can $\frac{7}{25}$ be exactly expressed decimally? how many places?

OPERATION.

$$\begin{array}{r} 25)\ 70\ (.28 \\ \underline{50} \\ 200 \\ 200 \end{array}$$

$25 = 5 \times 5$; hence, the fraction can be exactly expressed decimally.

Find the decimals and *number of places* in the following:

1. Express $\frac{9}{150}$ decimally.
2. Express $\frac{13}{140}$ decimally.
3. Express $\frac{11}{320}$ decimally.
4. Express $\frac{17}{1280}$ decimally.
5. Express $\frac{11}{370}$ decimally.
6. Express $\frac{17}{500}$ decimally.
7. Express $\frac{7}{250}$ decimally.
8. Express $\frac{31}{720}$ decimally.

CASE II.

168. *When the division does not terminate.*

1. Let it be required to reduce $\frac{1}{3}$ to its equivalent decimal.

ANALYSIS.—By annexing decimal ciphers to the numerator 1, and making the division, we find the equivalent decimal to be .3333 +, &c., giving 3's as far as we choose to continue the division.

OPERATION.

$$\begin{array}{r} 3)\underline{1.0000} \\ .3333+ \end{array}$$

The further the *division is continued*, the *nearer* the value of the decimal will approach to $\frac{1}{3}$, the *true value* of the common fraction. We express this approach to equality of value, by saying, that if the division be continued *without limit*, that is, to *infinity*, the value of the decimal will then become *equal* to that of the common fraction; thus,

$$.3333 +, \text{ continued to } \textit{infinity} = \tfrac{1}{3};$$

for, every annexation of a 3 brings the value *nearer* to $\frac{1}{3}$.

Also, $\quad .9999 +, \text{ continued to } \textit{infinity} = 1;$

for, every annexation of a 9 brings the value *nearer* to 1.

2. Find the decimal corresponding to the common fraction $\frac{2}{9}$.

ANALYSIS.—Annexing decimal ciphers and dividing, we find the decimal to be .2222 +, in which we see that the figure 2 is continually *repeated*.

OPERATION.

$$\begin{array}{r} 9)\underline{2.0000} \\ .2222+ \end{array}$$

169. A CIRCULATING DECIMAL is a decimal fraction in which a *single figure*, or a *set of figures*, is constantly repeated.

170. A REPETEND is a *single figure* or a set of figures, which is constantly repeated.

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168. Can one-third be exactly expressed decimally? What is the form of the quotient? To what does the value of this quotient approach? When does it become equal to one-third?

169. What is a circulating decimal?

170. What is a repetend?

171. A SINGLE REPETEND is one in which only a single figure is repeated; as

$\frac{2}{9} = .2222 +$, or $\frac{3}{9} = .3333 +$.

Such repetends are expressed by simply putting a mark over the first figure; thus,

.2222 +, is denoted by .'2, and .3333 + by .'3.

172. A COMPOUND REPETEND has the same set of figures circulating alternately; thus,

$\frac{19}{33} = .57\ 57 +$, and $\frac{5723}{9999} = .5723\ 5723 +$,

are compound repetends, and are distinguished by marking the first and last figures of the circulating period. Thus, .57 57 + is written .'57′, and .5723 5723 + is written .'5723′.

173. A PURE REPETEND is one which begins with the first decimal figure; as

.'3, .'5, .'473′, &c.

174. A MIXED REPETEND is one which has significant figures or ciphers between the repetend and the decimal point; or which has whole numbers at the left hand of the decimal point; such figures are called *finite figures*. Thus,

.0'733′, .4'73′, .3'573′, 6.'5,

are all mixed repetends; .0, .4, .3, and 6, are the *finite figures*.

175. SIMILAR REPETENDS are such as begin at equal distances from the decimal points; as .3'54′, 2.7'534′.

176. DISSIMILAR REPETENDS are such as begin at different distances from the decimal point; as .'253′, .47'52′.

177. CONTERMINOUS REPETENDS are such as end at equal distances from the decimal points; as .1'25′, .'354′.

---

171. What is a single repetend?
172. What is a compound repetend?
173. What is a pure repetend?
174. What is a mixed repetend?
175. What are similar repetends?
176. What are dissimilar repetends?

178. Similar and Conterminous Repetends are such as begin and end at the same distances from the decimal point; thus, 53.2˙753′, 4.6˙325′, and .4˙632′, are similar and conterminous repetends.

REDUCTION OF REPETENDS TO COMMON FRACTIONS.

CASE I.

179. *To reduce a pure repetend to its equivalent common fraction.*

Analysis.—This proposition is to be analyzed by examining the law of forming the repetends.

1st. $\frac{1}{9}$ =.1111+&c.=.˙1; and $\frac{4}{9}$ =.4444+&c.=.˙4 :

2d. $\frac{1}{99}$ =.010101+&c =.˙01′; and $\frac{27}{99}$ =.2727+&c.=.˙27′:

3d. $\frac{1}{999}$=.001001+&c.=.˙001′; and $\frac{324}{999}$=.324324+&c.=.˙324′.

&c. &c. &c. &c.

The above law for the formation of repetends does not depend on the multipliers 4, 27, and 324, but would be the same for *any other figures;* hence,

The value of any pure repetend is equal to the number denoting the repetend, divided by as many 9's as there are figures.

EXAMPLES.

1. What is the equivalent common fraction of the repetend 0.3?

Now, $\frac{3}{9} = \frac{1}{3} = 0.33333 +. = 0.\dot{3}$.

2. What is the equivalent common fraction of the repetend .˙162′?

We have, $\frac{162}{999} = \frac{18}{111}$ *Ans.*

3. What are the simplest equivalent common fractions of the repetends .˙6, .˙162′, 0.˙769230′, .˙945′, and .˙09′?

4. What are the least equivalent common fractions of the repetends .˙594405′, .˙36′, and .˙142857′?

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177. What are conterminous repetends?

178. What are similar and conterminous repetends?

179. How do you find a common fraction equivalent to a pure repetend?

CASE II.

180. *To reduce a mixed repetend to its equivalent common fraction.*

ANALYSIS.—A mixed repetend is composed of the finite figures which precede, and of the repetend itself; hence, its value must be equal to such finite figures plus the repetend.

When the repetend begins at the decimal point, the unit of the first figure is .1. But if the repetend begins at any place at the right of the decimal point, the unit value of the first figure will be diminished ten times for each place at the right, and hence, 0's must be annexed to the 9's which form the divisor; therefore,

To the finite figures, add the repetend divided by as many 9's as it contains places of figures, with as many 0's annexed to them as there are places of decimal figures preceding the repetend; the sum reduced to its simplest form will be the equivalent fraction sought.

EXAMPLES.

1. Required the least equivalent common fraction of the mixed repetend, 2.4‘18′.

Now,

$$2.4‘18' = 2 + \tfrac{4}{10} + ‘18' = 2 + \tfrac{4}{10} + \tfrac{18}{990} = 2\tfrac{23}{55}. \textit{ Ans.}$$

2. Required the least equivalent common fraction of the mixed repetend .5‘925′.

We have, $.5‘925' = \frac{5}{10} + \frac{925}{9990} = \frac{16}{27}$. *Ans.*

3. What is the least equivalent common fraction of the repetend .008‘497133′?

We have, $.008‘497133' = \frac{8}{1000} + \frac{497133}{999999000} = \frac{83}{9768}$.

4. Required the least equivalent common fractions of the mixed repetends .13‘8, 7.5‘43′, .04‘354′, 37.5‘4, .6‘75′, and .7‘5437′.

5. Required the least equivalent common fractions of the mixed repetends 0.7‘5, 0.4‘38′, .09‘3, 4.7‘543′, .009‘87′, and .4‘5.

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180. How do you find the value of a mixed repetend?

CASE III.

181. *To find the finite figures and the repetend corresponding to any common fraction.*

1. Find the finite figures and the repetend corresponding to the fraction $\frac{6}{560}$.

OPERATION.

$$\frac{6}{560}=\frac{3}{280}.$$

$$\frac{3}{280}=\frac{3}{2\times2\times2\times5\times7}.$$

$$280)3.000+(.010\grave{}714285'$$

ANALYSIS.—1st. Reduce the fraction to its lowest terms, and then find all the factors 2 and 5 of the denominator.

2d. Add decimal ciphers to the numerator and make the division.

3d. The number of *finite decimals* preceding the first figure of the repetend will be equal to the greatest number of factors 2 or 5 (Art. 167). In this example it is 3.

4th. When a remainder is found which is the same as a *previous dividend*, the second repetend begins.

5th. The number of figures in any repetend will never exceed the number, less 1, of the units in that factor of the denominator which does not contain 2 or 5. In the example, that number is 7, and the number of figures of the repetend, is 6. Hence,

Divide the numerator of the common fraction, reduced to its lowest terms, by the denominator, and point off in the quotient the finite decimals, if any, and the repetend.

EXAMPLES.

1. Required to find whether the decimal, equivalent to the common fraction $\frac{249}{29304}$, is finite or circulating: required the finite figures, if any, and the repetend.

OPERATION.

$$\frac{249}{29304}=\frac{83}{9768}$$

$$\frac{83}{9768}=\frac{83}{2\times2\times2\times1221}$$

$$9768)83.00+(.008\grave{}497133'$$

ANALYSIS.—We first reduce the fraction to its lowest terms, giving $\frac{83}{9768}$. We then search for the factors 2 and 5 in the denominator, and find that 2 is a factor 3 times; hence, we know that there are three finite decimals preceding the repetend. We next divide the numerator 83 by the denominator 9768, and

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181. How do you find the finite figures and the repetend corresponding to any common fraction?

note that the repetend *begins* at the fourth place. After the ninth division, we find the remainder 83; at this point the figures begin to repeat; hence, the repetend has 6 places.

2. Find the finite decimals, if any, and the repetend, if any, of the fraction $\frac{210}{1120}$.

3. Find the finite decimals, if any, and the repetend, if any, of the fraction $\frac{4}{1160}$.

4. Find the finite decimals, if any, and the repetend, if any, of the fractions $\frac{12}{123}$, $\frac{80}{135}$, $\frac{72}{135}$.

PROPERTIES OF THE REPETENDS.

182. There are some properties of the repetends which it is important to remark.

1. Any finite decimal may be considered as a circulating decimal by making ciphers to recur; thus,

$$.35 = .35\grave{}0 = .35\grave{}00' = .35\grave{}000' = .35\grave{}0000', \&c.$$

2. If any circulating decimal have a repetend of any number of figures, it may be changed to one having twice or thrice that number of figures, or any multiple of that number.

Thus, a repetend $2.3\grave{}57'$ having two figures, may be changed to one having 4, 6, 8, or 10 places of figures. For,

$$2.3\grave{}57' = 2.3\grave{}5757' = 2.3\grave{}575757' = 2.3\grave{}57575757', \&c.;$$

so, the repetend $4.16\grave{}316'$ may be written

$$4.16\grave{}316' = 4.16\grave{}316316' = 4.16\grave{}316316316', \&c. \&c.;$$

and the same may be shown of any other. Hence, two or more repetends, having a different number of places in each, may be reduced to repetends having the same number of places, in the following manner:

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182. How may a finite decimal be made circulating? When a repetend has a given number of places, to what other form may it be reduced? How? Into what form may any circulating decimal be transformed? To what form may two or more repetends be reduced?

Find the least common multiple of the number of places in each repetend, and reduce each repetend to such number of places.

Ex. 1. Reduce .13‘8, 7.5‘43′ .04‘354′, to repetends having the same number of places.

Since the number of places are now 1, 2, and 3, the least common multiple is 6, and hence each new repetend will contain 6 places; that is,

.13‘8 = .13‘888888′; 7.5‘43′ = 7.5‘434343′; and
0.4‘354′ = 0.4‘354354′.

Ex. 2. Reduce 2.4‘18′, .5‘925′, .008‘497133′, to repetends having the same number of places.

3. Any circulating decimal may be transformed into another having finite decimals and a repetend of the *same* number of figures as the first. Thus,

.‘57′ = .5‘75′ = .57‘57′ = .575‘75′ = .5757‘57′; and
3.4‘785′ = 3.47‘857′ = 3.478‘578′ = 3.4785‘785′;

and hence, *any two repetends may be made similar.*

These properties may be proved by changing the repetends into their equivalent common fractions.

4. Having made two or more repetends similar by the last article, they may be rendered conterminous by the previous one; thus, *two or more repetends may always be made similar and conterminous.*

1. Reduce the circulating decimals 165.‘164′, .‘04′, .03‘7 to such as are similar and conterminous.

2. Reduce the circulating decimals .5‘3, .4‘75′, and 1.‘757′, to such as are similar and conterminous.

5. If two or more circulating decimals, having several repetends of equal places, be added together, their sum will have a repetend of the same number of places; for, *every two sets of repetends will give the same sum.*

6. If any circulating decimal be multiplied by any number, the product will be a circulating decimal having the same number of places in the repetend; for, *each repetend will be taken the same number of times, and consequently must produce the same product.*

ADDITION OF CIRCULATING DECIMALS.

183. To add circulating decimals :

I. *Make the repetends, in each number to be added, similar and conterminous.*

II. *Write the places of the same unit value in the same column, and so many figures of the second repetend in each as shall indicate with certainty, how many are to be carried from one repetend to the other : then add as in whole numbers.*

NOTE.—If all the figures of a repetend are 9's, omit them and add to the figure next at the left.

EXAMPLES.

1. Add .12ˋ5, 4.ˋ163′, 1.ˋ7143′, and 2.ˋ54′ together.

Dissimilar.	*Similar.*	*Similar and Conterminous.*	
.12ˋ5 =	.12ˋ5	= .12ˋ555555555555′ - - -	5555
4.ˋ163′ =	4.16ˋ316′	= 4.16ˋ316316316316′ - - -	3163
1.ˋ7143′ =	1.71ˋ4371′	= 1.71ˋ437143714371′ - - -	4371
2.ˋ54′ =	2.54ˋ54′	= 2.54ˋ545454545454′ - - -	5454
	The true sum	= 8.54ˋ854470131697′	1 to carry.

2. Add 67.3ˋ45′, 9.ˋ651′, .ˋ25′, 17.4ˋ7, .ˋ5, together.
3. Add .ˋ475′, 3.75ˋ43′, 64.ˋ75′, .ˋ57′, .1ˋ788′, together.
4. Add .ˋ5, 4.3ˋ7, 49.4ˋ57′, .4ˋ954′, .ˋ7345′, together.
5. Add .ˋ175′, 42.ˋ57′, .3ˋ753′, .4ˋ954′, 3.7ˋ54′, together.
6. Add 165, .ˋ164′, 147.ˋ04′, 4.ˋ95′, 94.3ˋ7, 4.ˋ712345′ together.

SUBTRACTION OF CIRCULATING DECIMALS.

184. To subtract one circulating decimal from another.

I. *Make the repetends similar and conterminous.*

II. *Subtract as in finite decimals, observing that when the repetend of the lower line is the larger,* 1 *must be carried to the first right hand figure.*

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183. How do you add Circulating Decimals ?
184. How do you subtract Circulating Decimals ?

EXAMPLES.

1. From 11.4‘75′ take 3.45‘735′.

Dissimilar.	*Similar.*	*Similar and Conterminous.*	
11.4‘75′ =	11.47‘57′ =	11.47‘575757′ - - - -	575
3.45‘735′ =	3.45‘735′ =	3.45‘735735′ - - - -	735
The true difference	=	8.01‘840021′ 1 to carry.	

2. From 47.5‘3 take 1.‘757′.
3. From 17.‘573′ take 14.5‘7.
4. From 17.4‘3 take 12.34‘3.
5. From 1.12‘754′ take .4‘7384′.
6. From 4.75 take .37‘5.
7. From 4.794 take .1‘744′.
8. From 1.45‘7 take .3654.
9. From 1.4‘937′ take .1475.

MULTIPLICATION OF CIRCULATING DECIMALS.

185. To multiply one circulating decimal by another.

Change the circulating decimals into their equivalent common fractions, and then multiply them together; then, reduce the product to its equivalent circulating decimal.

EXAMPLES.

1. Multiply 4.25‘3 by .257.

OPERATION.

4.25‘3 $= 4 + \frac{25}{100} + \frac{3}{900} = 4 + \frac{225}{900} + \frac{3}{900} = \frac{228}{900} = \frac{3828}{900}$
$= \frac{1914}{450} = \frac{957}{225}$.

Also, $.257 = \frac{257}{1000}$; hence,

$\frac{957}{225} \times \frac{257}{1000} = \frac{245949}{225000} =$ 1.09310‘6;

and since $225000 = 5 \times 5 \times 5 \times 5 \times 5 \times 2 \times 2 \times 2 \times 9$; there will be five places of finite decimals, and one figure in the repetend (Art. 167).

NOTE.—*Much labor will be saved in this and the next rule by keeping every fraction in its lowest terms; and when two fractions are to be multiplied together, cancel all the factors common to both terms before making the multiplication.*

185. How do you multiply Circulating Decimals?

2. Multiply .375ˋ4 by 14.75.
3. Multiply .4ˋ253′ by 2.57.
4. Multiply .437 by 3.7ˋ5.
5. Multiply 4.573 by .3ˋ75′.
6. Multiply 3.45ˋ6 by .42ˋ5.
7. Multiply 1.ˋ456′ by 4.2ˋ3.
8. Multiply 45.1ˋ3 by .ˋ245′.
9. Multiply .4705ˋ3 by 1.7ˋ35′.

DIVISION OF CIRCULATING DECIMALS.

186. To divide one circulating decimal by another.

Change the decimals into their equivalent common fractions, and find the quotient of these fractions. Then change the quotient into its equivalent decimal.

EXAMPLES.

1. Divide 56.ˋ6 by 137.

OPERATION.

$$56.\grave{}6 = 56 + \tfrac{6}{9} = \tfrac{510}{9} = \tfrac{170}{3}.$$

Then, $\tfrac{170}{3} \div 137 = \tfrac{170}{3} \times \tfrac{1}{137} = \tfrac{170}{411} = .\grave{}41362530'$.

2. Divide 24.3ˋ18′ by 1.792.
3. Divide 8.5968 by .2ˋ45′.
4. Divide 2.295 by .ˋ297′.
5. Divide 47.345 by 1.ˋ76′.
6. Divide 13.5ˋ169533 by 4.ˋ297′
7. Divide .ˋ45′ by .ˋ118881′.
8. Divide .ˋ475′ by .3ˋ753′.
9. Divide 3.ˋ753′ by .ˋ24′.

CONTINUED FRACTIONS.

1. If we take any irreducible fraction, as $\frac{15}{29}$, and divide both terms by the numerator, it will take the form

$$\frac{15}{29} = \frac{1}{\frac{29}{15}} = \frac{1}{1 + \frac{14}{15}},\ \text{by making the division.}$$

If now, we divide both terms of $\frac{14}{15}$ by the numerator 14, we have

$$\frac{14}{15} = \frac{1}{1 + \frac{1}{14}}.$$

186. How do you divide Circulating Decimals?

If, now, we replace $\frac{14}{15}$ by its value, $\frac{1}{1+\frac{1}{14}}$, we shall have

$$\frac{15}{29}=\cfrac{1}{1+\cfrac{1}{1+\frac{1}{14}}};$$

a fraction of this form is called a *continued fraction;* hence,

187. A CONTINUED FRACTION *has* 1 *for its numerator, and for its denominator a whole number plus a fraction which also has a numerator of* 1, *and for a denominator, a whole number plus a similar fraction, and so on, until the numerator of the added fraction becomes* 1.

2. Reduce $\frac{15}{19}$ to the form of a continued fraction.

$$\frac{15}{19}=\frac{1}{1+\frac{4}{15}};\quad \frac{4}{15}=\frac{1}{3+\frac{3}{4}};\quad \frac{3}{4}=\frac{1}{1+\frac{1}{3}}:$$

hence,

$$\frac{15}{19}=\cfrac{1}{1+\cfrac{1}{3+\cfrac{1}{1+\frac{1}{3}}}}.$$

3. Reduce $\frac{829}{437}$ to the form of a continued fraction.

$$\frac{829}{437}=2+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{3+\frac{1}{19}}}}}.$$

4. Reduce $\frac{65}{149}$ to the form of a continued fraction.

$$\frac{65}{149}=\cfrac{1}{2+\cfrac{1}{3+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{1+\frac{1}{2}}}}}}.$$

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187. What is a Continued Fraction?

NOTE.—In a similar manner, any irreducible common fraction may be placed under the form of a continued fraction.

188. Let us now consider the last example. The fractions,

$$\frac{1}{2},\quad \frac{1}{2+\frac{1}{3}},\quad \frac{1}{2+\cfrac{1}{3+\frac{1}{2}}},\quad \&c.,$$

are called, the *first*, *second*, *third*, &c., approximating fractions; and the object in placing a common fraction under the form of a continued fraction is to find its *approximate value.*

If we stop at the first approximating fraction, $\frac{1}{2}$, the denominator 2 will be *less* than the *true* denominator; hence, the value of the first approximating fraction will be *too great;* that is, it will exceed the value of the given fraction.

If we stop at the second, the denominator 3 will be *less* than the true denominator; hence, $\frac{1}{3}$ will be greater than the number to be added to 2; therefore, $2 + \frac{1}{3}$ is *too small,* and $1 \div 1 + \frac{1}{3}$ is *too large:* that is, it is greater than the value of the given fraction.

Thus, every *odd* approximating fraction gives a value *too large*, and every *even* one gives a value *too small.*

EXAMPLES.

1. Place $\frac{21}{39}$ under the form of a continued fraction, and find the value of each of the approximating fractions.

2. Place $\frac{47}{65}$ under the form of a continued fraction, and find the value of each of the approximating fractions.

3. Place $\frac{17}{27}$ under the form of a continued fraction, and find the value of each approximating fraction.

4. Place $\frac{67}{85}$ under the form of a continued fraction, and find the value of each approximating fraction.

5 Place $\frac{37}{87}$ under the form of a continued fraction, and find the value of each approximating fraction.

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188. What is an approximating fraction? Is the first approximating fraction too large or too small? How is the second? How are all the odd [illegible] How are all the even ones?

RATIO AND PROPORTION.

189. Two numbers having the same unit, may be compared in two ways:

1st. By considering *how much* one is greater or less than the other, which is shown by their difference; and,

2d. By considering *how many times* one is contained in the other, which is shown by their quotient.

In comparing two numbers, one with the other, by means of their difference, the less is always taken from the greater.

In comparing two numbers, one with the other, by means of their quotient, one of them must be regarded as a *standard* which *measures* the other, and the quotient which arises by dividing by the standard, is called the *ratio.*

190. Every ratio is derived from two terms: the first is called the *antecedent,* and the second the *consequent;* and the two, taken together, are called a *couplet.* The *antecedent* will be regarded as the *standard.*

If the numbers 3 and 12 be compared by their difference, the result of the comparison will be 9; for, 12 exceeds 3 by 9. If they are compared by means of their quotient, the result will be 4; for, 3 is contained in 12, 4 times; that is, 3 *measuring* 12, gives 4.

191. The ratio of one number to another is expressed in two ways:

1st. By a colon; thus, 3 : 12; and is read, 3 is to 12; or, 3 measuring 12.

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189. In how many ways may two numbers, having the same unit, be compared with each other? If you compare by their difference, what do you do? If you compare by the quotient, how do you regard one of the numbers? What is the ratio?

190. From how many terms is a ratio derived? What is the first term called? What is the second called? Which is the standard?

191. How may the ratio of two numbers be expressed? How read?

2d. In a fractional form, as $\frac{12}{3}$; or, 3 measuring 12.

192. If two couplets have the same ratio, their terms are said to be proportional: the couplets,

$$4 \; : \; 20 \quad \text{and} \quad 1 \; : \; 5,$$

have the same ratio 5; hence, the terms are proportional, and are written,

$$4 \; : \; 20 \; :: \; 1 \; : \; 5,$$

by simply placing a double colon between the couplets. The terms are read,

4 is to 20 as 1 is to 5,

and taken together, they are called a *proportion:* hence,

A proportion is a comparison of the terms of two equal ratios

What are the ratios of the proportions?

6	:	24	: :	8	:	32
9	:	36	: :	10	:	40
8	:	72	: :	12	:	108
4	:	48	: :	5	:	60

193. The 1st and 4th terms of a proportion are called the *extremes;* the 2d and 3d terms, the *means.* Thus, in any proportion,

$$6 \; : \; 24 \; :: \; 8 \; : \; 32,$$

6 and 32 are the *extremes*, and 24 and 8 the *means:*

Since $$\frac{24}{6} = \frac{32}{8},$$

we shall have, by reducing to a common denominator,

$$\frac{24 \times 8}{6 \times 8} = \frac{32 \times 6}{6 \times 8}.$$

But since the fractions are equal, and have the same denominators, their numerators must be equal, viz.:

$$24 \times 8 = 32 \times 6; \text{ that is,}$$

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192. If two couplets have the same ratio, what is said of the terms? How are they written? How read? What is a proportion?

193. Which are the extremes of a proportion? Which the means? What is the product of the extremes equal to?

In any proportion, the product of the extremes is equal to the product of the means.

Thus, in the proportions,

$1 : 8 :: 2 : 16$; we have $1 \times 16 = 2 \times 8$.
$4 : 12 :: 8 : 24$; " " $4 \times 24 = 12 \times 8$;

194. Since, in any proportion, the product of the extremes is equal to the product of the means, it follows that,

1st. *If the product of the means be divided by one of the extremes, the quotient will be the other extreme.*

Thus, in the proportion,

$4 : 16 :: 6 : 24$, and $4 \times 24 = 16 \times 6 = 96$;

then, if 96, the product of the means, be divided by one of the extremes, 4, the quotient will be the other extreme, 24; or, if the product be divided by 24, the quotient will be 4.

2d. *If the product of the extremes be divided by either of the means, the quotient will be the other mean.*

Thus, if $4 \times 24 = 16 \times 6 = 96$ be divided by 16, the quotient will be 6; or if it be divided by 6, the quotient will be 16.

EXAMPLES.

1. The first three terms of a proportion are 5, 10, and 19? what is the fourth term?
2. The first three terms of a proportion are 6, 24, and 14: what is the fourth term?
3. The first, second and fourth terms of a proportion are 9, 12 and 16: what is the third term?
4. The first, third and fourth terms of a proportion are 16, 8, and 20: what is the second term?
5. The second, third and fourth terms of a proportion are 48, 90, and 45: what is the first term?

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194. If the product of the means be divided by one of the extremes, what will the quotient be? If the product of the means be divided by either extreme, what will the quotient be?

SIMPLE AND COMPOUND RATIO.

195. The ratio of two single numbers is called a *Simple Ratio*, and the proportion which arises from the equality of two such ratios, a *Simple Proportion*.

If the terms of one ratio be multiplied by the terms of another, antecedent by antecedent, and consequent by consequent, the ratio of the products is called a *Compound Ratio*. Thus, if the two ratios

$$3 : 6 \quad \text{and} \quad 4 : 12$$

be multiplied together, we shall have the compound ratio

$$3 \times 4 : 6 \times 12, \quad \text{or} \quad 12 : 72;$$

in which the ratio is equal to the product of the simple ratios.

A proportion formed from the equality of two compound ratios, or from the equality of a compound ratio and a simple ratio, is called a *Compound Proportion*.

196. *What part one number is of another.*

When the standard, or antecedent, is greater than the number which it measures, the ratio is a proper fraction, and is such a part of 1, as the number measured is of the standard.

NOTE.—The standard is generally preceded by the word *of*, and in comparing numbers, may be named second, as in examples 7, 8, 9, 10, and 11, but it must always be used as a divisor, and should be placed first in the statement.

1. What part of 25 is 5? that is, what part of the standard 25, is 5?

$$\tfrac{5}{25} = \tfrac{1}{5}; \quad \text{or} \quad 25 : 5 :: 1 : \tfrac{1}{5};$$

that is, the standard is to the number measured as 1 to $\frac{1}{5}$; or, the number measured is one-fifth of the standard.

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195. What is a simple ratio? What is the proportion called which comes from the equality of two simple ratios? What is a compound ratio? What is a compound proportion?

196. When the standard is greater than the consequent, what kind of a number is the ratio? What part is 3 of 4? 6 of 12? What part of 4 is 16? 12 of 36?

2. What part of 6 is 4?
3. What part of 10 is 5?
4. What part of 34 is 17?
5. What part of 450 is 300?
6. What part of 96 is 16?
7. 8 is what part of 12?
8. 16 is what part of 48?
9. 18 is what part of 90?
10. 15 is what part of 165?
11. 9 is what part of 11?

DIRECT AND INVERSE PROPORTION.

197. It often happens that two numbers which are compared with each other, undergo, or may undergo, certain *changes of value*, in which case they represent *variable* and not *fixed* quantities. Thus, when we say, that the amount of work done, in a single day, will be proportional to the number of men employed, we mean, that if we *increase* the number of men the amount of work done will also be *increased;* or, if we *diminish* the number of men employed, the work done will also be *diminished*. This is called *Direct Proportion.*

If we say that a barrel of flour will serve 12 men a certain time, and ask how long it will serve 24 men, there is a certain relation between the number of men and time; but that relation is such that the time will *decrease* if the number of men is *increased,* and will *increase* if the number of men is *decreased.* This is called *Inverse Proportion;* hence,

1. *Two numbers are directly proportional when they increase or decrease together; in which case their ratio is always the same.*

2. *Two numbers are inversely or reciprocally proportional when one increases as the other decreases; in which case their product is always the same.*

NOTE.—This is sometimes called *Reciprocal Proportion.*

198. If we refer to the numeration table of integral and decimal numbers (Art. 146), we see that the unit of the first

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197. When are two numbers directly proportional? When are two numbers inversely proportional? Does their product then vary?

198. What relation exists between the units of place in the integral and decimal numeration table? Give an example?

place, at the left of 1, is 1 ten; that is, a number *ten times* as great as 1. The unit of the first decimal place at the right, is 1 tenth, a number only one-tenth of 1. The unit of the second place, at the left, is one hundred times as great as 1; while the unit of the second place, at the right, is only one hundredth of 1; and similarly for other corresponding places; hence,

The units of the places, taken at equal distances from the unit 1, are inversely proportional.

CAUSE AND EFFECT.

199. SIMPLE PROPORTION is a comparison of the terms of two simple ratios. Hence, a simple proportion consists of four single terms, in which the ratio of the first to the second is equal to the ratio of the third to the fourth. If three of these terms are known, the fourth can be easily found (Art. 193).

200. Whatever produces *Effects*, as men at work, animals eating, time, goods purchased or sold, money lent producing interest, and the like, may be regarded as *Causes*.

CAUSES are of two kinds, simple and compound:

A SIMPLE CAUSE has but a single element, as men at work, a portion of time, goods purchased or sold, and the like.

A COMPOUND CAUSE is made up of two or more simple elements, such as men at work taken in connection with time, and the like.

201. The results of causes, as work done, provisions consumed, money paid, cost of goods, and the like, may be regarded as *effects*.

A SIMPLE EFFECT is one which has but a single element.

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199. What is simple proportion? Of what does it consist? How many terms must be known before the rest can be found?

200. What are causes? How many kinds of causes are there? What is a simple cause? What is a compound cause?

201. What are effects? What is a simple effect? What is a compound effect?

A COMPOUND EFFECT is one which arises from the multiplication of two or more elements.

202. Causes which are of the same kind, that is, which can be reduced to the same unit, may be compared with each other; and effects which are of the same kind may likewise be compared with each other. From the nature of causes and effects, we know that

1st Cause : 2d Cause : : 1st Effect : 2d Effect;
and, 1st Effect : 2d Effect : : 1st Cause : 2d Cause.

203. Simple causes and simple effects give rise to simple ratios.

Compound causes or compound effects give rise to compound ratios.

204. All questions involving *simple ratios* are classed under SIMPLE PROPORTION; and all questions involving *compound ratios*, either under INVERSE or COMPOUND PROPORTION.

SIMPLE PROPORTION.

205. 1. If 8 barrels of flour cost $56, what will 9 barrels cost at the same rate?

NOTE.—We shall denote the required term of a proportion by the letter x.

ANALYSIS.—The condition "at the same rate" requires that the *quantity*, 8 barrels of flour have the same ratio to the quantity 9 barrels, as $56, the cost of 8 barrels, to x dollars, the cost of 9 barrels.

STATEMENT.

bar. *bar.* $ $
8 : 9 : : 56 : x

---

202. What causes are of the same kind? What causes may be compared with each other? What do we infer from the nature of causes and effects?

203. What ratios arise from simple causes and simple effects? From what do compound ratios arise?

204. What questions fall under simple proportion? Questions involving compound ratios, give rise to what kinds of proportion?

205. How is example 1 stated? How is the fourth term found? Which are the causes and which the effects?

Since the product of the two extremes is equal to the product of the two means (Art. 193)

$8 \times x = 56 \times 9$; and if 8 times $x = 56 \times 9$ x must be equal to this product divided by 8: that is,

OPERATION.

$$\begin{array}{c|c} \cancel{8} & \cancel{56}^{7} \\ x & 9 \\ \hline \multicolumn{2}{c}{x = \$63.} \end{array}$$

The 4th term is equal to the product of the second and third terms divided by the first.

NOTE.—In this example, 8 barrels of flour and 9 barrels of flour are the *causes*, and their cost, \$56 and \$63, are the *effects*.

2. If 36 dollars will buy 9 yards of cloth, how many yards, at the same rate, can be bought for \$44?

ANALYSIS.—Thirty-six dollars, (being the cost of 9 yards of cloth,) is an effect, and \$44 is also an effect. The effect \$36, has the same ratio to the effect \$44, as the first cause to the second cause.

STATEMENT.

$$\begin{array}{ccccccc} \$ & & \$ & & yd. & & yd. \\ 36 & : & 44 & :: & 9 & : & x \end{array}$$

NOTE.—In this example a cause is required. The required term, in every example will be either an effect or cause.

OPERATION.

$$\begin{array}{c|c} {}^{4}\cancel{36} & \cancel{44}^{11} \\ x & \cancel{9} \\ \hline \multicolumn{2}{c}{x = 11yd.} \end{array}$$

206. Hence, we have the following:

RULE.—I. *Write the terms in the statement so that the causes shall compose one couplet and the effects the other, and so that the required term shall fall in the 4th place.*

II. *Multiply the second and third terms together and divide the product by the first term:* Or,

Multiply the third term by the ratio of the first and second.

NOTES.—1. If the first and second terms have different units, they must be reduced to the same unit.

2. If the third term is a compound denominate number, it must be reduced to its smallest unit.

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206. What is the rule for stating questions by cause and effect? What is the rule for finding the 4th term?

3. The preparation of the terms, and writing them in their proper places, is called the *statement*.

4. When the vertical line is used, the unknown term is always written at the left.

EXAMPLES.

1. If 8 hats cost $24, what will 110 hats cost, at the same rate?

2. If 2 barrels of flour cost $15, what will 12 barrels cost?

3. If I walk 168 miles in 6 days, how far should I walk, at the same rate, in 18 days?

4. If 8*lb.* of sugar cost $1,28, how much will 13*lb.* cost?

5. If 300 barrels of flour cost $2100, what will 125 barrels cost?

6. If 120 sheep yield 330 pounds of wool, how many pounds will 36 sheep yield?

7. If 80 yards of cloth cost $340, what will 650 yards cost?

8. What is the value of 4*cwt.* of sugar, at 5 cents a pound?

9. If 6 gallons of molasses cost $1,95, what will 6 hogsheads cost?

10. If 16 men consume 560 pounds of bread in a month, how much would 40 men consume?

11. If a man travels at the rate of 630 miles in 12 days how far will he travel in a leap year, Sundays excepted?

12. If 2 yards of cloth cost $3,25, what will be the cost of 3 pieces, each containing 25 yards?

13. If 3 yards of cloth cost 18*s.* New York currency, what will 36 yards cost?

14. If the penny loaf weighs 8 ounces when a bushel of wheat cost 7*s.* 6*d.*, what ought it to weigh when wheat is 8*s.* 4*d.* per bushel?

15. If 5*A.* 1*R.* 16*P.* of land, cost $150.5, what will 125*A.* 2*R.* 20*P.* cost?

16. If 13*cwt.* 2*qr.* of sugar cost $129,93, what will be the cost of 9*cwt.*?

17. The clothing of a regiment of 750 men cost £2834 5*s.*: what will it cost to cloth a body of 10500 men?

18. If $3yd.$ $2qr.$ of cloth cost \$15,75, how much will $8yd.$ $3qr.$ of the same cloth cost?

19. If .5 of a house cost \$201.5, what would .95 cost?

20. What will 26.25 bushels of wheat cost, if 3.5 bushels cost \$8.40?

21. If the transportation of 2.5 tons of goods 2.8 miles costs \$1,80, what is that per *cwt.*?

22. If $\frac{3}{4}$ of a yard of cloth cost \$2,16, what will $\frac{7}{8}$ of a yard cost?

23. If $\frac{5}{7}$ of an ounce cost $\$\frac{11}{12}$, what will $1\frac{1}{2}oz.$ cost?

24. What is the cost of $16\frac{4}{5}lb.$ of sugar, if $14\frac{2}{3}lb.$ cost $\$1\frac{5}{6}$?

25. If $\$19\frac{1}{3}$ will buy $14\frac{1}{2}$ yards of cloth, how much will $39\frac{3}{8}$ yards cost?

26. If $\frac{7}{8}$ of a barrel of cider cost $\frac{9}{11}$ of a dollar, what will $\frac{11}{14}$ of a barrel cost?

27. If $\frac{3}{16}$ of a ship cost \$2880, what will $\frac{15}{32}$ of her cost?

28. What will $116\frac{1}{4}$ yards of cloth cost, if 462 yards cost \$150,66?

29. If $7\frac{7}{11}$ barrels of fish cost $\$31\frac{1}{4}$, what will $32\frac{3}{5}$ barrels cost?

30. How much wheat can be bought for $\$96\frac{7}{8}$, if $2bu.$ $1pk.$ cost $\$1,93\frac{3}{4}$?

31. If $\frac{5}{8}$ of a yard of cloth cost $\$1\frac{5}{9}$, what will $7\frac{1}{2}$ yards cost?

32. What will be the cost of 37.05 square yards of pavement, if 47.5 yards cost \$72.25?

33. If 3 paces or common steps be equal to 2 yards, how many yards will 160 paces make?

34. If a person pays half a guinea a week for his board, how long can he board for £21?

35. If 12 dozen copies of a certain book cost \$54.72, what will 297 copies cost at the same rate?

36. If \$3618 worth of provisions will subsist an army of 9000 men for 90 days, if the army be increased by 4500 men, how much would last them the same time?

37. A grocer bought a *hhd.* of rum for 80 cents a gallon, to

which he added as much water as reduced its cost to 60 cents a gallon: how much water did he put in?

38. A man failing in business, pays 60 cents for every dollar which he owes; he owes A \$3570, and B \$1875; how much does he pay each?

39. A bankrupt's effects amount to \$2328,75, his debts amount to \$3726: what will his creditors receive on a dollar?

40. If a person drinks 80 bottles of wine in 3 months, how much does he drink in a week?

41. If $4\frac{5}{7}$ yards of cloth cost 14*s.* 8*d.*, New York currency, what will $40\frac{4}{5}$ yards cost?

42. If a grocer use a false balance giving only $14\frac{3}{4}$*oz.* for a pound, how much will $154\frac{7}{8}$*lbs.* of just weight give, when weighed by the false balance?

43. If a dealer in liquors use a gallon measure which is too small by $\frac{1}{2}$ pint, what will be the true measure of 100 of the false gallons?

44. After A has travelled 96 miles on a journey, B sets out to overtake him, and travels 23 miles as often as A travels 19 miles: how far will B travel before he overtakes him?

45. A person owning $\frac{5}{7}$ of a coal mine, sold $\frac{3}{4}$ of his share for \$9345; what was the value of the whole mine?

46. At what time between 6 and 7 o'clock, will the hour and minute hands be exactly together?

47. If a staff 5 feet long casts a shadow 7 feet, what is the height of a steeple, whose shadow is 196 feet at the same time of day?

48. Two persons are 279 miles apart, start at the same time and travel toward each other. A goes 5 miles an hour, and B 4 miles: how many miles must each travel before they meet?

49. A can do a piece of work in 3 days, B in 4 days, and C in 6 days: in what time will they all do it, working together?

50. A can build a wall in 15 days, but with the assistance of C, he can do it in 9 days: in what time can C do it alone?

51. A and B take a job for which they are to receive \$165.75; A works himself and employs 7 hands; B does the same and employs 6 hands: what should each receive?

52. A watch, which is 10 minutes too fast at 12 o'clock, on Monday, gains *3min. 10sec.* per day: what will be the time by the watch at a quarter past ten in the morning of the following Saturday?

53. There are two clocks, one of which gains 10 minutes, and the other loses $7\frac{1}{2}$ minutes every 24 hours. They are together at noon on Tuesday: what will be the difference of their times at 6 o'clock on Friday morning?

54. If 15 men can be boarded 1 week for \$46,25, what will it cost to board 5 men and 6 boys, the same time, the boys being boarded at half price?

55. Two persons, A and B, are on the opposite sides of a wood, which is 536 yards in circumference; they begin to travel in the same direction at the same moment; A goes at the rate of 11 yards per minute, and B at the rate of 34 yards in 3 minutes: how many times must the quicker one go round the wood before he overtakes the slower?

INVERSE PROPORTION.

207.—1. The floor of a room is 20 feet long: what must be its breadth in order that it may contain 360 square feet?

ANALYSIS.—The length of the floor multiplied by its breadth, will give the area or contents; hence, the area divided by the length will give the breadth.

OPERATION.

$$\frac{360}{20}=18ft.\text{ breadth.}$$

NOTE.—When the area or contents of the room are known or given, the length of the room is *inversely* proportional to its breadth (Art. 197). If the length of the room containing a given area be multiplied by any number, the breadth of the room must be divided

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207. What are the contents of a floor equal to? What is the breadth equal to? When the contents of a floor are given, in what proportion is the length to the breadth? If two numbers are inversely proportional, what is either equal to?

by the same number. If the length be divided by any number, the breadth must be multiplied by the same number. Thus, a room 40 feet long and 9 feet wide, or 10 feet one way and 36 feet the other, have the same area, viz., 360 square feet. Hence, when two numbers are inversely proportional:

Either is equal to their product divided by the other.

CAUSE AND EFFECT.

208. We may regard the length of a room as *one cause* of its contents, and its breadth as *another cause* of its contents; for, the contents being equal to the product of the length and breadth, is the *effect* of them both.

In the case of the two rooms, one 40 feet long and 9 feet wide, and the other 10 feet by 36, the *effects* are the same. The causes are compound, each being composed of two elements (Art. 200); and since the effects are equal the causes are equal (Art. 202); hence,

When the causes are equal, the elements are inversely proportional.

1. If 36 men, in 12 days, can do a certain work, in what time will 48 men do the same work?

ANALYSIS.—The first cause is compounded of 36 men and 12 days, and is equal to $36 \times 12 = 432$, the number of days it would require 1 man to do the work.

The second cause is compounded of 48 men and the number of days it would require them to do the same work, and is equal to $48 \times x$.

But since the effects are the same, viz., the work done, the *causes must be equal;* hence, *the products of the elements of the causes are equal.* Therefore, in the solution of all like examples,

STATEMENT.

$$\left.\begin{array}{ccc} \textit{men.} & & \textit{men.} \\ 36 & : & 48 \\ \textit{days.} & & \textit{days.} \\ 12 & : & x \end{array}\right\} :: \ 1 : 1$$

$$432 \ : \ 48 \times x \ :: \ 1 : 1$$

OPERATION.

$$\begin{array}{r|l} \cancel{4} & 9 \\ \cancel{48} & \cancel{36} \\ x & \cancel{12} \end{array}$$

Ans. $x = 9$ days.

Write the elements of the cause containing the unknown element on the left of the vertical line for a divisor, and the elements of the other cause on the right for a dividend.

NOTES.—1. Since the effects are equal, they may each be denoted by 1; hence, the causes are to each other as 1 to 1.

2. It is evident, that in this class of questions the elements of the causes are *inversely* proportional; and hence, such questions have generally been arranged under the head of "Rule of Three Inverse."

EXAMPLES.

1. If $3\frac{3}{4}$ yards of cloth will make a coat and vest, when the cloth is $1\frac{1}{2}$ yards wide, how much cloth will be needed which is $\frac{5}{8}$ yards in width?

2. If a piece of land $16\frac{4}{5}$ rods in length contains $3\frac{1}{2}$ acres, what would be the length of a piece containing twice that number of acres?

3. How many yards of carpeting that is three-fourths of a yard wide, will carpet a room 36 feet long and 30 feet in breadth?

4. If a man can perform a journey in 8 days, walking 9 hours a day, how many days will it require if he walks 10 hours a day?

5. If a family of 15 persons have provisions for 8 months, by how many must the family be diminished that the provisions may last 2 years?

6. A garrison of 4600 men have provision for 6 months: to what number must the garrison be diminished that the provisions may last 2 years and 6 months?

7. A certain amount of provisions will subsist an army of 9000 men for 90 days: if the army be increased by 6000, how long will the same provisions subsist it?

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208. What may we regard as causing or producing the contents of a room? When two causes are equal, how are the elements? NOTE.—If the effects are equal, by what may they be denoted?

8. If 6 men and 3 boys can do a piece of work in 330 days, how long will it take 9 men and 4 boys to do the same work, under the supposition that each boy does half as much as a man?

9. Four thousand soldiers were supplied with bread for 24 weeks, each man to receive 14*oz.* per day; but, by some accident, 210 barrels containing 200*lb.* each were spoiled: what must each man receive in order that the remainder may last the same time?

10. Suppose 4000 soldiers after losing 210 barrels of bread each containing 200*lb.*, were to subsist on 13*oz.* each a day for 24 weeks; had none been lost they would have received 14*oz.* a day: what was the weight of the whole, and how much did they receive?

11. Let us now suppose 4000 soldiers to lose one-fourteenth of their bread, then to receive 13*oz.* each a day for 24 weeks: what was the whole weight of their bread including the lost, and how much would each have received per day had none been spoiled?

12. If 4 men can do a piece of work in 80 days, how many days will 16 men require to do the same work?

13. If 21 pioneers make a trench in 18 days, how many days will 7 men require to make a similar trench?

14. A certain piece of grass was to be mowed by 20 men in 6 days; one-half the workmen being called away, it is required to find in what time the rest will finish it?

15. If a field of grain be cut by 10 men in 12 days, in how many days would it be cut by 20 men?

16. If 90 barrels of flour will subsist 100 men for 120 days, how long will it subsist 75?

17. If a traveller perform a journey in 35.5 days, when the days are 13.566 hours long, in how many days of 11.9 hours would he perform the same journey?

18. If 50 persons consume 600 bushels of wheat in a year, how long would it last 5 persons?

19. A certain work can be done in 12 days, by working 4

hours each day: how many days would it require to do the same work by working 9 hours a day?

20. If 120 men can build $\frac{1}{2}$ mile of wall in $15\frac{1}{4}$ days, how many men would it require to build the same wall in $40\frac{2}{3}$ days?

21. A garrison of 3600 men has just bread enough to allow 24*oz.* a day to each man for 34 days; but a siege coming on, the garrison was reinforced to the number of 4800 men. How many ounces of bread a day must each man be allowed, to hold out 45 days against the enemy?

22. If 3 horses or 5 colts eat a certain quantity of oats in 40 days, in what time will 7 horses and 3 colts consume the same quantity?

23. If a person can perform a journey in 24 days of $10\frac{1}{2}$ hours each, in what time can he perform the same journey, when the days are $12\frac{1}{4}$ hours long?

24. A piece of land 40 rods long and 4 rods wide, is equivalent to an acre: what is the breadth of a piece 15 rods long that is equivalent to an acre?

25. If a person travelling 12 hours a day finish one half of a journey in 10 days, in what time will he finish the remaining half, travelling 9 hours a day?

26. How many pounds weight can be carried 20 miles for the same money that $4\frac{1}{2}$ hundred weight can be carried 36 miles?

27. If 20 men can perform a piece of work in 12 days, working 9 hours a day, how many men will accomplish the same work in one half the time, working 10 hours a day?

28. If 72 horses eat a certain quantity of hay in $7\frac{1}{2}$ weeks, how many horses will consume the same in 90 weeks?

29. Bought 5000 planks, 15 feet long and $2\frac{1}{2}$ inches thick; how many planks are they equivalent to, of $12\frac{1}{2}$ feet long and $1\frac{3}{4}$ inches thick?

30. If 12 pieces of cannon, eighteen pounders, can batter down a castle in 3 hours, in what time would nine twenty-four pounders batter down the same castle, both pieces of cannon being fired the same number of times, and their balls flying with the same velocity?

COMPOUND PROPORTION.

209. Compound Proportion is a comparison of compound ratios when the terms are unequal.

It embraces that class of questions in which the causes are compound, or in which the effects are compound. In this class of questions, either a cause or a single element of a cause, may be required; or an effect, or a single element of an effect may be required.

1. If 8 men in 12 days can build 80 rods of wall, how much will 6 men do in 18 days?

STATEMENT.

$$\text{1st Cause} : \text{2d Cause} :: \text{1st Effect} : \text{2d Effect.}$$

$$\left.\begin{matrix}8\\12\end{matrix}\right\} : \left.\begin{matrix}6\\18\end{matrix}\right\} :: 80 : x$$

or $12\times 8 : 18\times 6 :: 80 : x$

Analysis.—In this example the second effect is required, and the statement may be read thus:

If 8 men in 12 days can build 80 rods of wall, 6 men in 18 days will build how many (or x) rods of wall?

OPERATION.

$$\begin{array}{r|l} \cancel{2}\ \cancel{8} & \cancel{6}\ 9 \\ \cancel{12} & \cancel{18} \\ x & \cancel{80}\ 10 \end{array}$$

Ans. $x = 90$ rods.

2. If a family of 12 persons, in 8 months, expend $864, how many months will $900 serve a family of 20 persons?

STATEMENT.

$$\left.\begin{matrix}12\\8\end{matrix}\right\} : \left.\begin{matrix}20\\x\end{matrix}\right\} :: \$864 : \$900.$$

or, $12\times 8 : 20\times x :: \$864 : \$900.$

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209. What is compound proportion? What questions does it embrace? What is always required?

ANALYSIS.—In this example, an element of the second cause is required, viz., the number of months which the money will last 20 men. The question is thus stated:

If 12 persons, in 8 months, expend $864, 20 persons in how many (or x) months will expend $900?

OPERATION.

$$\begin{array}{r|l} \cancel{20} & \cancel{12} \\ x & \cancel{8} \\ \cancel{8}\ \cancel{12}\ \cancel{864} & \cancel{900}\ \cancel{45}\ 5 \end{array}$$

Ans. $x = 5$ months.

3. If 24 men, in 6 days, working 7 hours a day, can build a wall 115 feet long 3 feet thick and 4 feet high, how long a wall can 36 men build in 12 days, working 14 hours a day, if the wall is 4 feet thick and 5 feet in height?

STATEMENT.

$$\left.\begin{array}{r}24\\6\\7\end{array}\right\} : \left.\begin{array}{r}36\\12\\14\end{array}\right\} :: \left.\begin{array}{r}115\\3\\4\end{array}\right\} : \left.\begin{array}{r}x\\4\\5\end{array}\right\}$$

or, $24\times6\times7 \ : \ 36\times12\times14 \ :: \ 115\times3\times4 \ : \ x\times4\times5.$

ANALYSIS.—In this example, an element of the second *effect* is required, viz., the length of the wall; and the question may be thus stated:

If 24 men, in 6 days, working 7 hours a day, can build a wall 115 feet long, 3 feet thick, and 4 feet high, 36 men in 12 days, working 14 hours a day, will build a wall how many (or x) feet long, 4 feet thick and 5 feet high?

OPERATION.

$$\begin{array}{r|l} & 3 \\ \cancel{2} & \cancel{6} \\ \cancel{24} & \cancel{36} \\ \cancel{6} & \cancel{12}\ 2 \\ \cancel{7} & \cancel{14} \\ x & \cancel{115}\ 25 \\ \cancel{4} & 3 \\ \cancel{5} & \cancel{4} \end{array}$$

Ans. $x = 450$*ft.*

210. Hence, we have the following

RULE.—I. *Arrange the terms in the statement so that the causes shall compose one couplet, and the effects the other, putting* x *in the place of the required element.*

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210. Give the rule for stating the question and finding the unknown part

II. ***Then if* x *fall in one of the extremes, make the product of the means a dividend, and the product of the extremes a divisor; but if* x *fall in one of the means, make the product of the extremes a dividend, and the product of the means a divisor.***

EXAMPLES.

1. If 2 men can dig 125 rods of ditch in 75 days, in how many days can 18 men dig 243 rods?

2. If 400 soldiers consume 5 barrels of flour in 12 days, how many soldiers will consume 15 barrels in 2 days?

3. If a person can travel 120 miles in 12 days of 8 hours each, how far will he be able to travel in 15 days of 10 hours each?

4. If a pasture of 16 acres will feed 6 horses for 4 months, how many acres will feed 12 horses for 9 months?

5. If 60 bushels of oats will feed 24 horses 40 days, how long will 30 bushels feed 48 horses?

6. If 82 men build a wall 36 feet long, 8 feet high, and 4 feet thick, in 4 days, in what time will 48 men build a wall 864 feet long, 6 feet high, and 3 feet wide?

7. If the freight of 80 tierces of sugar, each weighing $3\frac{1}{2}$ hundred weight, for 150 miles, cost $84, what must be paid for the freight of 30 hogsheads of sugar, each weighing 12 hundred weight, for 50 miles?

8. A family consisting of 6 persons, usually drink 15.6 gallons of beer in a week: how much will they drink in 12.5 weeks, if the number be increased to 9? — 292.5

9. If 12 tailors in 7 days can finish 14 suits of clothes, how many tailors in 19 days can finish the clothes of a regiment of 494 men? — 156 [illegible]

10. If a garrison of 3600 men eat a certain quantity of bread in 35 days, at 24 ounces per day to each man, how many men, at the rate of 14 ounces per day, will eat twice as much in 45 days?

11. A company of 100 men drank £20 worth of wine at 2*s.* 6*d.* per bottle: how many men, at the same rate, will £7 worth supply, when wine is worth 1*s.* 9*d.* per bottle?

12. If the wages of 13 men for $7\frac{1}{2}$ days, be $149,76, what will be the wages of 20 men for $15\frac{1}{3}$ days?

13. If a footman travel 264 miles in $6\frac{3}{5}$ days of $12\frac{1}{2}$ hours each, in how many days of $10\frac{2}{7}$ hours each will he travel $129\frac{3}{5}$ miles?

14. If 120 men in 3 days, of 12 hours each, can dig a trench of 30 yards long, 2 feet broad, and 4 feet deep, how many men would be required to dig a trench 50 yards long, 6 feet deep, and $1\frac{1}{2}$ yards broad, in 9 days of 15 hours each?

15. If 40 men can perform a piece of work in 12 days, how many men will perform another piece of work three times as large, in one-fifth part of the time?

16. A person having a journey of 500 miles to perform, walks 200 miles in 8 days, walking 12 hours a day: in how many days, walking 10 hours a day, will he complete the remainder of the journey?

17. If 1000 men, besieged in a town, with provisions for 28 days, at the rate of 18 ounces per day for each man, be reinforced by 600 men, how many ounces a day must each man have that the provisions may last them 42 days?

18. If a bar of iron *5ft.* long, $2\frac{1}{2}$*in.* wide, and $1\frac{3}{4}$*in.* thick, weigh *45lbs.* how much will a bar of the same metal weigh that is *7ft.* long, *3in.* wide, and $2\frac{1}{4}$*in.* thick?

19. If 5 compositors in 16 days, working 14 hours a day, can compose 20 sheets of 24 pages each, 50 lines in a page, and 40 letters in a line, in how many days, working 7 hours a day, can 10 compositors compose 40 sheets of 16 pages in a sheet, 60 lines in a page, and 50 letters in a line?

20. Fifty thousand bricks are to be removed a given distance in 10 days. Twelve horses can remove 18000 in 6 days: how many horses can remove the remainder in 4 days?

21. If 3 men, working 10 hours a day, can plant a field 150 rods by 240 rods, in 5 days, how many men, working 12 hours a day, can plant a field measuring 192 rods by 300 rods, in 4 days?

22. If 248 men, in $5\frac{1}{2}$ days of 11 hours each, dig a trench of 7 degrees of hardness, $232\frac{1}{2}$ yards long, $3\frac{2}{3}$ wide, and $2\frac{1}{3}$ deep, in how many days, of 9 hours long, will 24 men dig a trench of 4 degrees of hardness, $337\frac{1}{2}$ yards long, $5\frac{3}{5}$ wide, and $3\frac{1}{2}$ deep?

PRACTICE.

210* PRACTICE is an easy and concise method of applying the rules of arithmetic to questions which occur in trade and business. It is only a contraction of the RULE OF THREE when the first term is unity.

For example, if 1 yard of cloth cost half a dollar, what will 60 yards cost? This is a question which may be answered by the rule called Practice. The cost is obviously $30.

One number is said to be an *aliquot* part of another, when it forms an exact part of it: that is, when it is contained in that other an *exact* number of times. Hence, an aliquot part is an exact or even part.

For example, 25 cents is an aliquot part of a dollar. It is an exact fourth part, and is contained in the dollar four times. So also, 2 months, 3 months, 4 months, and 6 months, are all aliquot parts of a year.

TABLE OF ALIQUOT PARTS.

Cts.	Parts of $1.	Mo.	Parts of a year.	Days.	Parts of 1 mo.	Parts of £1.	Parts of 1 shilling.
$50 =$	$\frac{1}{2}$	$6 =$	$\frac{1}{2}$	$15 =$	$\frac{1}{2}$	$10s. = \frac{1}{2}$	$6\,d. = \frac{1}{2}$
$33\frac{1}{3} =$	$\frac{1}{3}$	$4 =$	$\frac{1}{3}$	$10 =$	$\frac{1}{3}$	$6s.\ 8d. = \frac{1}{3}$	$4\,d. = \frac{1}{3}$
$25 =$	$\frac{1}{4}$	$3 =$	$\frac{1}{4}$	$7\frac{1}{2} =$	$\frac{1}{4}$	$5s. = \frac{1}{4}$	$3\,d. = \frac{1}{4}$
$20 =$	$\frac{1}{5}$	$2 =$	$\frac{1}{6}$	$6 =$	$\frac{1}{5}$	$4s. = \frac{1}{5}$	$2\,d. = \frac{1}{6}$
$12\frac{1}{2} =$	$\frac{1}{8}$	$1 =$	$\frac{1}{12}$	$5 =$	$\frac{1}{6}$	$3s.\ 4d. = \frac{1}{6}$	$1\frac{1}{2}d. = \frac{1}{8}$
$6\frac{1}{4} =$	$\frac{1}{16}$		or $\frac{1}{3}$ of	$3 =$	$\frac{1}{10}$	$2s.\ 6d. = \frac{1}{8}$	$1\,d. = \frac{1}{12}$
$5 =$	$\frac{1}{20}$		3*mo.*			$1s.\ 8d. = \frac{1}{12}$	

210*. What is Practice? When is one number said to be an aliquot part of another? What is an aliquot part? What are the aliquot parts of a dollar expressed in the table? What of a year? What of a month? What of a pound? What of a shilling?

EXAMPLES.

1. What is the cost of 376 yards of cloth at $0,75, or $\frac{3}{4}$ of a dollar per yard?

ANALYSIS.—Had the cloth cost $1 per yard, the cost of the 376 yards would have been $376. Had it cost 50*cts.* a yard, the cost would have been $\frac{1}{2}$ of $376, or $188: had it been 25 *cts.* per yard, the cost would have been $\frac{1}{4}$ of $376, or $94; but the price being 75*cts.* per yard, the cost is 188+94 =$282.

OPERATION.

cts.		$	
50	$\frac{1}{2}$	376	
		188	cost at 50 cents.
25	$\frac{1}{4}$	94	cost at 25 cents.
75	$\frac{3}{4}$	$282	cost at $\frac{3}{4}$*doll.*

2. What is the cost of 196 yards of cotton, at 9*d.* per yard?

196*yd.* at 6*d.* or $\frac{1}{2}$*s.* = 98*s.*
196*yd.* at 3*d.* or $\frac{1}{4}$*s.* = 49*s.*
Therefore, 196*yd.* at 9*d.* or $\frac{3}{4}$*s.* = 146*s.* = £7 7*s.* *Ans.*

3. What is the cost of 475 yards of tape, at $\frac{1}{4}$*d.* per yard?

$\frac{1}{4}$*d.* - - 4)4715
12)1178$\frac{3}{4}$*d.* = *cost.*
20)98*s.* 2$\frac{3}{4}$*d.*
Ans. = £4 18*s.* 2$\frac{3}{4}$*d.*

4. What is the cost of 425 yards at 1 penny per yard?

1*d.* = $\frac{1}{12}$*s.* - 12)425
20)35*s.* 5*d.*
Ans. £1 15*s.* 5*d.*

5. What will be the cost of 354 yards at 1$\frac{1}{4}$*d.* per yard?

1*d.* = $\frac{1}{12}$*s.* - 12)354
4)29*s.* 6*d.*
$\frac{1}{4}$*d.* - - - - 7*s.* 4$\frac{1}{2}$*d.*
cost 36*s.* 10$\frac{1}{2}$*d.*
= £1 16*s.* 10$\frac{1}{2}$*d.*

6. What will be the cost of 4756 yards of cotton shirting, at 12$\frac{1}{2}$ cents per yard?

12$\frac{1}{2}$*cts.* = $\frac{1}{8}$ of $1. 8)4756
594$\frac{1}{2}$
Ans. $594,50.

7. What will be the cost of 3754 pairs of gloves, at 2*s.* 6*d.* per pair?

2*s.* 6*d.* $=\frac{1}{8}$£. - - - 8)3754

$469\frac{2}{8}$

Ans. £469 5*s.*

8. What will be the cost of 5320 bushels of wheat, at 3*s.* 6*d.* per bushel?

	5320
	3
at 3*s.* - -	15960
at $\frac{1}{2}$*s.* - -	2660
at 3*s.* 6*d.* -	18620*s.*
Ans.	£9310

9. What will be the cost of 435 yards of cloth, at £2 7*s.* per yard?

	435
	2
Cost at £2 - -	£870
5*s.* $=\frac{1}{4}$ of £.	108 15*s.*
2*s.* $=\frac{1}{10}$ of £.	43 10*s.*
Total cost	£1022 5*s.*

10. What will 'be the cost of 660 yards at 2*s.* 6*d.*?

2*s.* $=\frac{1}{10}$£. 10)660

6*d.* $=\frac{1}{4}$ of 2*s.* 4)66 costs at 2*s.*

16 10*s.*

Ans. £82 10*s.*

11. What will be the cost of 40*lbs.* of soap, at $6\frac{3}{4}$*cts.* per pound? *Ans.* \$2,70.

12. What will be the cost of 70 yards of tape, at $2\frac{1}{4}$*cts.* per yard? *Ans.* \$1,575.

13. What will 876 bushels of apples cost, at $62\frac{1}{2}$ cents a bushel? *Ans.* \$547,50.

14. What will 1000 quills cost, at $1\frac{1}{2}$ cents for 5 quills? *Ans.* \$3,00.

15. What will 9*T.* 13*cwt.* 20*lb.* of pewter cost, at £14 15*s.* 9*d.* per ton? *Ans.* £142 16*s.* $11\frac{17}{50}$*d.*

16. What will be the cost of 74*oz.* 2*pwt.* 12*gr.* of silver, at 4*s.* $11\frac{1}{2}$*d.* per *oz.*? *Ans.* £18 7*s.* 6*d.* $1\frac{3}{4}$*far.*

17. What is the cost of a pair of chased silver salts, weighing 7*oz.* 11*pwt.*, at 8*s.* $11\frac{1}{4}$*d.* per *oz.*?

Ans. £3 7*s.* 5*d.* $2\frac{19}{20}$*far.*

18. What will be the cost of $85\frac{1}{2}$ yards of cloth, at $\$9\frac{1}{2}$ per yard? *Ans.* \$812,25.

19. What will be the cost of 1848 yards of linen cloth, at $87\frac{1}{2}$ cents per yard? *Ans.* \$1617,00.

20. What will be the cost of $51\frac{1}{2}$ tons of hay, at \$12,50 per ton? *Ans.* \$643,75.

21. What will be the cost of 696 yards of broadcloth, at $\$4\frac{7}{8}$ per yard? *Ans.* \$3393,00.

22. What will be the cost of 725*A.* 2*R.* 19*P.* of land, at £2 11*s.* 9*d.* per acre? *Ans.* £1877 10*s.* $9\frac{39}{160}$*d.*

23. What will 6*gal.* 1*qt.* 1*pt.* 2*gi.* of wine cost, at 5*s.* 4*d.* per quart? *Ans.* £6 17*s.* 4*d.*

24. What will 51 acres of land be worth at £3 2*s.* 2*d.* per acre? *Ans.* £158 10*s.* 6*d.*

25. What will 15*cwt.* 2*qr.* 17*lb.* of sugar come to, at 1*s.* per pound? *Ans.* £78 7*s.*

26. What will 4 *E. E.* 3*qr.* 2*na.* of broadcloth cost, at £2 3*s.* 8*d.* per yard? *Ans.* £12 16*s.* $6\frac{1}{2}$*d.*

27. What will 1*hhd.* 2*gal.* 3*qt.* 1*pt.* 1*gi.* of molasses come to, at $12\frac{1}{2}$ cents per quart? *Ans.* \$32,953+.

28. What will be the cost of 27*bu.* 3*pk.* 6*qt.* 1*pt.* of wheat, at 10*s.* 2*d.* 3*far.* per bushel? *Ans.* £14 5*s.* 11*d.* $0\frac{63}{64}$*far.*

29. What will be the cost of 51*A.* 3*R.* 15*P.*, at £4 10*s.* per acre? *Ans.* £233 5*s.* $11\frac{1}{4}$*d.*

30. What will be the cost of 97*A.* 14*P.*, at £3 11*s.* 10*d.* per acre? *Ans.* £348 14*s.* $1\frac{17}{40}$*d.*

31. What will be the cost of $28\frac{1}{2}$ yards of cloth, at $\$4\frac{3}{4}$ per yard? *Ans.* \$135,375.

32. What will be the cost of 1*hhd.* 2*gal.* 3*qt.* 1*pt.* 1*gi.* of oil at $56\frac{1}{4}$ cents per quart? *Ans.* \$148,28+.

33. What is the cost of 27*bu.* 3*pk.* 6*qt.* 1*pt.* of wheat at $\$1\frac{3}{4}$ per bushel? *Ans.* \$48,918+.

34. What will be the cost of 514*yd.* 3*qr.* 2*na.*, at 17*s.* 9½*d.* per yard? *Ans.* £458 0*s.* $5\frac{13}{16}$*d.*

35. What will be the cost of 125 *E. E.* 1*qr.* 1*na.*, at £1 11*s.* 9½*d.* per ell? *Ans.* £199 1*s.* $10\frac{7}{8}$*d.*

PARTNERSHIP.

211. PARTNERSHIP is the joining together of two or more persons in trade, with an agreement to share the profits or losses.

PARTNERS are those who are united together in carrying on business.

CAPITAL, is the amount of money employed

DIVIDEND is the gain or profit:

Loss is the opposite of profit.

212. The Capital or Stock is a *cause* of the entire profit:

Each man's capital it the *cause* of his profit:

The entire profit or loss is the *effect* of the cause or capital:

Each man's profit or loss is the *effect* of his capital: hence,

Whole Stock : Each man's Stock
:: Whole profit or loss : Each man's profit or loss.

1. Mr. Jones and Mr. Wilson are partners in trade: Mr. Jones puts in, as capital, $1250, and Mr. Wilson, $750: at the end of a year there is a profit of $720: what is the share of each?

STATEMENT.

2000 : 1250 :: 720 : Jones' share.

OPERATION.

$$\begin{array}{r|l} {}^{4}\cancel{2000} & \cancel{1250}\ ^{25} \\ x & \cancel{720}\ ^{18} \end{array}$$

$x = \$450$ *Ans.*

2000 : 750 :: 720 : Wilson's share.

$$\begin{array}{r|l} {}^{4}\cancel{2000} & \cancel{750}\ ^{15} \\ x & \cancel{720}\ ^{18} \end{array}$$

$x = 270.$

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211. What is a partnership? What are partners? What is capital or stock? What is dividend? What is loss?

Hence, the following

RULE.—*As the whole stock is to each man's share, so is the whole gain or loss to each man's share of the gain or loss.*

EXAMPLES.

1. A, B, and C, entered into partnership with a capital of $7500, of which A put in $2500, B put in $3000, and C put in the remainder; at the end of the year their gain was $3000: what was each one's share?

2. C and D have a joint stock of $4200, of which A owns $3600, and B, $600: they gain in one year, $2000: what is each one's share of the profits?

3. A, B, C, and D, have £40,000 in trade, each an equal share; at the end of six months their profits amount to £16000: what is each one's share, allowing A to receive £50, and D, £30, out of the profits, for extra services?

4. Five persons have to share between them an estate of $20000; A is to have one-fourth, B one-eighth, C one-sixth, D one-eighth, and E what is left: what will be the share of each?

5. Three merchants loaded a vessel with flour; A loaded 500 barrels, B, 700 barrels, and C, 1000 barrels; in a storm at sea, it became necessary to throw overboard 440 barrels; what was each one's share of the loss?

6. A man bequeathed his estate to his four sons, in the following manner, viz.: to his first, $5000, to his second, $4500, to his third, $4500, and to his fourth, $4000. But on settling the estate, it was found that after paying the debts and expenses, only $12000 remained to be divided: how much should each receive?

7. A widow and her two sons receive a legacy of $4500, of which the widow is to have $\frac{1}{2}$, and the sons each $\frac{1}{4}$. But the oldest son dying, the whole is to be divided in the same proportion between the mother and youngest son: what will each receive?

8. Four persons engage jointly in a land speculation: D puts

in $5499 capital. They gain $15000, of which A takes $4320,50, B, $5245,75, and C, $3600,75: how much capital did A, B, and C put in, and what is D's share of the gain?

9. A steam-mill, valued at $4300, was entirely destroyed by fire. A owned $\frac{1}{4}$ of it, B $\frac{1}{3}$, and C the remainder; supposing it to have been insured for $2500, what was each one's share of the loss?

10. A copartnership is formed with a joint capital of $16970. A puts in $5 as often as B puts in $7, and as often as C puts in $8; their annual gain is equal to C's stock: what is each person's stock and gain?

11. A man failing in business is indebted to A, $475,50, to B, $362,12$\frac{1}{2}$, to C, $250,87$\frac{1}{2}$, and to D, $140. He is worth only $614,25: to how much is each entitled?

12. Brown, Smith & Co., produce dealers, were obliged to make an assignment on account of a falling off in prices. Their effects were valued at $2544, with which they can pay but twenty cents on the dollar: how much did they owe?

13. Four persons agreed to do a job of work for $270; A offered to do $\frac{3}{5}$ of it, B, $\frac{4}{9}$, C, $\frac{1}{3}$, and D, $\frac{7}{15}$: what should each receive for his work?

14. Three persons buying a piece of land for $4569, pay in such proportions that the first and second own $\frac{1}{2}$ of it, the second and third, $\frac{3}{5}$ of it, and the first and third $\frac{7}{10}$ of it: how much did each pay, and what part does each own?

COMPOUND PARTNERSHIP.

When the Cause of Profit or Loss is Compound.

213. When the partners employ their capital for *different* periods of time, the profit of each will depend on two circumstances; first, *on the amount of capital he puts in*; and secondly, *on the time it is continued in business.*

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213. When the partners employ their capital for different periods of time, on what will the profit depend? Will the cause, then, be simple or compound? To what will it be equal? Give the rule for finding each share?

Hence, the *cause* of the loss or gain will be *compound*, and the product of the elements, in each particular case, will be the cause of each man's gain or loss; and their sum will be the cause of the entire gain or loss.

1. A put in trade \$500 for 4 months, and B, \$600 for 5 months. They gained \$240: what was the share of each?

OPERATION.

$$\begin{array}{l} \text{A, } \$500 \times 4 = 2000 \\ \text{B, } 600 \times 5 = \underline{3000} \\ \qquad\qquad\quad 5000 : \begin{cases} 2000 \\ 3000 \end{cases} :: \ 240 \ : \begin{cases} \$96 \text{ A's.} \\ \$144 \text{ B's.} \end{cases} \end{array}$$

Hence, the following

RULE.—*Multiply each man's stock by the time he continued it in trade: then say, as the sum of the products is to each product, so is the whole gain or loss to each man's share of the gain or loss.*

EXAMPLES.

1. Three men hire a pasture for \$70,20: A put in 7 horses for 3 months; B, 9 horses for 5 months; and C, 4 horses for 6 months: what part of the rent should each pay?

2. A commenced business, with a capital of \$10000. Four months afterwards B entered into partnership with him, and put in 1500 barrels of flour. At the close of the year their profits were \$5100, of which B was entitled to \$2100: what was the value of the flour per barrel?

3. On the 1st of January, 1856, A commenced business with a capital of \$23000; two months afterwards he drew out \$1800; on the 1st of April B, entered into partnership with him, and put in \$13500; four months afterwards he drew out \$10000; at the end of the year their profits were \$8400: how much ought each to receive?

4. Three persons received interest to the amount of \$798. A put out \$4000 for 12 months, B, \$3000 for 15 months, and C, \$5000 for 8 months: to how much interest was each entitled?

5. C, D, and E, form a copartnership; C's stock is in trade 3 months, and he claims $\frac{1}{12}$ of the gain; D's stock is in 9 months; and E put in $756 for 4 months, and claims $\frac{1}{2}$ of the profits: how much did C and D put in?

6. A ship's company took a prize worth $20760, which was divided among them according to their pay and the time they had been on board. There were 4 officers receiving $40 a month each, and 12 midshipmen receiving $30 a month each, all of whom had been on board 6 months; there were also 110 sailors receiving $22 a month each, and who had been on board 5 months: what was the share of each?

7. Two persons form a partnership for one year and eight months. A, at first, put in $3000 for 9 months, and then $1000 more. B, at first, put in $4000, and at the end of the first year $500 more, but at the end of 15 months he drew out $2000. At the end of 12 months, C was admitted as a partner with $5500. The gain was $7400: how much should each receive?

8. Four persons together agreed to build a barn for $346,50. A worked 14 days, 12 hours each day; B, 18 days, 10 hours each day; C, 15 days, 11 hours each day; and D, 20 days, 9 hours each day: how much should each man receive?

9. In a certain school, premiums to the value of $27 are to be distributed in the following manner: The premiums are divided into three grades. The value of a premium of the first grade is twice the value of one of the second; and the value of one of the second grade twice that of the third. There are 6 to receive premiums of the first grade, 12 of the second, and 6 of the third: what will be the value of a single premium of each grade?

10. Three men take an interest in a mining company. A put in $480 for 6 months, B, a sum not named, for 12 months, and C, $320 for a time not named: when the accounts were settled, A received $600 for his stock and profits, B, $1200 for his, and C, $520 for his: what was B's stock, and C's time?

PERCENTAGE.

214. PERCENTAGE is an allowance made by the hundred, and is always a *part* of the number on which the allowance is made.

THE BASE of percentage, is the number on which the percentage is reckoned.

215. PER CENT means by the hundred: thus, 1 per cent means 1 for every hundred; 2 per cent, 2 for every hundred; 3 per cent, 3 for every hundred, &c. The *numbers* denoting the allowances, 1 per cent, 2 per cent, 3 per cent, &c., are called *rates*, and may be expressed decimally, as in the following

TABLE.

1 per cent is	.01	7 per cent is	.07
3 per cent is	.03	8 per cent is	.08
4 per cent is	.04	15 per cent is	.15
5 per cent is	.05	68 per cent is	.68
6 per cent is	.06	99 per cent is	.99

ALSO,

100 per cent is 1; for, $\frac{100}{100}$ is equal to 1.
150 per cent is 1.50; for, $\frac{150}{150}$ is equal to 1.50.
140 per cent is 1.40; for, $\frac{140}{140}$ is equal to 1.40.
200 per cent is 2; for, $\frac{200}{100}$ is equal to 2.
$\frac{1}{2}$ per cent is .005; for, $\frac{1}{100} \div 2$ is equal to .005.
$3\frac{1}{2}$ per cent is .035; for, $3\frac{1}{2} \div 100 = .03 + .005 = .035$.
$5\frac{3}{4}$ per cent is .0575; for, $5\frac{3}{4} \div 100 = .05 + .0075 = .0575$.
&c. &c. &c. &c.

EXAMPLES.

1. Write decimally, $9\frac{1}{2}$ per cent, and $8\frac{3}{4}$ per cent.
2. Write decimally, $12\frac{1}{2}$ per cent, and $9\frac{7}{8}$ per cent.
3. Write decimally, 208 per cent, 375 per cent, and 95 per cent?
4. Write decimally, $66\frac{2}{3}$ per cent.

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214. What is percentage? What is the base?

215. What does per cent mean? What do you understand by 3 per cent? What is the rate, or rate per cent?

216. *To find the percentage of any number.*

1. What is the percentage of $450, the rate being 6 per cent?

OPERATION.

$$\begin{array}{r} 450 \\ .06 \\ \hline \$27.00 \end{array} \textit{ Ans.}$$

ANALYSIS.—The rate being 6 per cent, is expressed decimally by .06. We are then to take .06 of the base, $450; this we do by multiplying $450 by .06, giving $27.

Hence, to find the percentage of a number,

Multiply the number by the rate expressed decimally, and the product will be the percentage.

EXAMPLES.

1. What is the percentage of $564, the rate being $5\frac{1}{8}$ per cent?

OPERATION.

$$\begin{array}{rl} 564 & \\ .05\frac{1}{8} & \\ \hline 188 & = \frac{1}{8} \text{ per cent.} \\ 2840 & = 5 \text{ per cent.} \\ \hline \$3028 & = 5\frac{1}{8} \text{ per cent.} \end{array}$$

NOTE.—When the rate cannot be reduced to an exact decimal, it is most convenient to multiply by the fraction, and then by that part of the rate which is expressed in exact decimals.

Find the percentage of the following numbers:

2. $\frac{1}{4}$ per cent of $1256.
3. $\frac{1}{2}$ per cent of $956,50.
4. $\frac{3}{4}$ per cent of 475 yards.
5. $\frac{7}{8}$ per cent of 324.5*cwt.*
6. $\frac{4}{5}$ per cent of 125.25*lb.*
7. $1\frac{3}{5}$ per cent of 750*bush.*
8. $4\frac{1}{2}$ per cent of $2000.
9. 9 per cent of 186 miles.
10. $10\frac{3}{8}$ per cent of 460 sheep.
11. $5\frac{1}{10}$ per cent of 540 tons.
12. $8\frac{2}{3}$ per cent of $3465,75.
13. $12\frac{1}{2}$ per cent of 126 cows.
14. 50 per cent of 320 bales.
15. $37\frac{1}{2}$ per cent of 1275*yds.*
16. 95 per cent of $4573.
17. 105 per cent of 2500*bar.*
18. $112\frac{1}{2}$ per cent of $4573.
19. 250 per cent of $5000.
20. 305 per cent of $1267,87\frac{1}{2}$.
21. 500 per cent of $3000.

22. What is the difference between $4\frac{3}{4}$ per cent of $1000 and $7\frac{1}{2}$ per cent of $1500?

---

216. How do you find the percentage of any number?

23. If I buy 895 gallons of molasses, and lose 17 per cent by leakage, how much have I left?

24. A grocer purchased 250 boxes of oranges, and found that he had lost in bad ones 18 per cent: how many full boxes of good ones had he left?

25. A capitalist wishes to invest $25000; he invests 20 per cent in bank stock, $37\frac{1}{2}$ per cent in railroad stock, and the remainder in bonds and mortgages: what per cent, and what amount did he invest in the latter?

26. A man bought a house and lot for $3250; in three years time it increased in value $87\frac{1}{2}$ per cent: what was its value then?

27. A farmer having $1572,75, purchased cows with 25 per cent of it, sheep with $12\frac{1}{2}$ per cent of it, and lent 50 per cent of it to a friend: how much had he left?

217. *To find the per cent which one number is of another.*

1. What per cent of 64 is 16?

OPERATION.

$$\frac{16}{64} = \frac{1}{4} = .25$$

ANALYSIS.—In this example 16 is the *percentage*, 64 is the *base*, and we wish to find the *rate*. Since the percentage is equal to the base multiplied by the rate (Art. 216), the *rate* is equal to the *percentage* divided by the *base*; hence, $\frac{16}{64} = \frac{1}{4} = .25$; therefore, the rate is 25 per cent: hence, to find what per cent one number is of another,

Divide the number denoting the percentage by the base, and the two first decimal places will express the rate per cent.

NOTES.—1. The base is generally preceded by the word *of*.

2. There are three parts in percentage: 1st. The base; 2d. The rate; and 3d, their product, which is the *percentage*.

3. The *percentage* divided by the *rate*, gives the *base*; the *percentage* divided by the *base*, gives the *rate*.

---

217. How do you find the per cent which one number is of another?

EXAMPLES.

1. What per cent of 10 dollars is 2 dollars?
2. What per cent of 32 dollars is 4 dollars?
3. What per cent of 40 pounds is 3 pounds?
4. Seventeen bushels is what per cent of 125 bushels?
5. Thirty-six tons is what per cent of 144 tons?
6. What per cent is $84 of $96?
7. What per cent is 275 of 440?
8. What per cent is 3 miles of 400 miles?
9. Eleven is what per cent of 800?
10. One hundred and four sheep is what per cent of a drove of 312 sheep?
11. A grocer has $325, and purchases sugars to the amount of $121,87$\frac{1}{2}$: what per cent of his money does he expend?
12. Out of a bin containing 450 bushels of oats, 56$\frac{1}{4}$ bushels were sold: what per cent is this of the whole?
13. A merchant goes to New York with $2500; he first lays out 20 per cent for groceries, and then expends $1875 for dry goods: what per cent of his money has he left?
14. Two persons invested in stocks $4500 each; one lost $562,50, and the other lost $405: what per cent more did one lose than the other?
15. A and B engage in different kinds of business with $5400 capital each; A gains $1350, and B loses $540 the first year: what per cent is B's money of A's?

218. *To find the base when the percentage is added to or subtracted from the base.*

1. Mr. Jones buys 8 hogsheads of sugar, sells them at an advance of 15 per cent, and receives $470: what did he pay for the sugar?

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218. How do you find the base when the percentage is subtracted from the base?

ANALYSIS.—The amount received, $470, arises from *adding* the percentage to the base; that is, *it arises from multiplying the base by* 1 + *plus the rate per cent;* hence, to find the base, in such cases,

OPERATION.

```
1.15) 470 ($400
      470
```

Divide the given number by 1 *plus the rate per cent, expressed decimally.*

2. A cask of wine, out of which 37 per cent had leaked, was found to contain 33.39 gallons: how many gallons did the cask contain?

ANALYSIS.—Thirty-seven per cent denotes .37 of the capacity of the cask; and hence, the part of the cask that is filled is denoted by 1 − .37 = .63. But .63 of the cask contains 33.39 gallons; therefore, the entire cask will contain as many gallons as .63 is contained times in 33.39, viz., 53; hence, to find the base, in such cases,

OPERATION.

1 −.37 per cent = .63 per cent.

```
.63) 33.39 (53 gallons.
     31 5
      1 89
      1 89
```

Divide the given number by the difference between 1 *and the rate per cent, expressed decimally.*

EXAMPLES.

1. A farmer bought 40 sheep, and after keeping them for one year, sold them at an advance of 55 per cent, and received $248: what did he pay for the sheep per head?

2. A merchant bought a lot of goods and marked them at an advance of 26 per cent: when sold, he found that they brought him $6835,50: what did the goods cost him?

3. A son, who inherited a fortune, spent 37½ per cent of it, when he found that he had only $31250 remaining: what was the amount of his fortune?

4. A grocer purchased a lot of teas and sugar, on which he lost 16 per cent. by selling them for $4200: what did he pay for the goods?

INTEREST.

219. INTEREST is an allowance made for the use of money that is borrowed.

PRINCIPAL is the money on which interest is paid.

AMOUNT is the sum of the Principal and Interest.

For example: If I borrow 1 dollar of Mr. Wilson for 1 year, and pay him 7 cents for the use of it; then,

1 dollar is the *principal*,

7 cents is the *interest*, and

1 +.07 = $1,07, the *amount*.

The RATE of interest is the *number* of cents paid for the use of 1 dollar for 1 year. Thus, in the above example, the *rate* is 7 per cent per annum.

NOTE.—The term per cent, means, *by the hundred;* and *per annum* means *by the year*. As interest is always reckoned by the year, the term per annum, is understood and omitted.

CASE I.

220. *To find the interest of any principal for one or more years.*

1. What is the interest of $3920 for 2 years, at 7 per cent?

ANALYSIS.—The rate of interest being 7 per cent, is expressed decimally by .07: hence, each dollar, in 1 year, will produce .07 of itself, and $3920 will produce .07 of $3920, or $274,40. Therefore, $274,40 is the interest for 1 year, and this interest multiplied by 2, gives the interest for 2 years: hence, the following

OPERATION.

$3920	
.07	rate.
$274,40	int. for 1 year.
2	No. of years.
$548,80	interest.

---

219. What is interest? What is principal? What is amount? What is rate of interest? What does per annum mean?

220. How do you find the interest of any principal for any number of years? Give the analysis.

RULE.—*Multiply the principal by the rate, expressed decimally, and the product by the number of years.*

EXAMPLES.

1. What is the interest of $675 for 1 year, at $6\frac{1}{2}$ per cent?

ANALYSIS.—We first find the interest at $\frac{1}{2}$ per cent, and then the interest at 6 per cent; the sum is the interest at $6\frac{1}{2}$ per cent.

OPERATION.

$675	
.06½	
3375	½ per cent.
4050	6 per cent.
$43,875	6½ per cent.

2. What is the interest of $871,25, for 1 year, at 7 per cent?

3. What is the interest of $535,50, for 7 years, at 6 per cent?

4. What is the interest of $1125,885, for 4 years, at 8 per cent?

5. What is the interest of $789,74, for 12 years, at 5 per cent?

6. What is the interest of $2500, for 7 years, at $7\frac{1}{2}$ per cent?

7. What is the interest of $3153,82, for 2 years, at $4\frac{1}{2}$ per cent?

8. What is the amount of $199,48, for 16 years, at 7 per cent?

9. What is the amount of $897,50, for 3 years, at 8 per cent?

10. What is the interest of $982,35, for 4 years, at $6\frac{3}{4}$ per cent?

11. What is the amount of $1500, for 5 years, at $5\frac{1}{4}$ per cent?

12. What is the interest of $1914,10, for 6 years, at $3\frac{1}{4}$ per cent?

13. What is the interest of $350, for 21 years, at 10 per cent?

14. What is the amount of 628,50, for 5 years, at $12\frac{1}{2}$ per cent?

15. What is the amount of $75,50, for 10 years, at 6 per cent?

16. What is the amount of $5040, for 2 years, at $7\frac{1}{2}$ per cent?

NOTE.—When there are years and months, and the months are aliquot parts of a year, *multiply the interest for* 1 *year by the years and by the months reduced to the fraction of a year.*

EXAMPLES.

1. What is the interest of $119,48 for 2 years 6 months, at 7 per cent?

2. What is the interest of $250,60 for 1 year 9 months, at 6 per cent?

3. What is the interest of $956 for 5 years 4 months, at 9 per cent?

4. What is the amount of $1575,20 for 3 years 8 months, at 7 per cent?

5. What is the amount of $5000 for 2 years 3 months, at $5\frac{1}{2}$ per cent?

6. What is the interest of $1508,20 for 4 years 2 months, at 10 per cent?

7. What is the interest of $75 for 6 years 10 months at $12\frac{1}{2}$ per cent?

8. What is the amount of $125 for 5 years 6 months, at $4\frac{3}{4}$ per cent?

CASE II.

221. *To find the interest on a given principal for any rate and time.*

1. What is the interest of $1752,96 at 6 per cent, for 2 years 4 months and 29 days?

ANALYSIS.—The interest for 1 year is the product of the principal multiplied by the rate. If the interest for 1 year be divided by 12, the quotient will be the interest for 1 month: if the interest for 1 month be divided by 30, the quotient will be the interest for 1 day.

The interest for 2 years is 2 times the interest for 1 year: the interest for 4 months, 4 times the interest for 1 month; and the interest for 29 days, 29 times the interest for 1 day.

---

221. How do you find the interest for any time and rate? How do you find the interest for years, months, and days by the second method?

OPERATION.

\$1752,96		
.06		
12)105,1776 int. for 1 *yr.*	\$105,1776 × 2 =	\$210,3552 2*yr.*
30)8,7648 int. for 1*mo.*	8,7648 × 4 =	35,0592 4*mo.*
,29216 int. for 1*da.*	0,29216 × 29 =	8,47264 29*da.*
	Total interest,	\$253,88704

Hence, we have the following

RULE.—I. *Find the interest for* 1 *year:*

II. *Divide this interest by* 12, *and the quotient will be the interest for* 1 *month:*

III. *Divide the interest for* 1 *month by* 30, *and the quotient will be the interest for* 1 *day.*

IV. *Multiply the interest for* 1 *year by the number of years, the interest for* 1 *month by the number of months, and the interest for* 1 *day by the number of days, and the sum of the products will be the required interest.*

NOTES.—1. In computing interest, the month is reckoned at 30 days.

2. This method of computing interest for days, is the one in general use. It supposes the year to be made up of 360 instead of 365 days, and hence the result is too large by 5 of the 365 equal parts into which the interest may be divided. But this difference belongs only to the interest for the *fractional part of a month*, and hence may be neglected without sensible error; but if entire accuracy is required, the interest for the days must be diminished by its $\frac{5}{365}$ part $= \frac{1}{73}$ part.

2D METHOD.

222. There is another rule resulting from the last analysis which is regarded as the best general method of computing interest.

RULE.—I. *Find the interest for* 1 *year and divide it by* 12: *the quotient will be the interest for* 1 *month.*

II. *Multiply the interest for* 1 *month by the time expressed in months and decimal parts of a month, and the product will be the required interest.*

NOTE.—Since a month is reckoned at 30 days, *any number of days*
... decimals of a month by dividing the days by 3

EXAMPLES.

1. What is the interest of $655, for 3 years 7 months and 13 days, at 7 per cent?

OPERATION.

3yrs. = 36*mos.*	$655	
7*mos.*	.07	
13*da.* = .4$\frac{1}{3}$*mos.*	12)45.85	int. for 1 year.
Time = 43.4$\frac{1}{3}$*mos.*	3.82083 +	int. for 1 month.
	43.4$\frac{1}{3}$	time in months.
	127361	
	1528332	
	1146249	
	1528332	
	165,951383 *Ans.*	

2. What is the interest of $358,50, for 1 year 8 months and 6 days, at 7 per cent?

3. What is the interest of $1461,75, for 4 years 9 months and 15 days, at 6 per cent?

4. What is the interest of $1200, for 2 years 4 months and 12 days, at 7½ per cent?

5. What is the interest of $4500, for 9 months and 20 days, at 5 per cent?

6. What is the interest of $156,25, for 10 months and 18 days, at 8 per cent?

7. What is the interest of $640, for 3 years 2 months and 9 days, at 6½ per cent?

8. What is the interest of $276,50, for 11 months and 21 days, at 10 per cent?

9. What is the amount of $378,42, for 1 year 5 months and 3 days, at 7 per cent?

10. What is the amount of $1250, for 7 months and 21 days, at 10½ per cent?

11. What is the interest of $6500, for 2 months and 10 days, at 9½ per cent?

12. What is the interest of $70,50, for 10 years and 10 months, at 5¼ per cent?

13. What is the amount of \$45, for 12 years and 27 days, at $6\frac{3}{4}$ per cent?

14. What will \$100 amount to in 15 years and 6 months, if put at interest at 4 per cent?

15. How much will \$475,50 gain in 5 years 9 months and 24 days, at 8 per cent?

16. What will be the interest of \$4560, for 14 months and 19 days, at 7 per cent?

17. What will \$128,$37\frac{1}{2}$ amount to in 10 months and 27 days, at 6 per cent?

18. What is the interest of \$264,52, for 2 years 8 months and 14 days, at 6 per cent?

19. What is the amount of \$76,50, for 1 year 9 months and 12 days, at 6 per cent?

20. What will be the interest for 3 years 3 months and 15 days, of \$241,60, at 7 per cent?

21. What is the interest of \$5600, for 30 days, at 7 per cent?

22. What will \$8450 amount to in 60 days, at 10 per cent?

23. What is the interest of \$4000, for 1 month and 6 days, at 9 per cent?

24. What will be the amount of \$87,60, from Sept. 9th, 1852, to Oct. 10th, 1853, at $6\frac{1}{2}$ per cent?

25. What will be due on a note of \$126,75, given July 8th, 1854, and payable April 25th, 1858, at 7 per cent?

26. What is the interest of \$350, from Jan. 1st, 1856, to 15th of Sept. next following, at $5\frac{1}{4}$ per cent?

27. Gave a note of \$560,40, March 14th, 1855, on interest, after 90 days: what interest was due Dec. 1st, 1856, at 10 per cent?

28. Find the interest of \$1256, for 11 months and 9 days, at 6 per cent.

29. What is the amount of \$745,40 at 5 per cent interest, being reckoned from the 5th day of the 10th month of 1850, to the 10th day of the 5th month of 1854?

30. Sept. 10th, James Trusty borrowed of Peter Credit \$250, and March 4th, 1853, he borrowed \$500 more, agreeing

to pay 7 per cent interest on the whole: what was the amount of his indebtedness Jan. 1st, 1854?

31. Ordered dry goods of A. T. Stewart & Co., at different times, to the following amounts, viz., Jan. 1st, 1854, $254; March 15th, 1854, $154,60; April 20th, 1854, $424,25; and June 3d, 1854, $75,50. I bought on time at 6 per cent interest: what was the whole amount of my indebtedness on the first day of Sept. following?

32. If I borrow $475,75 of a friend at 7 per cent, what will I owe him at the end of 8 months and a half?

33. In settling with a merchant, I gave my note for $127,28, due in 1 year 9 months, at 6 per cent: what must be paid when the note falls due?

34. A person buying a piece of property for $4500, agreed to pay for it in three equal annual instalments, with interest at $6\frac{1}{2}$ per cent: what was the entire amount of money he paid?

35. A mechanic hired a journeyman for 9 months at $40 a month, to be paid monthly; but deferred paying him until 1 year 4 months and 15 days after his time was out; what should he then pay him, allowing him 7 per cent interest?

36. A person owning a part of a woollen factory, sold his share for $9000. The terms were, one-third cash, on delivery of the property, one-half of the remainder in 6 months, and the rest in 12 months, with $7\frac{1}{2}$ per cent interest: what was the whole amount paid?

NOTES.

$382,50 Chicago, January, 1st, 1856.

1. For value received I promise to pay on the 10th day of June next, to C. Hanford or order, the sum of three hundred and eighty-two dollars and fifty cents, with interest from date, at 7 per cent.

$612 Baltimore, January 1st, 1856.

2. For value received I promise to pay on the 4th of July, 1858, to Wm. Johnson or order, six hundred and twelve dollars with interest at 6 per cent from the 1st of March, 1856.

John Liberal.

$3120 Charleston, July 3d, 1855.

3. Six months after date, I promise to pay to C. Jones or order, three thousand one hundred and twenty dollars with interest from the 1st of January last, at 7 per cent.

Joseph Springs.

$786,50 New York, July 7th, 1851.

4. Twelve months after date, I promise to pay to Smith & Baker or order, seven hundred and eighty-six $\frac{50}{100}$ dollars for value received with interest from December 3d, 1851, at 8 per cent. *Silas Day.*

$4560,72 Cincinnati, March 10th, 1856.

5. Nine months after date, for value received, I promise to pay to Redfield, Wright & Co. or order, four thousand five hundred and sixty $\frac{72}{100}$ dollars with interest after 6 months, at 7 per cent. *Frederick Stillman.*

$1854,83 Boston, July 17th, 1856.

6. Eleven months after date, for value received, we promise to pay to the order of Fondy, Burnap & Co., one thousand eight hundred and fifty-four $\frac{83}{100}$ dollars with interest from May 18th, 1856, at 6 per cent. *Palmer & Blake.*

POUNDS, SHILLINGS AND PENCE.

223. To find the interest, when the principal is pounds, shillings and pence.

I. *Reduce the shillings and pence to the decimal of a pound* (Art. 164).

II. *Then find the interest as though the sum were dollars and cents; after which reduce the decimal part of the answer to shillings and pence* (Art. 165).

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223. How do you find the interest when the principal is pounds, shillings and pence?

EXAMPLES.

1. What is the interest, at 6 per cent, of £27 15*s.* 9*d.* for 2 years?

£27 15*s.* 9*d.* = £27.7875.

£27.7875 × .06 × 2 = £3.3345 interest.

£3.3345 = £3 6*s.* 8¼*d.* *Ans.*

2. What is the interest on £203 18*s.* 6*d.*, at 6 per cent, for 3 years 8 months 16 days?

3. What is the interest of £215 13*s.* 8*d.*, at 6 per cent, for 3 years 6 months and 9 days?

4. What is the interest of £1543 10*s.* 6*d.*, for 2 years and a half, at 4 per cent?

5. What is the amount of £1047 3*s.*, for 1*yr.* 4*mo.* 15*da.*, at 6 per cent.?

6. What is the interest on £511 1*s.* 4*d.*, at 6 per cent per annum, for 6*yr.* 6*mo.*?

7. What is the interest on £161 15*s.* 3*d.*, at 6 per cent, for 8*mo.* 13*da.*?

PROBLEMS IN INTEREST.

224. In every question of interest there are four parts: 1st. Principal; 2d. Rate; 3d. Time; and 4th. The amount of Interest. If any three of these parts are known, the 4th can be found.

1. At what rate per cent must $325 be put at interest for 1 year and 6 months, to produce an interest of $34,125?

ANALYSIS.—The principal, multiplied by the rate expressed decimally, multiplied by the time in years, is equal to the interest (Art. 221); and when the time is expressed in months and decimals of a month, the same product is equal to 12 times the interest (Art. 221). Hence,

OPERATION.

Principal, Rate, Time, in months.	12 Interest.

224. How many parts are there in every question of interest? How many of these must be known before the remainder can be found? How do you find the interest when you know the Principal, Rate, and Time? How do you find the Principal when you know the interest, rate, and time? How do you apply the formula to any case?

I. *When the time is in months, the product of the principal rate and time will be equal to* 12 *times the interest.*

II. *When two of these parts and the interest are given,* 12 *times the interest divided by the product of the given parts will be equal to the other part.*

NOTE.—Let this formula be written on the black board, or slate, and all the examples worked by it.

To apply the rule to the above example, place \$325 for the principal, x for the rate, 18 (months) for the time, and \$34,125 for the interest. Cancelling and dividing, we find $x = .07$; or, the rate is 7 per cent.

OPERATION.

$$\begin{array}{r|r} \$325 & 2 \\ x & \cancel{12} \\ 3\ \cancel{18} & \$34{,}125 \end{array}$$

$$x = \frac{34{,}125 \times 2}{325 \times 3} = .07$$

Ans. 7 per cent.

EXAMPLES.

1. What principal, at 6 per cent, will in 9 months give an interest of \$178,9552?

2. The interest for 2 years and 6 months, at 7 per cent, is \$76,965: what is the principal?

3. What sum must be invested, at 6 per cent, for 10 months and 15 days, to produce an interest of \$327,3249?

4. If my salary is \$1500 a year, what sum invested at 5 per cent, will pay it?

5. What sum put at interest for 4 years and 3 months, at 7 per cent, will gain \$283,3914?

6. The interest of \$2100 for 3 years 1 month and 18 days is \$460,60: what is the rate per cent?

7. A man invests \$5426 in Railroad stock, and receives a semi-annual dividend of \$244,17: what is the rate per cent?

8. A person owning property valued at \$2470,80, rents it for 1 year and 10 months for \$452,98: what per cent does it pay?

9. At what rate per cent must \$3456 be loaned for 2 years 7 months and 24 days, to gain \$503,712?

10. If I build a hotel at a cost of \$56000, and rent it for \$7000 a year, what per cent do I receive for the investment?

11. The interest on $1119,48, at 7 per cent, is $195,909: what is the time?

12. A man received $47,25, for the use of $1750; the rate of interest being 9 per cent: what was the time?

13. How long will it take $7500 to amount to $7850, at $3\frac{1}{2}$ per cent per annum?

14. How long will it take $500 to double itself, at 6 per cent, simple interest?

15. Wishing to commence business, a friend loaned me $3720, at $6\frac{1}{2}$ per cent, which I kept until it amounted to $5009,60: how long did I retain it?

16. I borrowed $700 of my neighbor for 1 year and 8 months, at 6 per cent; at the end of the time he borrowed of me $750: how long must he keep it to cancel the amount I owed him?

PARTIAL PAYMENTS.

225. We shall now give the rule established in New York (See Johnson's Chancery Reports, Vol. I., page 17,) for computing the interest on a bond or note, when partial payments have been made. The same rule is also adopted in Massachusetts, and in most of the other states.

RULE.—I. *Compute the interest on the principal to the time of the first payment, and if the payment exceed this interest, add the interest to the principal, and from the sum subtract the payment: the remainder forms a new principal.*

II. *But if the payment is less than the interest, take no notice of it until other payments are made, which in all, shall exceed the interest computed to the time of the last payment: then add the interest, so computed, to the principal, and from the sum subtract the sum of the payments: the remainder will form a new principal on which interest is to be computed as before.*

---

225. What is the rule for partial payments?

ANALYSIS.—The gain or loss will be equal to the difference between the cost and the amount received from the sale. If this difference be divided by the cost, the quotient will denote the per cent on the cost; if it be divided by the amount of the sale, the quotient will denote the per cent on the sale.

OPERATION.

$200 − 170 = $30

$30 ÷ $200 = .15.

Ans. 15 per cent.

RULE.—*Divide the gain or loss by the number on which the per cent is reckoned.*

EXAMPLES.

1. Bought a quantity of goods for $348,50, and sold the same for $425: what per cent did I make on the amount received?

2. Bought a piece of cotton goods for 6 cents a yard, and sold it for $7\frac{1}{2}$ cents a yard: what was my gain per cent?

3. If I buy rye for 90 cents a bushel, and sell it for $1,20, and wheat for $1,12$\frac{1}{2}$ a bushel, and sell it for $1,50 a bushel, upon which do I make the most per cent?

4. If paper that cost $2 a ream, be sold for 18 cents a quire, what is gained per cent?

5. How much per cent would be made upon a hogshead of sugar weighing 13*cwt.* 3*qr.* 14*lb.*, that cost $8 per *cwt.* if sold at 10 cents per pound?

6. A hardware merchant bought 45*T.* 16*cwt.* 25*lb.* of iron, at $75 per ton, and sold it for $78,50 per ton: what was his whole gain, and how much per cent did he make?

7. If 25 per cent be gained on flour when sold at $10 a barrel, what per cent would be gained when sold at $11,60 a barrel?

NOTE.—In this class of examples, first find the cost, as in Art. 246: then find the gain, or loss, and then divide by the number on which the per cent is reckoned.

8. A lumber dealer sold 25650 feet of lumber at $19,20 a thousand, and gained 20 per cent: how much would he have gained or lost had he sold it at $15 a thousand?

9. A man sold his farm for \$3881,25, by which he gained $12\frac{1}{2}$ per cent on its cost: what was its cost, and what would he have gained or lost per cent if he had sold it for \$3277,50?

10. If a merchant sell tea at 66 cents a pound, and gain 20 per cent, how much would he gain per cent if he sold it at 77 cents a pound?

11. Sold 5520 bushels of corn at 50 cents a bushel, and lost 8 per cent: how much per cent would have been gained had it been sold at 60 cents a bushel?

12. A grocer bought 3 hogsheads of sugar, each weighing $1412\frac{1}{2}$ pounds; he sold it at 11 cents a pound, and gained $37\frac{1}{2}$ per cent: what was its cost, and for how much must he sell it to gain 50 per cent on the cost?

INSURANCE.

249. INSURANCE is an agreement, generally in writing, by which individuals or companies bind themselves to exempt the owners of certain property, such as ships, goods, houses, &c., from loss or hazard.

The POLICY is the written agreement made by the parties.

250. The BASE of insurance is the *value* of the property insured.

251. PREMIUM is the amount paid by him who owns the property to those who insure it, as a compensation for their risk. The premium is generally so much per cent on the property insured.

252. There are four cases which may arise in questions of Insurance. The principles on which these cases depend have already been considered, and reference is made to the articles.

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249. What is insurance? What is a policy?

250. What is the base of insurance?

251. What is a premium?

252. How many cases are there which arise in insurance? What are they?

1. To find the Premium (Art. 216).
2. To find the Rate (Art. 217).
3. To find the base, or sum insured (Art. 218).
4. To insure on both the base and premium.

253. *To find the premium:*

1. What would be the premium on a cargo of goods, valued at \$39854, the insurance being made at $4\frac{1}{2}$ per cent?

ANALYSIS.—This is simply a case of finding the percentage when the base and rate are given (Art. 216).

OPERATION.
39854
.045
\$1793,430

EXAMPLES.

1. What would be the premium for insuring a ship and cargo, valued at \$147674, at $3\frac{1}{2}$ per cent?

2. What would be the insurance on a ship, valued at \$47520, at $\frac{1}{2}$ per cent? At $\frac{1}{3}$ per cent?

3. What would be the insurance on a house, valued at \$16800, at $1\frac{1}{2}$ per cent? At $\frac{3}{4}$ per cent?

4. A merchant owns $\frac{2}{3}$ of $\frac{3}{4}$ of a ship, valued at \$24000, and insures his interest at $2\frac{1}{2}$ per cent: what does he pay for his policy?

5. What will it cost to insure a store worth \$5640, at $\frac{3}{4}$ per cent, and the stock worth \$7560, at $\frac{5}{8}$ per cent?

6. A carriage maker shipped 15 carriages worth \$425 each: what must he pay to obtain an insurance upon them at 75 cents on a hundred dollars?

7. A merchant imported 150*hhd.* of molasses, at 35 cents a gallon: he gets it insured for $3\frac{1}{2}$ per cent on the selling price of 50 cents a gallon: if the whole should be destroyed, and he get the amount of insurance, how much would he gain?

8. If I get my house and furniture, valued at \$3640, insured at $4\frac{1}{2}$ per cent, what would be my actual loss if they were destroyed?

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253. How do you find the premium?

9. The ship Astoria is valued at $20450, and her cargo at $25600; being bound on a voyage from New York to Canton, insured $12000 on the vessel, at the St. Nicholas Office, at $2\frac{3}{4}$ per cent, and $18500 on the cargo, at the Howard Office, at $3\frac{1}{4}$ per cent: if the vessel founder at sea, what will be the loss to the owner?

10. Shipped from New York to the Crimea 5000 barrels of flour worth $10,50 a barrel. The premium paid was $2887,50: what was the rate per cent. of the insurance?

11. Paid $120 for insurance on my dwelling, valued at $7500: what was the rate per cent.?

12. A merchant imported 225 pieces of broadcloth, each piece containing 40 yards, at $3,50 a yard: he paid $1323 for insurance: what was the rate per cent.?

13. A merchant paid $1320 insurance on his vessel and cargo, which was $5\frac{1}{2}$ per cent on the amount insured: how much did he insure?

14. A man pays $51 a year for insurance on his storehouse, at $1\frac{1}{2}$ per cent, and $126,45 on the contents, at $2\frac{1}{4}$ per cent: what amount of property does he get insured?

15. A person shipped 15 pianos, valued at $275 each. He insures them at 3 per cent, and also insures the premium at the same rate: what insurance must he pay?

16. If a store and its contents are valued at $16750, and they are insured at $1\frac{3}{4}$ per cent, how much must be paid to cover the base of insurance and the premium?

LIFE INSURANCE.

254. INSURANCE for a term of years, or for the entire continuance of life, is a contract on the part of an authorized association to pay a certain sum, specified in the policy of insurance, on the happening of an event named therein, and for which the association receives a certain premium, generally in the form of an annual payment.

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254. What is a life insurance?

255. To enable the company to fix their premiums at such rates as shall be both fair to the insured and safe to the association, they must know the *average* duration of life from its commencement to its extreme limit. This average is called the "*Expectation of Life*," and this is determined by collecting from many sources the most authentic information in regard to births and deaths. The "Carlisle Table" shows the expectation of life from birth to 100 years, and is considered the most accurate. It is much used in England, and is in general use in this country.

By the "Expectation of Life," must be understood the *average age* of any number of individuals. Thus, if 100 infants be taken, some dying in infancy, some in childhood, some in youth, some in middle life, and some in old age; the average ages of all is found to be 38.72 years. At 10 years of age, the average age is 48.82 years; at 20, it is 41.46 years; at 30, it is 34.34 years; at 40, it is 27.61 years; at 50, it is 21.11 years; at 60, it is 14 years; at 70, it is 9.19 years; at 80, it is 5.51 years; at 90, it is 3.28 years; and at 100, it is 2.28 years.

256. From the above facts, and the value of money (which is shown by the rate of interest), a company can calculate with great exactness the amount which they should receive annually, for an insurance on a life for any number of years, or during its entire continuance.

Among the principal life insurance companies in the United States, are the New York Life Insurance and Trust Company, the Girard Life Insurance, Annuity and Trust Company of Philadelphia, and the Massachusetts Hospital Life Insurance and Trust Company of Boston. The rates of insurance, in these companies, differ but little.

The PREMIUM for life insurance is generally at so much per annum on $100; and is always paid in advance.

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255. What is necessary to enable a company to fix their premiums? How is the expectation determined? What do you understand by the expectation of life?

256 What may be calculated from the necessary facts?

EXAMPLES.

1. A person, 20 years of age, finds that the premium, per annum, is $1.36 on $100: what must he pay to insure his life for 1 year for $8950?

2. A man, aged 40 years, wishes to insure his life for 5 years, and finds that the annual rate is $1.86 for $100: how much premium must he pay per annum on $12500?

3. A person, 38 years of age, obtains an insurance on his life for 5 years, at the rate of $1,75 per annum on $100: how much is the annual premium on $15000?

4. A person going to Europe, expecting to return in 2 years, effects an insurance on his life at $\frac{1}{2}$ of $\frac{4}{5}$ per cent. premium on $100; he insures for $5000: what is the annual premium?

5. What will be the annual premium for insuring a person's life, who is 60 years of age, for $2000, at the rate of $4,91 on $100?

6. A person, at the age of 50 years, obtained an insurance at $4\frac{3}{5}$ per cent. per annum on each $100; he insured for $1500, and died at the age of 70. How much more was the insurance than the payments, without reckoning interest?

7. A gentleman, 47 years of age, going to China as ambassador, obtains an insurance on his life for $10000, by paying a premium of $2.71 per annum on every $100, and dies at the middle of the third year: reckoning simple interest on his payments at 7 per cent, what is gained by the insurance?

ENDOWMENTS.

257. AN ENDOWMENT is a certain sum to be paid at the expiration of a given time, in case the person on whose life it is taken shall live till the expiration of the time named.

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257. What is an endowment? What does the table of endowments show? What may be found from the table?

The following table shows the value of an endowment purchased for $100, at the several periods mentioned in the column of ages, the endowment to be paid if the person attains the age of 21 years.

TABLE OF ENDOWMENTS.

Age.	Sums to be paid at 21, if alive.	Age.	Sum to be paid at 21, if alive.	Age.	Sum to be paid at 21, if alive.
Birth, - -	$376,84	5 years, -	$210,53	13 years, -	$144,12
3 months, -	344,28	6 " -	198,83	14 " -	137,86
6 " -	331,46	7 " -	188,83	15 " -	131,83
9 " -	318,90	8 " -	179,97	16 " -	125,97
1 year, - -	306,58	9 " -	171,91	17 " -	120,31
2 " - -	271,03	10 " -	164.46	18 " -	114,89
3 " - -	243,69	11 " -	157,43	19 " -	109,70
4 " - -	225,42	12 " -	150,64	20 " -	104,74

This table shows that if $100 be paid at the birth of a child, he will be entitled to receive $376,84, if he lives to attain the age of 21 years. If $100 be paid when he is ten years old, he will be entitled to receive $164,46, if he lives to attain the age of 21 years. And similarly for other ages. We can easily find by proportion

1*st.* How much must be paid, at any age under 21, to purchase a given endowment at 21 ; and

2*d.* What endowment a sum paid at any age under 21, will purchase ?

EXAMPLES.

1. What endowment, at 21, can be purchased for $250, paid at the age of 10 years ?

2. What endowment, at 21, can be purchased for $360, paid at the age of 5 years ?

3. If my child is 7 years old, and I purchase an endowment for $650, what will he receive if he attains the age of 21 years?

4. If, at the birth of a daughter, I purchase an endowment for $350, what will she receive if she attain the age of 21 years ?

ANNUITIES.

258. **An Annuity** is a fixed sum of money to be paid at regular periods, either for a limited time, or forever, in consideration of a given sum paid in hand.

The Present Value of an annuity is that sum which being put at compound interest would produce the sums necessary to pay the annuity.

The purchaser of an annuity should pay more than the present value; for the seller cannot afford to take the money of the purchaser, invest it, re-invest the interest, and pay over the entire proceeds.

Knowing the rate of interest on money, and the present value of an annuity, a close estimate may be made of the price it ought to sell for.

TABLE

Showing the PRESENT VALUE OF AN ANNUITY *of* $1, *from* 1 *to* 30 *years, at different rates of interest.*

Years.	5 per cent.	6 per cent.	Years.	5 per cent.	6 per cent.
1	0.952381	0.943396	16	10.837770	10.105895
2	1.859410	1.833393	17	11.274066	10.477260
3	2.723248	2.673012	18	11.689587	10.827603
4	3.545950	3.465106	19	12.085321	11.158116
5	4.329477	4.212364	20	12.462216	11.469921
6	5.075692	4.917324	21	12.821153	11.764077
7	5.786373	5.582381	22	13.163003	12.041582
8	6.463213	6.209794	23	13.488574	12.303379
9	7.107822	6.801692	24	13.798642	12.550358
10	7.721735	7.360087	25	14.093945	12.783356
11	8.306414	7.886875	26	14.375185	13.003166
12	8.863252	8.388844	27	14.643034	13.210534
13	9.393573	8.852683	28	14.898127	13.406164
14	9.898641	9.294984	29	15.141074	13.590721
15	10.379658	9.712249	30	15.372451	13.764831

To find the present value of an annuity for any rate, and for any time, we simply multiply the present value of an annuity

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258. What is an annuity? What is the present value of an annuity? How do you find the present value of an annuity for a given rate and time? How do you find what annuity a given sum will produce, at a given rate and for a given time?

of $1 for the same rate and time, by the annuity, and the product will be its present value.

Thus, the present value of an annuity of $600 for 8 years, at 6 per cent, is

$$\$6.209794 \times 600 = \$3725.8764\text{; that is,}$$

$$\text{pres. val. of } \$1 \times \text{annuity} = \text{pres. val., hence,}$$

$$\text{annuity} = \frac{\text{pres. val.}}{\text{pres. val. of } \$1}\text{; therefore,}$$

I. To find what sum will produce a certain annuity at a given rate, and for a given time.

Multiply the present value of an annuity of $1, at the given rate and for the given time, by the given annuity; the product will be that sum.

II. To find what annuity a given sum will produce at a given rate and for a given time.

Divide the given sum, or present value by the present value of $1, for the given rate and time, and the quotient will be the annuity.

EXAMPLES.

1. What is the present value of an annuity of $550, at 5 per cent, for 21 years?

2. What would be the cost of an annuity of $860 a year, for 16 years, if the company borrowed the money at 5 per cent. compound interest, and charged 25 dollars a year for doing the business?

3. What is the present value of an annuity of $1500 a year, at 5 per cent, for 30 years?

4. For what sum could Mr. Jones purchase an annuity for 28 years, of $1250, if he paid 30 dollars a year for investing his money and paying over the interest; the money yielding 6 per cent?

5. What annuity would $27560 purchase for 24 years, the purchaser to pay $35 a year for loaning the money and collecting the inte[illegible] [illegible]ich is reckoned at 6 per cent?

6. Mr. Jones having a small fortune of $25000, and calculating that he will live about 20 years, purchases an annuity at six per cent, with an agreement that he will pay $20 a year for doing the business: what was his annual income from the investment?

ASSESSING TAXES.

259. A Tax is a certain sum required to be paid by the inhabitants of a town, county, or state, for the support of government. It is generally collected from each individual, in proportion to the amount of his property.

In some states, however, every white male citizen over the age of twenty-one years, is required to pay a certain tax. This tax is called a poll-tax; and each person so taxed is called a *poll.*

260. In assessing taxes, the first thing to be done is to make a complete inventory of all the property in the town, on which the tax is to be laid. If there is a poll-tax, make a full list of the polls and multiply the number by the tax on each poll, and subtract the product from the whole tax to be raised by the town; the remainder will be the amount to be raised on the property. Having done this, *divide the whole tax to be raised by the amount of taxable property, and the quotient will be the tax on* $1. Then multiply this quotient by the inventory of each individual, and the product will be the tax on his property.

EXAMPLES.

1. A certain town is to be taxed $4280; the property on which the tax is to be levied is valued at $1000000. Now there are 200 polls, each taxed $1,40. The property of A is valued at $2800, and he pays 4 polls,

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259. What is tax? How is it generally collected? What is a poll-tax?

260. What is the first thing to be done in assessing a tax? If there is a poll-tax, how do you find the amount? How then do you find the per cent of tax to be levied on a dollar? How do you find the tax to be raised on each individual?

B's at $2400, pays 4 polls,
C's at $2530, pays 2 "
D's at $2250, pays 6 "
E's at $7242, pays 4 polls,
F's at $1651, pays 6 "
G's at $1600,80, pays 4 "

What will be the tax on one dollar, and what will be A's tax, and also, that of each on the list?

First, $1,40 × 200 = $280, amount of poll-tax.

$4280 − $280 = $4000, amount to be levied on property.

Then, $4000 ÷ $1000000 = 4 mills on $1.

Now, to find the tax of each, as A's, for example,

A's inventory, - - -	$2800
	,004
	11,20
4 polls, at $1,40 each, - -	5,60
A's whole tax, - - -	$16,80

In the same manner, the tax of each person in the township may be found.

ASSESSMENT TABLE.

261. Having found the per cent, or the amount to be raised on each dollar, form a table showing the amount which certain sums would produce at the same rate per cent. Thus, after having found, as in the last example, that four mills are to be raised on every dollar, we can, by multiplying in succession by the numbers 1, 2, 3, 4, 5, 6, 7, 8, &c., form the following

TABLE.

$		$	$		$	$		$
1	gives	0,004	20	gives	0,080	300	gives	1,200
2	"	0.008	30	"	0.120	400	"	1,600
3	"	0,012	40	"	0,160	500	"	2,000
4	"	0.016	50	"	0,200	600	"	2,400
5	"	0.020	60	"	0,240	700	"	2,800
6	"	0.024	70	"	0,280	800	"	3,200
7	"	0,028	80	"	0,320	900	"	3,600
9	"	0,032	90	"	0.360	1000	"	4,000
9	"	0.036	100	"	0,400	2000	"	8,000
10	"	0.040	200	"	0.800	3000	"	12,000

261. How do you form the Assessment table?

This table shows the amount to be raised on each sum in the columns under $'s.

NOTE.—If you wish the tax on a sum not named in the Table, as $25, it is equal to the *sum* of the taxes on $20 and $5: and similarly for other numbers.

1. To find the amount of B's tax from this table.

B's tax on $2000, is	- -	$8,000
B's tax on 400, is	- -	1,600
B's tax on 4 polls, at $1,40,	-	5,600
B's total tax is	- -	$15,200

2. To find the amount of C's tax from the table.

C's tax on $2000, is	- -	$8,000
C's tax on 500, is	- -	2,000
C's tax on 30, is	- -	120
C's tax on 2 polls, is	- -	2,800
C's total tax is	- -	$12,920

In a similar manner, we might find the taxes to be paid by D, E, &c.

EXAMPLES.

1. In a county embracing 350 polls, the amount of property on the tax list is $318200; the amount to be raised is as follows: for state purposes, $1465,50; for county purposes, $350,25; and for town purposes, $200,25. By a vote of the county, a tax is levied on each poll of $1,50: how much per cent will be laid upon the property?

2. In a county embracing a population of 98415 persons, a tax is levied for town, county, and state purposes, amounting to $100406. Of this sum, a part is to be raised by a tax of 25 cents on each poll, and the remainder by a tax of two mills on the dollar: what was the amount of property on the tax list?

3. In a county embracing a population of 56450 persons, a tax is levied for town, county, and state purposes, amounting to $87467; the personal and real estate is valued at $4890300. Each poll is taxed 25 cents: what per cent is the tax, and how

much will a man's tax be, who pays for 5 polls, and whose property is valued at $5400?

What is B's tax, who was assessed for 2 polls, and whose property was valued at $3760,50?

4. A banking corporation, consisting of 40 persons, was taxed $957,50; their property was valued at $125000, and each poll was assessed 50 cents each: what per cent was their tax, and what was a man's tax, who paid for 1 poll, and whose share was assessed for $2000?

5. What sum must be assessed to raise a net amount of $5674,50, allowing $2\frac{1}{2}$ per cent commission on the money collected (Art. 218)?

6. Allowing 4 per cent. for collection, what sum must be assessed to raise $21346,75 net?

7. In a certain township, it becomes necessary to levy a tax of $4423,2475, to build a public hall. The taxable property is valued at $916210, and the town contains 150 polls, which are each assessed 50 cents. What amount of tax must be raised to build the hall, and pay 5 per cent. for collection, and what is the tax on a dollar?

What is a person's tax who pays for 3 polls, and whose personal property is valued at $2100, and his real estate at $3000?

What is G s tax, who is assessed for 1 poll, and $1275,50?

What is H's tax, who is assessed for 1 poll, and $2456?

8. The people of a school district wish to build a new school house, which shall cost $2850. The taxable property of the district is valued at $190000: what will be the tax on a dollar, and what will be a man's tax, whose property is valued at $7500?

How much is Mr. Merchant's tax, whose personal and real estate are assessed for $1200?

9. In a school district, a school is supported by a rate-bill. A teacher is employed for 6 months, at $60 a month; the fuel and other contingencies amount to $66. They drew $41,60 public money, and the whole number of days attendance was 7688: what was D's tax, who sent 148 days?

What was F's tax, who sent $184\frac{1}{2}$ days?

CUSTOM HOUSE BUSINESS.

259. PERSONS who bring goods, or merchandise, into the United States, from foreign countries, are required to land them at particular places or ports, called Ports of Entry, and to pay a certain amount on their value, called a *Duty*. This duty is imposed by the General Government, and must be the same on the same articles of merchandise, in every part of the United States.

Besides the duties on merchandise, vessels employed in commerce are required, by law, to pay certain sums for the privilege of entering the ports. These sums are large or small, in proportion to the size or tonnage of vessels. The moneys arising from duties and tonnage, are called *revenues*.

260. The revenues of the country are under the general direction of the Secretary of the Treasury, and to secure their faithful collection, the government has appointed various officers at each port of entry or place where goods may be landed.

261. The office established by the government at any port of entry, is called a *Custom House*, and the officers attached to it are called Custom House Officers.

262. All duties levied by law on goods imported into the United States, are collected at the various custom houses, and are of two kinds—*Specific* and *Ad valorem*.

SPECIFIC DUTY is a certain sum on a particular kind of goods named; as so much per square yard on cotton or woollen cloths, so much per ton weight on iron, or so much per gallon on molasses.

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259. What is a port of entry? What is a duty? By whom are duties imposed? What charges are vessels required to pay? What are the moneys arising from duties and tonnage called?

260. Under whose direction are the revenues of the country?

261. What is a custom house? What are the officers attached to it called?

262. Where are the duties collected? How many kinds are there, and what are they called? What is a specific duty? An ad valorem duty?

AD VALOREM DUTY is such a per cent on the actual cost of the goods in the country from which they are imported. Thus, an ad valorem duty of 15 per cent on English cloths, is a duty of 15 per cent on the cost of cloths imported from England.

263. The laws of Congress provide, that the cargoes of all vessels freighted with foreign goods or merchandise, shall be weighed or gauged by the custom house officers at the port to which they are consigned. As duties are only to be paid on the articles, and not on the boxes, casks, and bags which contain them, certain deductions are made from the weights and measures, called *Allowances.*

GROSS WEIGHT is the whole weight of the goods, together with that of the hogshead, barrel, box, bag, &c., which contains them.

NET WEIGHT is what remains after all deductions are made.

DRAFT is an allowance from the gross weight on account of waste, where there is not actual tare.

	lb.				*lb.*
On	112			it is	1,
From	112	to	224	"	2,
"	224	to	336	"	3,
"	336	to	1120	"	4,
"	1120	to	2016	"	7,
Above	2016	any	weight	"	9,

consequently, 9*lb.* is the greatest draft generally allowed.

TARE is an allowance made for the weight of the boxes, barrels, or bags containing the commodity, and is of three kinds, 1st. Legal tare, or such as is established by law; 2d. Customary tare, or such as is established by the custom among merchants; and 3d. Actual tare, or such as is found by re-

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263. What do the laws of Congress direct in relation to foreign goods? Why are deductions made from their weight? What are these deductions called? What is gross weight? What is net weight? What is draft? What is the greatest draft allowed? What is tare? What are the different kinds of tare? What allowances are made on liquors?

moving the goods and actually weighing the boxes or casks in which they are contained.

On liquors in casks, *customary tare* is sometimes allowed on the supposition that the cask is not full, or what is called its *actual wants;* and then an allowance of 5 per cent for leakage.

A tare of 10 per cent is allowed on porter, ale, and beer, in bottles, on account of breakage, and 5 per cent on all other liquors in bottles. At the custom house, bottles of the common size are estimated to contain $2\frac{3}{4}$ gallons the dozen. For tables of Tare and Duty, see Ogden on the Tariff of 1842.

EXAMPLES.

1. What is the net weight of 25 hogsheads of sugar, the gross weight being 66*cwt.* 3*qr.* 14*lb.*; tare 11*lb.* per hogshead?

		cwt.	*qr.*	*lb.*	
		66	3	14	gross.
$25 \times 11 = 275lb.$	- -	2	3		tare.
	Ans.	64	0	14	net.

2. If the tare be 4*lb.* per hundred, what will be the tare on 6*T.* 2*cwt.* 3*qr.* 14*lb.*?

$$\begin{array}{rcl} \text{Tare for } 6T. \text{ or } 120cwt. & = & 480lb. \\ 2cwt. & = & 8 \\ 3qr. & = & 3 \\ 14lb. & = & 0\frac{14}{25} \\ \hline \text{Tare} \quad - \quad - & & 491\frac{14}{25}lb. \end{array}$$

3. What will be the cost of 3 hogsheads of tobacco at $9,47 per *cwt.* net, the gross weight and tare being of

		cwt.	*qr.*	*lb.*			*lb.*
No. 1	- -	9	3	24	- -	tare	146
" 2	- -	10	2	12	- -	"	150
" 3	- -	11	1	24	- -	"	158

4. At 21 cents per *lb.*, what will be the cost of 5*hhd.* of coffee, the tare and gross weight being as follows:

		cwt.	qr.	lb.		lb.
No. 1	- -	6	2	14	- -	tare 94
" 2	- -	9	1	20	- -	" 100
" 3	- -	6	2	22	- -	" 88
" 4	- -	7	2	24	- -	" 89
" 5	- -	8	0	13	- -	" 100

5. What is the net weight of 18*hhd.* of tobacco, each weighing gross 8*cwt.* 3*qr.* 14*lb.*; tare 16*lb.* to the *cwt.*?

6. In 4*T.* 3*cwt.* 3*qr.* gross, tare 20*lb.* to the *cwt.*, what is the net weight?

7. What is the net weight and value of 80 kegs of figs, gross weight 7*T.* 11*cwt.* 3*qr.*, tare 12*lb.* per *cwt.*, at $2,31 per *cwt.*

8. A merchant bought 19*cwt.* 1*qr.* 24*lb.* gross of tobacco in leaf, at $24,28 per *cwt.*; and 12*cwt.* 3*qr.* 19*lb.* gross in rolls, at $28,56 per *cwt.*; the tare of the former was 149*lb.*, and of the latter 49*lb.*: what did the tobacco cost him net?

9. A grocer bought 17¼*hhd.* of sugar, each 10*cwt.* 1*qr.* 14*lb.*, draft 7*lb.* per *cwt.*, tare 4*lb.* per *cwt.* What is the value at $7,50 per *cwt.* net?

10. In 29 parcels, each weighing 3*cwt.* 3*qr.* 14*lb.* gross, draft 8*lb.* per *cwt.*, tare 4*lb.* per *cwt.* how much net weight, and what is the value at $7,50 per *cwt.* net?

11. A merchant bought 7 hogsheads of molasses, each weighing 4*cwt.* 3*qr.* 14*lb.* gross, draft 7*lb.* per *cwt.*, tare 8*lb.* per hogshead, and damage in the whole 99¾*lb.*: what is the value at $8,45 per *cwt.* net?

12. The net value of a hogshead of Barbadoes sugar was $22,50; the custom and fees $12,49, freight $5,11, factorage $1,31; the gross weight was 11*cwt.* 1*qr.* 15*lb.*, tare 11⅓*lb.* per *cwt.*: what was the sugar rated at per *cwt.* net, in the bill of parcels?

13. In 7*hhd.* of sugar, each weighing 3*cwt.* 2*qr.* 14*lb.* gross, tare 21*lb.* per *cwt.*, what is the value at $6,25 per *cwt.*?

14. I have imported 87 jars of Lucca oil, each containing 47 gallons: what did the freight come to at \$1,19 per *cwt.* net, reckoning 1*lb.* in 11*lb.* for tare, and 9*lb.* of oil to the gallon?

15. A grocer bought 5*hhd.* of sugar, each weighing 13*cwt.* 1*qr.* 12*lb.*, at $7\frac{1}{2}$ cents a pound; the draft was $1\frac{1}{2}$*lb.* per *cwt.*, and the tare $5\frac{1}{2}$ per cent: what was the cost of the net weight?

16. A wholesale merchant receives 450 bags of coffee, each weighing 76 pounds; the tare was 8 per cent, and the invoice price $10\frac{1}{2}$ cents per pound. He sold it at an advance of $33\frac{1}{3}$ per cent: what was his whole gain, and what his selling price?

17. A merchant imported 176 pieces of broadcloth, each piece measuring $46\frac{1}{4}$*yd.*, at \$3,25 a yard: what will be the duty at 30 per cent?

18. What is the duty on 54*T.* 13*cwt.* 3*qr.* 20*lb.* of iron, invoiced at \$45 a ton, and the duty $33\frac{1}{3}$ per cent?

19. What is the ad valorem duty, at 25 per cent, on 3*hhd.* of molasses, at 35 cents a gallon, an allowance of 2 per cent being made for leakage?

20. If I import 50 chests of tea, each weighing 140 pounds, invoiced at 60 cents a pound, a deduction of 10*lb.* per *cwt.* being made for tare: what were the governmental duties, at 40 per cent ad valorem?

21. What will be the duty on 225 bags of coffee, each weighing gross 160*lb.*, invoiced at 6 cents per *lb.*; 2 per cent being the legal rate of tare, and 20 per cent the duty?

22. What duty must be paid on 275 dozen bottles of claret, estimated to contain $2\frac{3}{4}$ gallons per dozen, 5 per cent being allowed for breakage, and the duty being 35 cents per gallon?

23. A merchant imports 175 cases of indigo, each case weighing 196*lb.* gross: 15 per cent is the customary rate of tare, and the duty 5 cents per *lb.* What duty must he pay on the whole?

24. What is the tare and duty on 75 casks of Epsom salts, each weighing gross 2*cwt.* 2*qr.* 24*lb.*, and invoiced at $1\frac{7}{8}$ cents per *lb.*, the customary tare being 11 per cent, and the rate of duty 20 per cent?

EQUATION OF PAYMENTS.

264. EQUATION OF PAYMENTS is a process of finding the average time of payment of several sums due at different times, so that no interest shall be gained or lost.*

1. B owes Mr. Jones $57: $16 is to be paid in 6 months; $18 in 7 months; and $24 in 10 months: what is the average time of payment so that no interest shall be gained or lost?

OPERATION.

$$\begin{array}{l}\$15 \times 6 = 90 \\ \$18 \times 7 = 126 \\ \underline{\$24 \times 10 = 240} \\ 57 \qquad 57)456(8 \\ \qquad\qquad\quad 456\end{array}$$

ANALYSIS.—The interest of $15 for 6 months, is the same as the interest of $1 for 90 months: the interest of $18 for 7 months is the same as the interest of $1 for 126 months; and the interest of $24 for 10 months is the same as the interest of $1 for 240 months; hence, the sum of these products, 466, is the number of months it would take $1 to produce the required interests. Now, the sum of the payments, $57, will produce the same interest in *one fifty-seventh part of the time*; that is, in 8 months: hence, to find the average time of payment:

Multiply each payment by the time before it becomes due, and divide the sum of the products by the sum of the payments: the quotient will be the average time.

EXAMPLES.

1. A merchant owes $1200, of which $200 is to be paid in 4 months, $400 in 10 months, and the remainder in 16 months: if he pays the whole at once, at what time must he make the payment?

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264. What is equation of payments? How do you find the average time of payment?

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* The mean time of payment is sometimes found by first finding the *present* value of each payment; but the rule here given has the sanction of the best authorities in this country and England.

2. A owes B $2400; one-third is to be paid in 6 months, one-fourth in 8 months, and the remainder in 12 months: what is the mean time of payment?

3. A merchant has due him $4500; one-sixth is to be paid in 4 months, one-third in 6 months, and the rest in 12 months: what is the equated time for the payment of the whole?

4. A owes B $1200, of which $240 is to be paid in three months, $360 in five months, and the remainder in ten months: what is the average time of payment?

5. Mr. Swain bought goods to the amount of $3840, to be paid for as follows, viz.: one-fourth in cash, one-fourth in 6 months, one-fourth in 7 months, and the remainder in one year: what is the average time of payment?

6. A flour merchant bought at one time 150 barrels of flour, at $8 a barrel; 15 days afterwards he bought 176 barrels, at $8,50 a barrel; 25 days after that he bought 200 barrels, at $9 a barrel: how many days after the first purchase would be the equated time of payment?

7. A man bought a farm for $5000, for which he agreed to pay $1000 down, $1200 in 3 months, $800 in 8 months, $1500 in 10 months, and the remainder in one year: if he pays the whole at once, what would be the average time of payment?

NOTES.—1. In finding the equated time of payments for several sums, due at different times, *any day may be assumed as the one* from which we reckon.

2. If one of the payments is due on the day from which the equated time is reckoned, its corresponding product will be nothing, but the payment must still be added in finding the sum of the payments.

8. A person owes three notes: the first is for $200, payable July 1st; the second for $150, payable August 1st; and the third for $250, payable August 15th: what is the average time, reckoned from July 1st?

NOTES.—1. May the equated time be reckoned from any day?

2. If one of the payments is due on the day from which the equated time is reckoned, what will be the value of the corresponding product?

9. E. BOND, Bought of TRUST & Co.

1856, Aug.	1,	450 yds.	muslin,	at 10 cents,	- -	$45,00
" "	16,	800 "	calico	" 12½ "	- -	100,00
" Sept.	5,	720 "	bombazine	80	- - -	576,00
" Oct.	1,	300 "	cloth,	at 3,50	- - -	1050,00

On what day does the whole amount fall due?

10. Mr. Johnson sold, on a credit of 8 months, the following bills of goods:

April 1st, a bill of $4350,
May 7th, a bill of 3750,
June 5th, a bill of 2550.

At what time will the whole become due?

11. A purchased of B the following bill of goods, on different times of credit:

May 1st,	1857, a bill amounting to	$800	on 3 months.
June 1st,	" " " "	700	" 3 "
" 15th,	" " " "	900	" 4 "
July 25th,	" " " "	1000	" 6 "

What is the equated time for the payment of the whole, and on what day, reckoned from Aug. 1st, is the bill due?

12. A person purchased the following bills of goods, on different times of credit:

Jan. 1st,	1855, a bill amounting to	$367,20	on 4 months.
" 28th,	" " " "	901,80	" 3 "
Feb. 24th,	" " " "	826,38	" 5 "
March 30th,	" " " "	854,88	" 6 "
May 1st,	" " " "	396,50	" 4 "

When is the average time of payment of the whole?

ALLIGATION.

265. Alligation is that branch of Arithmetic which treats of all questions relating to the mixing or compounding of two or more ingredients of different values. It is divided into two parts: Alligation Medial and Alligation Alternate.

ALLIGATION MEDIAL.

266. Alligation Medial teaches the method of finding the price or quality of a mixture of several simple ingredients whose prices or quantities are known.

1. A grocer would mix 200 pounds of lump sugar, worth 13 cents a pound, 400 pounds of Havana, worth 10 cents a pound, and 600 New Orleans, worth 7 cents a pound: what should be the price of the mixture?

OPERATION.

$$\begin{array}{l} 200 \times 13 = 26,00 \\ 400 \times 10 = 40,00 \\ \underline{600 \times \ \ 7 = 42,00} \\ 1200 \qquad)108,00(9 \text{ cts.} \end{array}$$

Analysis.—The quantity, 200*lb.*, at 13 cents a pound, costs \$26; 400 pounds, at 10 cents a pound, costs \$40; and 600*lb.* at 7 cents a pound, costs \$42: hence, the entire mixture, consisting of 1200*lb.*, costs \$108. Now, the price of the mixture will be as many cents as 1200 is contained times in 10800 cents: viz., 9 times. Hence, to find the price of the mixture,

Rule.—*Multiply the price or quality of a unit of each simple by the number of such units: take the sum of their products and divide it by the whole number of units: the quotient will be the price or quality of a unit of the mixture.*

EXAMPLES.

1. If 1 gallon of molasses, at 75 cents, and 3 gallons, at 50 cents, be mixed with 2 gallons, at 37½, what is the mixture worth a gallon?

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265. What is Alligation? Into how many parts is it divided? What are they?

266. What is Alligation Medial? How do you find the price of the mixture?

2. If teas at $37\frac{1}{2}$, 50, $62\frac{1}{2}$, 80, and 100 cents per pound, be mixed together, what will be the value of a pound of the mixture?

3. If 5 gallons of alcohol, worth 60 cents a gallon, and 3 gallons worth 96 cents a gallon, be diluted by 4 gallons of water, what will be the price of one gallon of the mixture?

4. A farmer sold 50 bushels of wheat at $2 a bushel; 60 bushels of rye, at 90 cents; 36 bushels of corn, at $62\frac{1}{2}$ cents; and 50 bushels of oats, at 39 cents a bushel: what was the average price per bushel of the whole?

5. During the seven days of the week, the thermometer stood as follows: 70°, 73°, $73\frac{1}{2}$°, 77°, 70°, $80\frac{1}{2}$°, and 81°: what was the average temperature for the week?

6. If gold 18, 21, 17, 19, and 20 carats fine, be melted together, what will be the fineness of the compound?

7. A grocer bought 34*lb.* of sugar at 5 cents a pound, 102*lb.* at 8 cents, 136*lb.* at 10 cents a pound, and 34*lb.* at 12 cents a pound. He mixed it together, and sold the mixture so as to make 50 per cent on the cost: what did he sell it for per pound?

8. A merchant sold 8*lb.* of tea, 11*lb.* of coffee, and 25*lb.* of sugar, at an average of 15 cents a pound. The tea was worth 30 cents a pound; the coffee, 25 cents a pound; and the sugar, 7 cents a pound: did he gain or lose, and how much?

ALLIGATION ALTERNATE.

267. ALLIGATION ALTERNATE teaches the method of finding what proportion of several simples, whose prices or qualities are known, must be taken to form a mixture of any required price or quality. It is the reverse of Alligation Medial, and may be proved by it.

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267. What is Alligation Alternate? How may Alligation Alternate be proved?

268. How do you find the proportional parts?

CASE I.

268. *To find the proportional parts.*

1. A miller would mix wheat, worth 12 shillings a bushel; corn, worth 8 shillings; and oats, worth 5 shillings, so as to make a mixture worth 7 shillings a bushel: what are the proportional parts of each?

OPERATION.

		A.	B.	C.	D.	E.
7s.	oats, 5s.	$\frac{1}{2}$	$\frac{1}{2}$	5	1	6 or 3
	corn, 8s.		1		2	2 " 1
	wheat, 12s.	$\frac{1}{5}$		2		2 " 1

ANALYSIS.—On every bushel put into the mixture, whose price is *less* than the mean price, there will be a *gain;* on every bushel whose price is *greater* than the mean price, there will be a *loss;* and since there is to be neither gain nor loss by the mixture, the gains and losses must balance each other.

A bushel of oats, when put into the mixture, will bring 7 shillings, giving a gain of 2 shillings; and to gain 1 shilling, we must take half as much, or $\frac{1}{2}$ a bushel, which we write in column A.

On 1 bushel of wheat there will be a loss of 5 shillings; and to make a loss of 1 shilling, we must take $\frac{1}{5}$ of a bushel, which we write in column A: $\frac{1}{2}$ and $\frac{1}{5}$ are called *proportional numbers.*

Again: comparing the oats and corn, there is a gain of 2 shillings on every bushel of oats, and a loss of 1 shilling on every bushel of corn: to *gain* 1 shilling on the oats, and *lose* 1 shilling on the corn, we must take $\frac{1}{2}$ a bushel of the oats, and 1 bushel of the corn: these numbers are written in column B. Two simples, thus compared, are called a *couplet:* in one, the price of unity is less than the mean price, and in the other it is greater.

If, every time we take $\frac{1}{2}$ a bushel of oats we take $\frac{1}{5}$ of a bushel of wheat, the gain and loss will balance; and if every time we take $\frac{1}{2}$ a bushel of oats we take 1 bushel of corn, the gain and loss will balance: hence, *if the proportional numbers of a couplet be multiplied by any number, the gain and loss denoted by the products will balance.*

When the proportional numbers, in any column, are fractional (as in columns A and B), multiply them by the least common multiple of their denominators, and write the products in new columns C and D. Then, add the numbers in columns C and D, standing opposite each simple, and if their sums have a common factor, reject it: the last result will be the proportional numbers.

NOTE.—The answers to the last, and to all similar questions, will be infinite in number, for two reasons:

1st. If the proportional numbers in column E be multiplied by any number, integral or fractional, the products will denote proportional parts of the simples.

2d. If the proportional numbers of *any couplet* be multiplied by any number, the gain and loss in that couplet will still balance, and the proportional numbers in the final result will be changed.

RULE.—I. *Write the prices or qualities of the simples in a column, beginning with the lowest, and the mean price or quality at the left.*

II. *Opposite the first simple write the part which must be taken to gain* 1 *of the mean price, and opposite the other simple of the couplet write the part which must be taken to lose* 1 *of the mean price, and do the same for each simple.*

III. *When the proportional numbers are fractional, reduce them to integral numbers, and then add those which stand opposite the same simple: if the sums have a common factor, reject it: the result will denote the proportional parts.*

EXAMPLES.

1. What proportions of coffee, at 8 cents, 10 cents, and 14 cents per pound, must be mixed together so that the compound shall be worth 12 cents per pound?

2. A merchant has teas worth 40 cents, 65 cents, and 75 cents a pound, from which he wishes to make a mixture worth 60 cents a pound: what is the smallest quantity of each that he can take and express the parts by whole numbers?

3. A farmer sold a number of colts at $50 each, oxen at $40, cows at $25, calves at $10, and realized an average price of $30 per head: what was the smallest number he could sell of each?

4. What is the smallest quantity of water that can be denoted by a whole number, that must be mixed with wine worth 14*s*. and 15*s*. a gallon, to form a mixture worth 13*s*. a gallon?

CASE II.

269. *When the quantity of one of the simples is given.*

1. A farmer would mix rye worth 80 cents a bushel, and corn worth 75 cents a bushel, with 66 bushels of oats worth 45 cents a bushel, so that the mixture shall be worth 50 cents a bushel: how much must be taken of each sort?

OPERATION.

		A.	B.	C.	D.	E.	F.
50	45	$\frac{1}{5}$	$\frac{1}{5}$	6	5	11	66
	75		$\frac{1}{25}$		1	1	6
	80	$\frac{1}{30}$		1		1	6

ANALYSIS.—Find the proportional parts, as in Case I: they are 11, 1 and 1. But we are to take 66 bushels of oats in the mixture; hence, each proportional number is to be taken 6 times; that is, as many times as there are units in the quotient of 66 ÷ 11.

RULE.—I. *Find the proportional numbers as in Case I, and write the given simples opposite its proportional number.*

II. *Multiply the given simple by the ratio which its proportional number bears to each of the others, and the products will denote the quantities to be taken of each.*

NOTE.—If we multiply the numbers in either or both of the columns C or D by any number, the proportion of the numbers in column E will be changed. Thus, if we multiply column D by 12, we shall have 60 and 12, and the numbers in column E become 66, 12 and 1, numbers which will fulfil the conditions of the question.

EXAMPLES.

1. What quantity of teas at 12*s.* 10*s.* and 6*s.* must be mixed with 20 pounds, at 4*s.* a pound, to make the mixture worth 8*s.* a pound?

2. How many pounds of sugar, at 7 cents and 11 cents a pound, must be mixed with 75 pounds, at 12 cents a pound, so that the mixture may be worth 10 cents a pound?

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269. How do you find the proportional parts when the quantity of one simple is given?

3. How many gallons of oil, at 7*s*., 7*s*. 6*d*. and 9*s*. a gallon, must be mixed with 24 gallons of oil, at 9*s*. 6*d*. a gallon, so as to form a mixture worth 8*s*. a gallon?

4. Bought 10 knives at \$2 each: how many must be bought at \$$\frac{3}{4}$ each, that the average price of the whole shall be \$$1\frac{1}{4}$?

5. A grocer mixed 50*lb*. of sugar worth 10 cents a pound, with sugars worth $9\frac{1}{3}$ cents, $7\frac{1}{3}$ cents, 7 cents, and 5 cents a pound, and found the mixture to be worth 8 cents a pound how much did he take of each kind?

CASE III.

270. *When the quantity of the mixture is given.*

1. A silversmith has four sorts of gold, viz., of 24 carats fine, of 22 carats fine, of 20 carats fine, and of 15 carats fine: he would make a mixture of 42 ounces of 17 carats fine: how much must he take of each sort?

OPERATION.

		A.	B.	C.	D.	E.	F.	G.	H.
17	15	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	7	5	3	15	30
	20			$\frac{1}{3}$			2	2	4
	22		$\frac{1}{5}$			2		2	4
	24	$\frac{1}{7}$			2			2	4

Proportional Parts:

$$15 + 2 + 2 + 2 = 21; \quad 42 \div 21 = 2.$$

RULE.—I. *Find the proportional parts as in Case I.*

II. *Divide the quantity of the mixture by the sum of the proportional parts, and the quotient will denote how many times each part is to be taken. Multiply this quotient by the parts separately, and each product will denote the quantity of the corresponding simple.*

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270. How do you find the proportional parts when the quantity of the mixture is given?

NOTE.—We may, as in the other cases, multiply each couplet by any number we please, which will merely change the relation of the proportional parts, and consequently give different proportions of the ingredients. Hence, there is an *infinite number of answers*, if we employ fractions, and often many answers to similar questions, in whole numbers.*

EXAMPLES.

1. A grocer has teas at *5s.*, *6s.*, *8s.*, and *9s.* a pound, and wishes to make a compound of *88lb.* worth *7s.* a pound: how much of each sort must be taken?

2. A liquor dealer wishes to fill a hogshead with water, and two kinds of brandy, at $2,50 and $3,00 per gallon, so that the mixture may be worth $2,25 a gallon: in what proportions must he mix them?

3. A person sold a number of sheep, calves, and lambs, 40 in all, for $48: how many did he sell of each, if he received for each calf $$1\frac{3}{4}$, each sheep $$1\frac{1}{4}$, and each lamb $$\frac{3}{4}$?

4. A merchant sold 20 stoves for $180; for the largest size he received $20, for the middle size, $7, and for the small size 6: how many did he sell of each kind?

5. A vintner has wines at *4s.*, *6s.*, *8s.*, and *10s.* per gallon; he wishes to make a mixture of 120 gallons, worth *5s.* per gallon: what quantity must he take of each?

6. A tailor has 24 garments, worth $144. He has coats, pantaloons and vests, worth $12, $5 and $2 each, respectively: how many has he of each?

7. A jeweler melted together four sorts of gold, of 24, 22, 20 and 15 carats fine, so as to produce a compound of *42oz.* of 17 carats fine: how much did he take of each sort?

8. A man paid $70 to 3 men for 35 days labor: to the first he paid $5 a day, to the second, $1 a day, and to the third, $$\frac{1}{2}$ a day: how many days did each labor?

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* See an admirable article on Alligation, published by Professor D. Wood, in the June number of the New York Teacher for 1855. By his permission, I have used such parts of it as seemed appropriate to a Text Book.

COINS AND CURRENCIES.

271. Coins are pieces of metal, of gold, silver, or copper, of fixed values, and impressed with a public stamp prescribed by the country where they are made. These are called specie, and are generally declared to be a legal tender in payment of debts. The Constitution of the United States provides, that gold and silver only, shall be a legal tender.

The coins of a country, and those of foreign countries having a fixed value established by law, together with bank notes redeemable in specie, make up what is called the *Currency*.

272. A Foreign coin may be said to have four values:

1st. The intrinsic value, which is determined by the amount of pure metal which it contains :

2d. The Custom House or legal value, which is fixed by law:

3d. The mercantile value, which is the amount it will sell for in open market:

4th. The Exchange value, which is the value assigned to it in buying and selling bills of exchange between one country and another.

Let us take, as an example, the English pound sterling, which is represented by the gold sovereign. Its intrinsic value, as determined at the Mint in Philadelphia, compared with our gold eagle, is $4,861. Its legal or custom house value is $4,84. Its commercial value, that is, what it will bring in Wall-street, New York, varies from $4,83 to $4,86, seldom reaching either the lowest or highest limit. The exchange value of the English pound, is $4,44$\frac{4}{9}$, and was the legal value before the change in our standard. This change raised the legal value of the pound to $4,84, but merchants and dealers in exchange preferred to retain the old value, which became nominal, and to add the difference in the form of a *premium on exchange*, which is explained in Art. 287. For the values of the various coins, see Table, page 391.

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271. What are coins? What are they called? What is declared in

EXCHANGE.

273. Exchange is a term which denotes the payment of money by a person residing in one place to a person residing in another. The payment is generally made by means of a bill of exchange.

274. A Bill of Exchange is an open letter of request from one person to another, desiring the payment to a third party named therein, of a certain sum of money to be paid at a specified time and place. There are always three parties to a bill of exchange, and generally four:

1. He who writes the open letter of request, is called the *drawer* or *maker* of the bill:

2. The person to whom it is directed is called the *drawee:*

3. The person to whom the money is ordered to be paid is called the *payee;* and

4. Any person who purchases a bill of exchange is called the *buyer* or *remitter.*

275. Bills of exchange are the proper money of commerce. Suppose Mr. Isaac Wilson of the city of New York, ships 1000 bags of cotton, worth £6000, to Samuel Johns & Co. of Liverpool; and at about the same time William James of New York orders goods from Liverpool, of Ambrose Spooner, to the amount of six thousand pounds sterling. Now, Mr. Wilson draws a bill of exchange on Messrs. Johns & Co. in the following form, viz.:

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regard to them? What is provided by the Constitution of the United States? What do you understand by Currency?

272. How many values may a coin be said to have? What is the intrinsic value? What is its Custom House value? What is the mercantile value? What is the exchange value?

273. What is Exchange? How is the payment generally made?

274. What is a Bill of Exchange? How many parties are there to a bill of exchange? Name them.

275. How do bills of exchange aid commerce? Name all the parties to the bill in this example.

Exchange for £6000. New York, July 30th, 1846.

Sixty days after sight of this my first Bill of Exchange (second and third of the same date and tenor unpaid*) pay to David C. Jones or order, six thousand pounds sterling, with or without further notice. ISAAC WILSON.

Messrs. Samuel Johns & Co.,
Merchants, Liverpool.

Let us now suppose that Mr. James purchases this bill of David C. Jones for the purpose of sending it to Ambrose Spooner of Liverpool, whom he owes. We shall then have all the parties to a bill of exchange; viz., Isaac Wilson, the *maker* or *drawer;* Messrs. Johns & Co., the *drawees;* David C. Jones, the *payee;* and William James, the *buyer* or *remitter.*

276. A bill of exchange is called an *inland bill,* when the drawer and drawee both reside in the same country; and when they reside in different countries, it is called a *foreign bill.* Thus, all bills in which the drawer and drawee reside in the United States, are inland bills; but if one of them resides in England or France, the bill is a foreign bill.

277. The time at which a bill is made payable varies, and is a matter of agreement between the drawer and buyer. They may either be drawn *at sight,* or at a certain number of days *after sight,* or at a certain number of days *after date.*

278. DAYS OF GRACE are a certain number of days granted to the person who pays the bill, after the time named in the bill

276. What is an inland bill? What is a foreign bill? Are bills drawn between one state and another inland, or foreign?

277. How is the time determined at which a bill is made payable? How are bills always drawn?

* Three bills are generally drawn for the same amount, called the first, second, and third, and together they form a set. One only is paid, and then the other two are of no value. This arrangement avoids the accidents and delays incident to transmitting the bills.

has expired. In the United States and Great Britain three days are allowed.

279. In ascertaining the time when a bill payable so many days after sight, or after date, actually falls due, the day of presentment, or the day of the date, is not reckoned. When the time is expressed in months, *calendar months* are always understood.

If the month in which a bill falls due is shorter than the one in which it is dated, it is a rule not to go on into the next month. Thus, a bill drawn on the 28th, 29th, 30th, or 31st of December, payable two months after date, would fall due on the last of February, except for the days of grace, and would be actually due on the third of March.

ENDORSING BILLS.

280. In examining the bill of exchange drawn by Isaac Wilson, it will be seen that Messrs. Johns & Co. are requested to pay the amount to David C. Jones or order; that is, either to Jones or to any other person named by him. If Mr. Jones simply writes his name on the back of the bill, he is said to endorse it in *blank*, and the drawees must pay it to any rightful owner who presents it. Such rightful owner is called the *holder*, and Mr. Jones is called the *endorser*.

If Mr. Jones writes on the back of the bill, over his signature, "Pay to the order of William James," this is called *a special endorsement*, and William James is the *endorsee*, and he may either endorse in blank or write over his signature, "Pay

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278. What are days of grace? How many days of grace are allowed in this country and in Great Britain?

279. In ascertaining the time when a bill is payable, what days are reckoned? When the time is expressed in months, what kind of months is understood? If the month in which the bill falls due is shorter than that in which it is drawn, what rule is observed?

280. What is an endorsement in blank? What is the person making it called? What is a special endorsement? What is the effect of an endorsement? How may a bill drawn to bearer be transferred?

to the order of Ambrose Spooner," and the drawees, Messrs. Johns & Co., will then be bound to pay the amount to Mr. Spooner.

A bill drawn payable to bearer, may be transferred by mere delivery.

ACCEPTANCE.

281. When the bill drawn on Messrs. Johns & Co. is presented to them, they must inform the holder whether or not they will pay it at the expiration of the time named. Their agreement to pay it is signified by writing across the face of the bill, and over their signature the word "accepted," and they are then called the *acceptors*.

LIABILITIES OF THE PARTIES.

282. The drawee of a bill does not become responsible for its payment until after he has accepted. On the presentation of the bill, if the drawee does not accept, the holder should immediately take means to have the drawer and all the endorsers notified. Such notice is called a *protest*, and is given by a public officer called a *notary*, or *notary public*. If the parties are not notified in a reasonable time, they are not responsible for the payment of the bill.

If the drawer accepts the bill, and fails to make the payment when it becomes due, the parties must be notified as before, and this is called *protesting the bill for non-payment*. If the endorsers are not notified in a reasonable time, they are not responsible for the amount of the bill.

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281. What is an acceptance? How is it made?

282. When does the drawee of a bill become responsible for its payment? If the drawee does not accept, what must the holder do? What is such notice called? By whom is it made? If the parties to the bill are not notified, what is the consequence? If the drawee accepts the bill and fails to make the payment, what must then be done? If the bill is not protested, what will be the consequence?

PAR OF EXCHANGE—COURSE OF EXCHANGE.

283. The intrinsic *par of exchange*, is a term used to compare the coins of different countries with each other, with respect to their intrinsic values, that is, with reference to the amount of pure metal in each. Thus, the English sovereign, which represents the pound sterling, is intrinsically worth $4,861 in our gold, taken as a standard, as determined at the Mint in Philadelphia. This, therefore, is the value at which the sovereign must be reckoned, in estimating the par of exchange.

284. The *commercial par of exchange* is a comparison of the coins of different countries according to their market value. Thus, as the market value of the English sovereign varies from $4,83 to $4,85 (Art. 272), the commercial par of exchange will fluctuate. It is, however, always determined when we know the value at which the foreign coin sells in our market.

285. The *course of exchange* is the variable price which is paid at one place for bills of exchange drawn on another. The course of exchange differs from the intrinsic par of exchange, and also from the commercial par, in the same way that the market price of an article differs from its natural price. The commercial par of exchange would at all times determine the course of exchange, if there were no fluctuations in trade.

286. When the market price of a foreign bill is *above* the commercial par, the exchange is said to be at a *premium*, or in favor of the foreign place, because it indicates that the foreign

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283. What do you understand by the intrinsic par of exchange? What is the intrinsic value of the English sovereign?

284. What is the commercial par of exchange? What is the commercial value of the English sovereign?

285. What do you understand by the course of exchange? How does it differ from the intrinsic par and the commercial par? What causes it to differ from the commercial par?

286. What is said when the price of a foreign bill is above the commercial par? When is it below it? To whom is a favorable state of exchange advantageous? To whom is it injurious?

place has sold more than it has bought, and that specie must be shipped to make up the difference. When the market price is *below* this par, exchange is said to be *below par*, or in favor of the place where the bill is drawn. Such place will then be a creditor, and the debt must be paid in specie or other property. It should be observed that a favorable state of exchange is advantageous to the buyer but not to the seller, whose interest, as a dealer in exchange, is identified with that of the place on which the bill is drawn.

287. It was stated in Art. 272, that the exchange value of the pound sterling is $\$4,44\frac{4}{9} = 4,4444+$; that is, this value is the basis on which the bills of exchange are drawn. Now this value being below both the commercial and intrinsic value, the drawers of bills increase the course of exchange so as to make up this deficiency.

For example, if we add to the exchange value of the pound, 9 per cent, we shall have its commercial value, very nearly.

Thus, exchange value,	- -	$= \$4,4444+$
Nine per cent,	- - - -	$= {,}3999+$
which gives	- - -	$\$4,8443$

and this is the average of the commercial value, very nearly. Therefore, when the course of exchange is at a premium of 9 per cent, it is at the commercial par, and as between England and this country it would stand near this point, but for the fluctuations of trade and other accidental circumstances.

INLAND BILLS.

288. We have seen that inland bills are those in which the drawer and drawee both reside in the same country (Art. 276).

EXAMPLES.

1. A merchant at New Orleans wishes to remit to New York $8465, and exchange is $1\frac{1}{2}$ per cent premium. How much must he pay for such a bill?

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287. What is the exchange value of the pound sterling?

288. What are inland bills?

2. A merchant in Boston wishes to pay in Philadelphia \$8746,50; exchange between Boston and Philadelphia is $1\frac{1}{4}$ per cent below par. What must he pay for a bill?

3. A merchant in Philadelphia wishes to pay \$9876,40 in Baltimore, and finds exchange to be 1 per cent below par: what must he pay for the bill?

ENGLAND.

289. It has already been stated that the exchanges between this country and England are made in pounds, shillings, and pence, and that the exchange value of the pound sterling is $\$4,44\frac{4}{9}$, and that the premiums are all reckoned from this standard.

EXAMPLES.

1. A merchant in New York wishes to remit to Liverpool £1167 10*s.* 6*d.*, exchange being at $8\frac{1}{2}$ per cent premium. How much must he pay for the bill in Federal money?

First, £1167 10*s.* 6*d.* - -	=	£1167.525
For multiply by $8\frac{1}{2}$ per cent, -		.085
the product is the premium - -	=	99.239625
the product added gives - -		£1266.764625

which reduced to dollars and cents at the rate of $\$4,44\frac{4}{9}$ to the pound, gives the amount which must be paid for the bill in dollars and cents.

2. A merchant has to remit £36794 8*s.* 9*d.* to London, how much must he pay for a bill in dollars and cents, exchange being $7\frac{3}{4}$ per cent premium?

3. A merchant in New York wishes to remit to London \$67894,25, exchange being at a premium of 9 per cent. What will be the amount of his bill in pounds shillings and pence?

NOTE.—Add the amount of the premium to the exchange value of the pound, viz. $\$4,44\frac{4}{9}$, which in this case gives \$4,84444, and then divide the amount in dollars by this sum, and the quotient will be the amount of the bill in pounds and the decimals of a pound.

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289. In what currency are the exchanges between this country and England made? What is the exchange value of the pound sterling?

4. A merchant in New York owes £1256 18*s.* 9*d.* in London; exchange at a nominal premium of $7\frac{1}{2}$ per cent: how much money, in Federal currency, will be necessary to purchase the bill?

5. I have $947,86 and wish to remit to London £364 18*s.* 8*d.*, exchange being at $8\frac{1}{4}$ per cent: how much additional money will be necessary?

FRANCE.

290. The accounts in France, and the exchange between France and other countries, are all kept in francs and centimes, which are hundredths of the franc. We see from the table that the value of the franc is 18.6 cents, which gives very nearly, 5 francs and 38 centimes to the dollar. The rate of exchange is computed on the value 18.6 cents, but is often quoted by stating the value of the dollar in francs. Thus, exchange on Paris is said to be 5 francs 40 centimes, that is, one dollar will buy a bill on Paris of 5 francs and 40 hundreds of a franc.

EXAMPLES.

1. A merchant in New York wishes to remit 167556 francs to Paris, exchange being at a premium of $1\frac{1}{2}$ per cent. What will be the cost of his bill in dollars and cents?

Commercial value of the franc,	- -	18.6 cents,
Add $1\frac{1}{2}$ per cent,	- - - - -	.279
Gives value for remitting,	- -	18.879 cents;

then, $167556 \times 18.879 = \$31632{,}89724$,

which is the amount to be paid for the bill?

2. What amount, in dollars and cents, will purchase a bill on Paris for 86978 francs, exchange being at the rate of 5 francs and 2 centimes to the dollar?

First, $86978 \div 5.02 = \$17326{,}274 +$, the amount.

Is this bill above or below par? What per cent?

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290. In what currency are the exchanges with France conducted? What is a centime? What is the value of a franc? What is meant when exchange on Paris is quoted at 5 francs 40 centimes?

3. How much money must be paid to purchase a bill of exchange on Paris for 68097 francs, exchange being 3 per cent below par?

4. A merchant in New York wishes to remit $16785,25 to Paris; exchange gives 5 francs 4 centimes to the dollar: how much can he remit in the currency of Paris?

HAMBURG.

291. Accounts and exchanges with Hamburg are generally made in the marc banco, valued, as we see in the table, at 35 cents.

EXAMPLES.

1. What amount in dollars and cents will purchase a bill of exchange on Hamburg for 18649 marcs banco, exchange being at 2 per cent premium?

2. What amount will purchase a bill for 3678 marcs banco, reckoning the exchange value of the marc banco at 34 cents? Will this be above or below the par of exchange?

ARBITRATION OF EXCHANGE.

292. Arbitration of Exchange is the method by which the currency of one country is changed into that of another, through the medium of one or more intervening currencies, with which the first and last are compared.

293. When there is but one intervening currency it is called *Simple Arbitration*; and when there is more than one it is called *Compound Arbitration*. The method of performing this is called the *Chain Rule*.

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291. In what are accounts kept at Hamburgh? What is the value of the marc banco?

292. What is arbitration of exchange?

293. When there is but one intervening currency, what is the exchange called? When there is more than one, what is it called?

294. The principle involved in arbitration of exchange is simply this: To pass from one system of values through several others, and find the true proportion between the first and last.

1. Suppose it were required to exchange 109150 pence into dollars, by first changing them into shillings, then into pounds, and then into dollars.

OPERATION.

$$109150d. = 109150 \times \tfrac{1}{12}s.$$
$$109150d. = £109150 \times \tfrac{1}{12} \times \tfrac{1}{20}.$$
$$109150d. = \$109150 \times \tfrac{1}{12} \times \tfrac{1}{20} \times 4.444 = \$201,924.$$

ANALYSIS.—Since 12 pence make 1 shilling, there will be one-twelfth as many shillings as pence: since 20 shillings make 1 pound, there will be one-twentieth as many pounds as shillings; and since there are $4.444 in a pound, there will be as many dollars as result from taking the pounds, 4.444 times; that is, $201,924.

2. Let it be required to remit $6570 to London, by the way of Paris, exchange on Paris being 5 francs 15 centimes for $1, and the exchange from Paris to London, 25 francs and 80 centimes for £1: what will be the value of the remittance at London?

OPERATION.

$$\$6570 = 6570 \times 5.15 \text{ francs.}$$
$$\$6570 = \$6570 \times 5.15 \times \tfrac{1}{25.80} = £131\ 3s.\ 10\tfrac{3}{4}d.$$

ANALYSIS.—Since 5 francs and 15 centimes are equal to $1, there will be as many francs at Paris as are equal to 6570 taken 5.15 times; and since £1 at London is equal to 25.80 francs, there will be as many pounds as 25.80 is contained times in the last product; that is, £131 3s. 10¾d. Hence, the following, which is called the Chain Rule:

Multiply the sum to be remitted by the following quotients: By a certain amount at the second place divided by its equivalent

294. What principle is involved in the arbitration of exchange? What is the chain rule? Give the rule.

at the first; by a certain amount at the third place by its equivalent at the second; by a certain amount at the fourth place divided by its equivalent at the third, and so on to the last place.

EXAMPLES.

1. A merchant wishes to remit \$4888,40 from New York to London, and the exchange is at a premium of 10 per cent. He finds that he can remit to Paris at 5 francs 15 centimes to the dollar, and to Hamburg at 35 cents per marc banco. Now, the exchange between Paris and London is 25 francs 80 centimes for £1 sterling, and between Hamburg and London $13\frac{3}{4}$ marcs banco for £1 sterling. How had he better remit?

1st. *To London direct.*

The amount to be remitted is \$4888,40. The exchange value of £1 is \$4,444, and since the exchange is at a premium of 10 per cent, the value of £1 is \$4,444 + ,4444 = \$4,8884: hence,

$$\$4888{,}40 \times \frac{1}{4.8884} = £1000:$$

hence, if he remits direct he will obtain a bill for £1000.

2d. *Exchange through Paris.*

$$4888{,}40 \times \frac{\overset{1.03}{\cancel{5.15}}}{1} \times \frac{1}{\underset{5.16}{\cancel{25.80}}} = £975{,}7852 = £975\ 15s.\ 8\tfrac{1}{4}d.$$

ANALYSIS.—Since 5.15 francs are equal to 1 dollar, the first multiplier will be this amount divided by \$1; and since £1 is equal to 25.80 francs, the second multiplier will be £1 divided by this amount. Then, by dividing by 5 and multiplying, we find that the amount remitted by the second method would be worth, at London, £975 15*s*. $8\frac{1}{4}d$.

3d. *Exchange through Hamburg.*

$$\$4888{,}40 \times \frac{1}{.35} \times \frac{1}{13.75} = £1015.771 = £1015\ 15s.\ 5d.$$

ANALYSIS.—Since 1 marc banco is equal to 35 cents, it is 35 hundredths of a dollar: hence, the first multiplier is 1 marc banco

divided by .35, and the second, 1 divided by 13.75. By this course of exchange the remittance at London would be worth £1015 15*s*. 5*d*.

Hence, the best way to remit is through Hamburg, then through Paris, and the least advantageous, direct.

2. A merchant in London has sold goods in Amsterdam to the amount of 824 pounds Flemish, which could be remitted to London at the rate of 34*s*. 4*d*. Flemish per pound sterling. He orders it to be remitted circuitously at the following rates, viz.: to France at the rate of 48*d*. Flemish per crown; thence to Vienna at 100 crowns for 60 ducats; thence to Hamburg at 100*d*. Flemish per ducat; thence to Lisbon at 50*d*. Flemish per crusado of 400 reas; and lastly, from Lisbon to England at 5*s*. 8*d*. per milrea: does he gain or lose by the circular exchange?

48*d*. Flemish	= 1 crown,
100 crowns	= 60 ducats,
1 ducat	= 100*d*. Flemish,
50*d*. Flemish	= 400 reas,
1 milrea or 1000 reas	= 68*d*. sterling.

$$824 \times \frac{1}{\underset{8}{\cancel{48}}} \times \frac{\overset{\cancel{10}}{\cancel{60}}}{\cancel{100}} \times \frac{\cancel{100}}{1} \times \frac{\overset{\cancel{8}}{\cancel{400}}}{\cancel{50}} \times \frac{\overset{17}{\cancel{68}}}{\underset{\underset{25}{\cancel{100}}}{\cancel{1000}}} = \frac{824 \times 17}{25} = \frac{14008}{25}$$

$$= £560\ 6s.\ 4\tfrac{4}{5}d.$$

The direct exchange would give,

$$824 \times \frac{£1}{34s.\ 4d.\ \text{Flemish}} = 824 \times \tfrac{240}{412} = £480 \text{ sterling.}$$

Hence, the amount gained by circuitous exchange would be £80 6*s*. $\frac{4}{5}$*d*.

GENERAL AVERAGE.

295. AVERAGE is a term of commerce and navigation, to signify a contribution by individuals, where the goods of a particular merchant are thrown overboard in a storm, to save the ship from sinking, or where the masts, cables, anchors, or other furniture of the ship are cut away or destroyed for the preservation of the whole. In these and like cases, where any sacrifices are deliberately made, or any expenses voluntarily incurred, to prevent a total loss, such sacrifice or expense is the proper subject of a general contribution, and ought to be ratably borne by the owners of the ship, the freight, and the cargo, so that the loss may fall proportionably on all. The amount sacrificed is called the *jettison.*

296. Average is either *general* or *particular;* that is, it is either chargeable to all the interests, viz., the ship, the freight, and the cargo, or only to some of them. As when losses occur from ordinary wear and tear, or from the perils incident to the voyage, without being *voluntarily* incurred; or when any particular sacrifice is made for the sake of the *ship only* or the *cargo only*, these losses must be borne by the parties immediately interested, and are consequently defrayed by *a particular* average. There are also some small charges called *petty* or *accustomed* averages, one-third of which is usually charged to the ship, and two-thirds to the cargo.

No general average ever takes place, except it can be shown that *the danger was imminent, and that the sacrifice was made indispensable, or supposed to be so by the captain and officers, for the safety of the ship.*

297. In different countries different modes are adopted of valuing the articles which are to constitute a general average.

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295. What does the term average signify?

296. How many kinds of average are there? What are the small charges called? Under what circumstances will a general average take place

In general, however, the value of the freight is held to be the clear sum which the ship has earned after seamen's wages, pilotage, and all such other charges as came under the name of petty charges, are deducted; one-third, and in some cases one-half, being deducted for the wages of the crew.

The goods lost, as well as those saved, are valued at the price they would have brought in ready money at *the place of delivery*, on the ship's arriving there, freight, duties, and all other charges being deducted: indeed, they bear their proportions, the same as the goods saved. The ship is valued at the price she would bring on her arrival at the port of delivery. But when the loss of masts, cables, and other furniture of the ship is compensated by general average, it is usual, as the new articles will be of greater value than the old, to deduct one-third, leaving two-thirds only to be charged to the amount to be contributed.

EXAMPLES.

1. The vessel Good Intent, bound from New York to New Orleans, was lost on the Jersey beach the day after sailing. She cut away her cables and masts, and cast overboard a part of her cargo, by which another part was injured. The ship was finally got off, and brought back to New York.

AMOUNT OF LOSS.

Goods of A cast overboard, - - -		$500
Damage of the goods of B by the jettison, -		200
Freight of the goods cast overboard, - -		100
Cable, anchors, mast, &c., worth -	$300	200
Deduct one-third, - - -	100	
Expenses of getting the ship off the sands,		56
Pilotage and port duties going in and out of the harbor, commissions, &c., -		100
Expenses in port, - - - - - -		25
Adjusting the average, - - - - -		4
Postage, - - - - - - - -		1
	Total loss,	$1186

ARTICLES TO CONTRIBUTE.

Goods of A cast overboard, - - - - - -	$ 500
Value of B's goods at N. O., deducting freight, &c.,	1000
" of C's " " " "	500
" of D's " " " "	2000
" of E's " " " "	5000
Value of the ship, - - - - - - -	2000
Freight, after deducting one-third, - - - - -	800
	$11,800

Then, total value : total loss : : 100 : per cent of loss.
$11800 : 1180 : : 100 : 10;

hence, each loses 10 per cent on the value of his interest in the cargo, ship, or freight. Therefore, A loses $50; B, $100; C, $50; D, $200; E, $500; the owners of the ship, $280—in all $1180. Upon this calculation the owners are to lose $280; but they are to receive their disbursements from the contribution, viz., freight on goods thrown overboard, $100; damages to ship, $200; various disbursements in expenses, $180; total, $480; and deducting the amount of contribution, they will actually receive $200. Hence, the account will stand:

The owners are to receive - - - - - -		$200
A loses $500, and is to contribute $50; hence, he receives - - - - - - - -		450
B loses $200, and is to contribute $100; hence, he receives - - - - - - - -		100
Total to be received, - - - -		$750
C, D, and E, have lost nothing, and are to pay	C	$ 50
	D	200
	E	500
Total actually paid, - - - - -		$750;

297. How is the freight valued? How much is charged on account of the seamen's wages? How is the cargo valued? Does the part lost bear its part of the loss? How is the ship valued? When parts of the ship are lost, how are they compensated for? How do you explain the example?

so that the total to be paid is just equal to the total loss, as it should be, and A and B get their remaining and injured goods, and the three others get theirs in a perfect state, after paying their ratable proportion of the loss.

TONNAGE OF VESSELS.

298. There are certain custom house charges on vessels, which are made according to their tonnage. The tonage of a vessel is the number of tons weight she will carry, and this is determined by measurement.

[From the "Digest," by Andrew A. Jones, of the N. Y. Custom House].

Custom house charges on all ships or vessels entering from any foreign port or place.

Ships or vessels of the United States, having three-fourths of the crew and all the officers American citizens, *per ton*,	$0,06
Ships or vessels of nations entitled by treaty to enter at the same rate as American vessels, - - - - - -	,06
Ships or vessels of the United States not having three-fourths the crew as above, - - - - - - - - -	,50
On foreign ships or vessels other than those entitled by treaty,	,50
Additional tonnage on foreign vessels, denominated light money, - - - - - - - - - - - -	,50

Licensed coasters are also liable once in each year to a duty of 50 cents per ton, being engaged in a trade from a port in one state to a port in another state, other than an adjoining state, unless the officers and three-fourths of the crew are American citizens; to ascertain which, the crews are always liable to an examination by an officer.

A foreign vessel is not permitted to carry on the coasting trade; but having arrived from a foreign port with a cargo consigned to more than one port of the United States, she may proceed coastwise with a certified manifest until her voyage is completed.

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298. What is the tonnage of a vessel? What are the custom house charges on the different classes of vessels trading with foreign countries? To what charges are coasters subject?

299. The government estimate the tonnage according to one rule, while the ship carpenter who builds the vessel uses another.

GOVERNMENT RULE.—I. *Measure, in feet, above the upper deck the length of the vessel, from the fore part of the main stem to the after part of the stern-post. Then measure the breadth taken at the widest part above the main wale on the outside, and the depth from the under side of the deck-plank to the ceiling in the hold.*

II. *From the length take three-fifths of the breadth and multiply the remainder by the breadth and depth, and the product divided by 95 will give the tonnage of a single decker; and the same for a double decker, by merely making the depth equal to half the breadth.*

CARPENTERS' RULE.—*Multiply together the length of the keel, the breadth of the main beam, and the depth of the hold, and the product divided by 95 will be the carpenters' tonnage for a single decker; and for a double decker, deduct from the depth of the hold half the distance between decks.*

EXAMPLES.

1. What is the government tonnage of a single decker, whose length is 75 feet, breadth 20 feet, and depth 17 feet?

2. What is the carpenters' tonnage of a single decker, the length of whose keel is 90 feet, breadth 22 feet 7 inches, and depth 20 feet 6 inches?

3. What is the carpenters' tonnage of a steamship, double decker, length 154 feet, breadth 30 feet 8 inches, and depth, after deducting half between decks, 14 feet 8 inches?

4. What is the government tonnage of a double decker, the length being 103 feet, breadth 25 feet 6 inches?

5. What is the carpenters' tonnage of a double decker, its length 125 feet, breadth 25 feet 6 inches, depth of hold 34 feet, and distance between decks 8 feet?

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299. What is the government rule for finding the tonnage? What the ship-builders' rule?

INVOLUTION.

300. A POWER is the product of equal factors. The equal factor is called the *root* of the power.

The *first power* is the *equal factor* itself, or the *root:*

The *second power* is the product of the root by itself:

The *third power* is the product when the root is taken 3 times as a factor:

The *fourth power*, when it is taken 4 times:

The *fifth power*, when it is taken 5 times, &c.

301. The number denoting how many times the root is taken as a factor, is called the *exponent* of the power. It is written a little at the right and over the root: thus, if the equal factor or root is 3,

$$\begin{aligned} 3 &= 3 \text{ the 1st power of 3.} \\ 3^2 = 3 \times 3 &= 9 \text{ the 2d power of 3.} \\ 3^3 = 3 \times 3 \times 3 &= 27 \text{ the 3d power of 3.} \\ 3^4 = 3 \times 3 \times 3 \times 3 &= 81 \text{ the 4th power of 3.} \\ 3^5 = 3 \times 3 \times 3 \times 3 \times 3 &= 243 \text{ the 5th power of 3.} \end{aligned}$$

INVOLUTION *is the process of finding the powers of numbers.*

NOTE.—1. There are three things connected with every power: 1st, The root; 2d, The exponent: and 3d, The power or result of the multiplication.

2. In finding a power, the root is always the 1st power: hence, *the number of multiplications is* 1 *less than the exponent.*

RULE.—*Multiply the number by itself as many times less* 1 *as there are units in the exponent, and the last product will be the power.*

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300. What is a power? What is the root of a power? What is the first power? What is the second power? The third power?

301. What is the exponent of the power? How is it written? What is Involution? How many things are connected with every power? How find the power of a number?

EXAMPLES.

Find the power of the following numbers:

1. The square of 4?
2. The square of 15?
3. The square of 26?
4. The square of 142?
5. The square of 463?
6. The square of 1340?
7. The square of 24.6?
8. The square of .526?
9. The square of 3.125?
10. The square of .0524?
11. The square of 246.25?
12. The square of $\frac{3}{4}$?
13. The square of $\frac{6}{7}$?
14. The square of $\frac{7}{9}$?
15. The square of $\frac{13}{14}$?
16. The square of $\frac{35}{84}$?
17. The square of $\frac{125}{247}$?
18. The square of $2\frac{4}{5}$?
19. The square of $7\frac{5}{8}$?
20. The square of $15\frac{9}{11}$?
21. The square of $225\frac{9}{10}$?
22. The cube of 6?
23. The cube of 24?
24. The cube of 72?
25. The cube of 125?
26. The cube of 136?
27. The 4th power of 12?
28. The 5th power of 9?
29. The value of $(4.25)^3$?
30. The value of $(1.8)^4$?
31. The value of $(32.4)^3$?
32. The value of $(.45)^5$?
33. The value of $(\frac{15}{16})^3$?
34. The cube of $(\frac{5}{6})$?
35. The 4th power of $\frac{3}{8}$?
36. The cube of $14\frac{2}{3}$?
37. The value of $(2\frac{1}{4})^5$?
38. The value of $(\frac{25}{27})^4$?
39. The value of $(24\frac{3}{5})^3$?
40. The value of $(.25)^6$?
41. The value of $(142.5)^3$?
42. The value of $(3.205)^2$?

EVOLUTION.

302. Evolution is the process of finding the factor when we know the power.

The Square Root of a number is the factor which multiplied by itself *once* will produce the number.

The Cube Root of a number is the factor which multiplied by itself *twice* will produce the number.

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302. What is Evolution? What is the square root of a number? What is the cube root of a number? How do you denote the square root of a number? How the cube root?

Thus, 8 is the square root of 64, because $8 \times 8 = 64$; and 3 is the cube root of 27, because $3 \times 3 \times 3 = 27$.

The sign $\sqrt{\ }$ is called the radical sign. When placed before a number it denotes that its square root is to be extracted. Thus, $\sqrt{36} = 6$.

We denote the cube root by the same sign with 3 written over it: thus, $\sqrt[3]{27}$, denotes the cube root of 27, which is equal to 3. The small figure 3, placed over the radical, is called the *index* of the root.

EXTRACTION OF THE SQUARE ROOT.

303. The *square root* of a number is one of the two equal factors of that number. To extract the square root is to find this factor. The first ten numbers and their squares are

1,	2,	3,	4,	5,	6,	7,	8,	9,	10.
1,	4,	9,	16,	25,	36,	49,	64,	81,	100.

The numbers in the first line are the square roots of those in the second. The numbers 1, 4, 9, 16, 25, 36, &c., having *exact* factors, are called *perfect squares*.

A perfect square is a number which has two *exact factors*.

NOTE.—The square root of a number less than 100 will be less than 10, while the square root of a number greater than 100 will be greater than 10.

304. *To find the square root of a number.*

1. What is the square of $36 = 3$ tens $+ 6$ units?

ANALYSIS.—The square of 36 is found by taking 36 thirty-six times; and this is done by first taking it 6 units times and then 3 tens times, and adding the products. 36 taken 6 units times, gives $3 \times 6 + 6^2$: and taken 3 tens times gives $3^2 + 3 \times 6$, and their sum is $3^2 + 2(3 \times 6) + 6^2$: that is,

OPERATION.

$$\begin{array}{r} 3 + 6 \\ 3 + 6 \\ \hline 3 \times 6 + 6^2 \\ 3^2 + 3 \times 6 \quad\quad \\ \hline 3^2 + 2(3 \times 6) + 6^2 \end{array}$$

303. What is the square root of a number? What is a perfect square? How many perfect squares are there between 1 and 100? How do the roots of numbers less than 100 compare with 10?

304. Into what parts may every number be decomposed? When so decomposed, what is its square equal to?

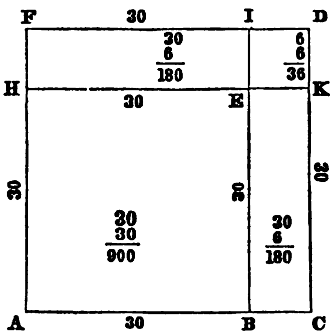
The square of a number is equal to the square of the tens, plus twice the product of the tens by the units, plus the square of the units.

The same may be shown by a figure:

Let the line AB represent the 3 tens or 30, and BC the six units.

Let AD be a square on AC, and AE a square on the ten's line AB.

Then ED will be a square on the unit line 6, and the rectangle EF will be the product of HE, which is equal to the ten's line, by IE, which is equal to the unit line. Also, the rectangle BK will be the product of EB, which is equal to the ten's line, by the unit line, BC. But the whole square on AC is made up of the square AE, the two rectangles FE and EC and the square ED: hence, it is equal to



The square of the tens plus twice the product of the tens by the units, plus the square of the units.

1. Let it now be required to extract the square root of 2025.

ANALYSIS.—Since the number contains more than two places of figures, its root will contain tens and units. But as the square of one ten is one hundred, it follows that the square of the tens of the required root must be found in the two figures on the left of 25. Hence, we point off the number into periods of two figures each.

We next find the greatest square contained in 20, which is 4 tens or 40. We then square 4 tens which gives 16 hundred, and then place 16 under the second period, and subtract; this takes away the square of the tens, and leaves 425, *which is twice the product of the tens by the units plus the square of the units.*

OPERATION.

```
    2̇0 2̇5(45
    16
85)42 5
   42 5
```

If now, we double the divisor and then divide this remainder exclusive of the right hand figure, (since that figure cannot enter into the product of the tens by the units) by it, the quotient will be the units figure of the root. If we annex this figure to the augmented divisor, and then multiply the whole divisor thus increased by it, the product will be twice the tens by the units plus the square of the units; and hence, we have found both figures of the root

This process may also be illustrated by the figure.

Suppose AC = 45. Then, subtracting the square of the tens is taking away the square AE, and leaves the two rectangles FE and BK, together with the square ED on the unit line.

The two rectangles FE and BK representing the product of units by tens, can be expressed by no figures less than tens.

If then, we divide the number 42, at the left of 5, by twice the tens, that is, by twice AB or BE, the quotient will be BC or EK, the unit of the root.

Then, placing BC or 5, in the root, and also annexing it to the divisor doubled, and then multiplying the whole divisor 85 by 5, we obtain the two rectangles FE and CE, together with the square ED.

305. Hence, for the extraction of the square root, we have the following

RULE.—I. *Separate the given number into periods of two figures each, by setting a dot over the place of units, a second over the place of hundreds, and so on for each alternative figure at the left.*

II. *Note the greatest square contained in the period on the left, and place its root on the right after the manner of a quotient in division. Subtract the square of this root from the first period, and to the remainder bring down the second period for a dividend.*

III. *Double the root thus found for a trial divisor and place it on the left of the dividend. Find how many times the trial divisor is contained in the dividend, exclusive of the right-hand*

---

305. What is the first step in extracting the square root of numbers? What is the second? What is the third? What the fourth? What the *fifth*? Give the entire rule?

figure, and place the quotient in the root and also annex it to the divisor.

IV. *Multiply the divisor thus increased, by the last figure of the root; subtract the product from the dividend, and to the remainder bring down the next period for a new dividend.*

V. *Double the whole root thus found, for a new trial divisor, and continue the operation as before, until all the periods are brought down.*

EXAMPLES.

1. What is the square root of 425104?

OPERATION.

$$\begin{array}{r} 4\dot{2}\ 5\dot{1}\ 0\dot{4}(652 \\ 36 \\ \hline 125)651 \\ 625 \\ \hline 1302)2604 \\ 2604 \end{array}$$

ANALYSIS.—We first place a dot over the 4, making the right-hand period 04. We then put a dot over the 1, and also over the 2, making three periods.

The greatest perfect square in 42 is 36, the root of which is 6. Placing 6 in the root, subtracting its square from 42, and bringing down the next period 51, we have 651 for a dividend, and by doubling the root we have 12 for a trial divisor. Now, 12 is contained in 65, 5 times. Place 5 both in the root and in the divisor; then multiply 125 by 5; subtract the product and bring down the next period.

We must now double the whole root 65 for a new trial divisor; or we may take the first divisor after having doubled the last figure 5; then dividing, we obtain 2, the third figure of the root.

NOTES.—1. The left-hand period may contain but one figure; each of the others will contain two.

2. If any trial divisor is greater than its dividend, the corresponding quotient figure will be a cipher.

3. If the product of the divisor by any figure of the root exceeds the corresponding dividend, the quotient figure is too large and must be diminished.

4. There will be as many figures in the root as there are periods in the given number.

5. If the given number is not a perfect square there will be a remainder after all the periods are brought down. In this case, periods of ciphers may be annexed, forming new periods, each of which will give one decimal place in the root.

2. What is the square root of 758692?

OPERATION.

```
       75 86 92(871.029+.
       64
      167)11 86
          11 69
     1741)17 92
          17 41
   174202)510000
          348404
  1742049)16159600
          15678441
            481159 Rem.
```

NOTE.—After using all the periods of the given number, we annex periods of decimal ciphers, each of which gives one decimal place in the quotient.

306. *To extract the square root of a fraction:*

1. What is the square root of .6?

NOTE.—We first annex one cipher to make *even* decimal places; for, one decimal multiplied by itself will give two places in the product. We then extract the root of the first period, and to the remainder annex a decimal period, and so on, till we have found a sufficient number of decimal places.

OPERATION.

```
       .60(.774+
        49
   147)1100
       1029
  1544)7100
       6176
        924 rem.
```

2. What is the square root of $\frac{16}{25}$?

NOTE.—The square root of a fraction is equal to the square root of the numerator divided by the square root of the denominator.

OPERATION.

$$\sqrt{\tfrac{16}{25}} = \frac{\sqrt{16}}{\sqrt{25}} = \tfrac{4}{5}.$$

3. What is the square root of $\frac{3}{4}$?

NOTE.—When the terms are not perfect squares, reduce the common fraction to a decimal, and then extract the square root of the decimal.

OPERATION.

$\frac{3}{4} = .75;$

$\sqrt{\tfrac{3}{4}} = \sqrt{.75} = .8545+$

---

306. How do you extract the square root of a decimal number? How of a common fraction?

RULE.—I. *If the fraction is a decimal, point off the periods from the decimal point to the right, annexing ciphers if necessary, so that each period shall contain two places, and then extract the root as in integral numbers.*

II. *If the fraction is a common fraction, and its terms perfect squares, extract the square root of the numerator and denominator separately; if they are not perfect squares, reduce the fraction to a decimal, and then extract the square root of the result.*

EXAMPLES.

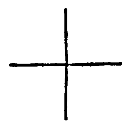
What are the square roots of the following numbers?

1. Square root of 49?
2. Square root of 144?
3. Square root of 225?
4. Square root of 2304?
5. Square root of $\frac{36}{81}$?
6. Square root of $\frac{225}{2304}$?
7. Square root of .0196?
8. Square root of 6.25?
9. Square root of 278.89?
10. Square root of 6275025?
11. Square root of 7994?
12. Square root of .205209?
13. Square root of $\frac{7}{8}$?
14. Square root of $\frac{15}{16}$?
15. Square root of $\frac{1}{40}$?
16. Square root of .60794?
17. Value of $\sqrt{.022201}$?
18. Value of $\sqrt{25.1001}$?
19. Value of $\sqrt{196.425}$?
20. Value of $\sqrt{1.5}$?
21. Value of $\sqrt{\frac{2809}{6241}}$?
22. Value of $\frac{\sqrt{9}}{\sqrt{49}}$?
23. Value of $\sqrt{\frac{2}{25}}$?
24. Value of $\sqrt{135}$?
25. Value of $\sqrt{19000}$?
26. Value of $\sqrt{.784}$?
27. Square root of 5647.5225?
28. Square root of 160048.0036?

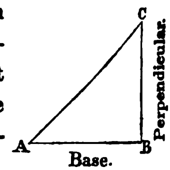
APPLICATIONS IN SQUARE ROOT.

307. A triangle is a plain figure which has three sides and three angles.

If a straight line meets another straight line, making the adjacent angles equal, each is called a right angle; and the lines are said to be perpendicular to each other.

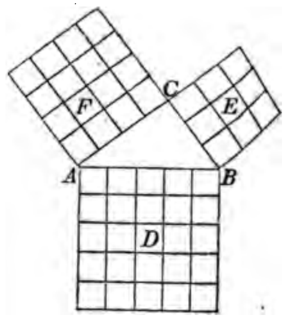


307. What is a triangle? What is a right angle?



308. A right angled triangle is one which has one right angle. In the right angled triangle ABC, the side AC opposite the right angle B, is called the *hypothenuse ;* the side AB the *base ;* and the side BC the *perpendicular.*

309. In a right angled triangle the square described on the hypothenuse is equal to the sum of the squares described on the other two sides.



Thus, if ACB be a right angled triangle, right angled at C, then will the large square, D, described in the hypothenuse AB, be equal to the sum of the squares F and E described on the sides AC and CB. This is called the carpenter's theorem. By counting the small squares in the large square D, you will find their number equal to that contained in the small squares F and E. In this triangle the hypothenuse AB = 5, AC = 4, and CB = 3. Any numbers having the same ratio, as 5, 4 and 3, such as 10, 8 and 6 ; 20, 16 and 12, &c., will represent the sides of a right angled triangle.

310. *When the base and perpendicular are known, to find the hypothenuse.*

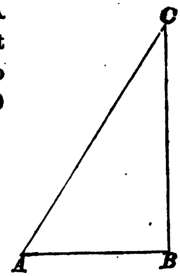
---

308. What is a right angled triangle ? Which side is the hypothenuse?

309. In a right angled triangle, what is the square on the hypothenuse equal to ?

310. How do you find the hypothenuse when you know the base and perpendicular ?

1. Wishing to know the distance from A to the top of a tower, I measured the height of the tower and found it to be 40 feet; also the distance from A to B, and found it 30 feet: what was the distance from A to C?



$$AB = 30;\ AB^2 = 30^2 = 900$$
$$BC = 40;\ BC^2 = 40^2 = 1600$$
$$AC^2 = AB^2 + BC^2 = 900 + 1600$$
$$AC = \sqrt{2500} = 50 \text{ feet.}$$

RULE.—*Square the base and square the perpendicular, add the results, and then extract the square root of their sum.*

311. *To find one side when we know the hypothenuse and the other side.*

1. The length of a ladder which will reach from the middle of a street 80 feet wide to the eaves of a house, is 50 feet: what is the height of the house?

ANALYSIS.—Since the square of the length of the ladder is equal to the sum of the squares of half the street and the height of the house, the square of the length of the ladder diminished by the square of half the street will be equal to the square of the height of the house: hence,

RULE.—*Square the hypothenuse and the known side, and take the difference; the square root of the difference will be the other side.*

EXAMPLES.

1. A general having an army of 117649 men, wished to form them into a square: how many should he place on each front?

2. In a square piece of pavement there are 48841 stones, of equal size, one foot square: what is the length of one side of the pavement?

3. In the centre of a square garden, there is an artificial circular pond covering an area of 810 square feet, which is $\frac{1}{10}$

---

311. When you know the hypothenuse and one side, how do you find the other side?

of the whole garden: how many rods of fence will enclose the garden?

4. Let it be required to lay out 67*A.* 2*R.* of land in the form of a rectangle, the longer side of which is to be three times as great as the less: what is its length and width?

5. A farmer wishes to set out an orchard of 3200 dwarf pear trees. He has a field which is twice as long as it is wide which he appropriates to this purpose, setting the trees 12 feet apart each way: how many trees will there be in a row, each way, and how much land will they occupy?

6. There is a wall 45 feet high built upon the bank of a stream 60 feet wide: how long must a ladder be that will reach from the outside of the stream to the top of the wall?

7. A boy having lodged his kite in the top of a tree, finds that by letting out the whole length of his line, which he knows to be 225 feet, it will reach the ground 180 feet from the foot of the tree: what is the height of the tree?

8. There are two buildings standing on opposite sides of the street, one 39 feet, and the other 49 feet from the ground to the eaves. The foot of a ladder 65 feet long rests upon the ground at a point between them, from which it will touch the eaves of either building: what is the width of the street?

9. A tree 120 feet high was broken off in a storm, the top striking 40 feet from the roots, and the broken end resting upon the stump: allowing the ground to be a horizontal plane, what was the height of the part standing?

10. What will be the distance from corner to corner, through the centre of a cube, whose dimensions are 5 feet on a side?

11. Two vessels start from the same point, one sails due north at the rate of 10 miles an hour, the other due west at the rate of 14 miles an hour: how far apart will they be at the end of 2 days, supposing the surface of the earth to be a plane?

12. How much more will it cost to fence 10 acres of land, in the form of a rectangle, the length of which is four times its breadth, than if it were in the form of a square, the cost of the fence being $2,50 a rod?

13. What is the diameter of a cylindrical reservoir containing 9 times as much water as one 25 feet in diameter, the heights being the same?*

14. If a cylindrical cistern 8 feet in diameter will hold 120 barrels, what must be the diameter of a cistern of the same depth to hold 1500 barrels?

15. If a pipe 3 inches in diameter will discharge 400 gallons in 3 minutes, what must be the diameter of a pipe that will discharge 1600 gallons in the same time?

16. What length of rope must be attached to a halter 4 feet long that a horse may feed over $2\frac{1}{2}$ acres of ground?

17. Three men bought a grindstone, which was four feet in diameter: how much must each grind off to use up his share of the stone?

CUBE ROOT.

312. The CUBE ROOT of a number is one of three equal factors of the number.

To extract the cube root of a number is to find a factor which multiplied into itself twice, will produce the given number.

Thus, 2 is the cube root of 8; for, $2 \times 2 \times 2 = 8$: and 3 is the cube root of 27; for, $3 \times 3 \times 3 = 27$.

1,	2,	3,	4,	5,	6,	7,	8,	9,
1	8	27	64	125	216	343	512	729

The numbers in the first line are the cube roots of the corresponding numbers of the second. The numbers of the second line are called *perfect cubes*. A number is a *perfect cube* when

312. What is the cube root of a number? When is a number a perfect cube? How many perfect cubes are there between 1 and 1000?

* NOTE.—If two volumes have the same altitude, their contents will be to each other in the same proportion as their bases; and if the bases are similar figures (that is, of like form,) they will be to each other as the *squares of their diameters*, or other like dimensions.

it has three exact factors. By examining the numbers of the two lines we see,

1st. That the cube of units cannot give a higher order than hundreds.

2d. That since the cube of one ten (10) is 1000 and the cube of 9 tens (90), 729000, *the cube of tens will not give a lower denomination than thousands, nor a higher denomination than hundreds of thousands.*

Hence, if a number contains more than three figures, its cube root will contain more than one; if it contains more than six, its root will contain more than two, and so on; every additional three figures giving one additional figure in the root, and the figures which remain at the left hand, although less than three, will also give a figure in the root. This law explains the reason for pointing off into periods of three figures each.

313. Let us now see how the cube of any number, as 16, is formed. Sixteen is composed of 1 ten and 6 units, and may be written, $10 + 6$. To find the cube of $16 = 10 + 6$, we must multiply the number by itself twice.

To do this we place the number thus,		$16 =$	$10 +$	6
			$10 +$	6
product by the units, - - -			$60 +$	36
product by the tens, - - -		$100 +$	60	
Square of 16 - - - -		$100 +$	$120 +$	36
Multiply again by 16, - - -			$10 +$	6
product by the units, - - -		$600 +$	$720 +$	216
product by the tens, - - -	$1000 +$	$1200 +$	360	
Cube of 16 - - - -	$1000 +$	$1800 +$	$1080 +$	216

1. By examining the parts of this number, it is seen that the first part 1000 is the *cube of the tens;* that is,

$$10 \times 10 \times 10 = 1000.$$

---

313. Of how many parts is the cube of a number composed? What are they?

2. The second part 1800 is *three times the square of the tens multiplied by the units;* that is,

$$3 \times (10)^2 \times 6 = 3 \times 100 \times 6 = 1800.$$

3. The third part 1080 is *three times the square of the units multiplied by the tens;* that is,

$$3 \times 6^2 \times 10 = 3 \times 36 \times 10 = 1080.$$

4. The fourth part is the *cube of the units;* that is,

$$6^2 = 6 \times 6 \times 6 = 216.$$

1. What is the cube root of the number 4096?

ANALYSIS.—Since the number contains more than three figures, we know that the root will contain at least units and tens.

OPERATION.

$$\dot{4}\ 09\dot{6}(16$$
$$1$$
$$1^2 \times 3 = 3)3\ 0\quad(9\text{-}8\text{-}7\text{-}6$$
$$16^3 = 4\ 096.$$

Separating the three right-hand figures from the 4, we know that the cube of the tens will be found in the 4; and 1 is the greatest cube in 4.

Hence, we place the root 1 on the right, and this is the tens of the required root. We then cube 1 and subtract the result from 4, and to the remainder we bring down the first figure 0 of the next period.

We have seen that the second part of the cube of 16, viz., 1800, is *three times the square of the tens multiplied by the units;* and hence, it can have no significant figure of a less denomination than hundreds. It must, therefore, make up a part of the 30 hundreds above. But this 30 hundreds also contains all the hundreds which come from the 3d and 4th parts of the cube of 16. If it were not so, the 30 hundreds, divided by three times the square of the tens, would give the unit figure exactly.

Forming a divisor of three times the square of the tens, we find the quotient to be ten; but this we know to be too large. Placing 9 in the root and cubing 19, we find the result to be 6859. Ther trying 8 we find the cube of 18 still too large; but when we take 6 we find the exact number. Hence, the cube root of 4096 is 16.

314. Hence, to find the cube root of a number:

RULE.—I. *Separate the given number into periods of three figures each, by placing a dot over the place of units, a second*

over the place of thousands, and so on over each third figure to the left: the left hand period will often contain less than three places of figures.

II. *Note the greatest perfect cube in the first period, and set its root on the right, after the manner of a quotient in division. Subtract the cube of this number from the first period, and to the remainder bring down the first figure of the next period for a dividend.*

III. *Take three times the square of the root just found for a trial divisor, and see how often it is contained in the dividend, and place the quotient for a second figure of the root. Then cube the figures of the root thus found, and if their cube be greater than the first two periods of the given number, diminish the last figure, but if it be less, subtract it from the first two periods, and to the remainder bring down the first figure of the next period for a new dividend.*

IV.—*Take three times the square of the whole root for a second trial divisor, and find a third figure of the root. Cube the whole root thus found and subtract the result from the first three periods of the given number when it is less than that number, but if it is greater, diminish the figure of the root: proceed in a similar way for all the periods.*

EXAMPLES.

1. What is the cube root of 20796875?

OPERATION.

$$\begin{array}{lr}
 & \dot{2}0\ 79\dot{6}\ 87\dot{5}(275 \\
2^3 = & 8 \\
2^2 \times 3 = & 12)127 \\
\text{First two periods, - - -} & \dot{2}0\ 79\dot{6} \\
(27)^3 = 27 \times 27 \times 27 = & 19\ 683 \\
3 \times (27)^2 = & 2217)11\ 13\dot{8} \\
\text{First three periods, - - -} & \dot{2}0\ 79\dot{6}\ 87\dot{5} \\
(275)^3 = 275 \times 275 \times 275 = & 20\ 796\ 875
\end{array}$$

---

314. What is the rule for extracting the cube root of a number?

Find the cube roots of the following numbers:

1. Cube root of 1728?
2. Cube root of 117649?
3. Cube root of 46656?
4. Cube root of 15069223?
5. Cube root of 5735339?
6. Cube root of 48228544?
7. Cube root of 84604519?
8. Cube root of 28991029248?

315. To extract the cube root of a decimal fraction:

Annex ciphers to the decimal, if necessary, so that it shall consist of 3, 6, 9, &c., decimal places. Then put the first point over the place of thousandths, the second over the place of millionths, and so on over every third place to the right; after which extract the root as in whole numbers.

NOTES.—1. There will be as many decimal places in the root as there are periods in the given number.

2. The same rule applies when the given number is composed of a whole number and a decimal.

3. If in extracting the root of a number there is a remainder after all the periods have been brought down, periods of ciphers may be annexed by considering them as decimals.

EXAMPLES.

Find the cube roots of the following numbers:

1. Cube root of 8.343?
2. Cube root of 1728.729?
3. Cube root of .0125?
4. Cube root of 19683.46656?
5. Cube root of .387420489?
6. Cube root of .000003375?
7. Cube root of .0066592?
8. Value of $\sqrt[3]{81.729}$?

316. To extract the cube root of a common fraction,

I. *Reduce compound fractions to simple ones, mixed numbers to improper fractions, and then reduce the fraction to its lowest terms.*

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314. What is the rule for extracting the cube root of a number?

315. How do you extract the cube root of a decimal fraction? How many decimal places will there be in the root? Will the same rule apply when there is a whole number and a decimal? If in extracting the root of any number you find a decimal, how do you proceed?

II. *Extract the cube root of the numerator and denominator separately, if they have exact roots; but if either of them has not an exact root, reduce the fraction to a decimal, and extract the root as in the last case.*

EXAMPLES.

Find the cube roots of the following fractions:

1. Cube root of $\frac{64}{125}$?
2. Cube root of $\frac{343}{729}$?
3. Cube root of $31\frac{15}{343}$?
4. Cube root of $91\frac{1}{8}$?
5. Cube root of $\frac{343}{512}$?
6. Cube root of $\frac{729}{15625}$?
7. Cube root of $\frac{19683}{262144}$?
8. Cube root of $\frac{13824}{42875}$?
9. Cube root of $7\frac{6}{7}$?
10. Cube root of $56\frac{3}{5}$?

APPLICATIONS.

1. What must be the dimensions of a cubical bin, that its volume or capacity may be 19683 feet?

2. If a cubical body contains 6859 cubic feet, what is the length of one side: what the area of its surface?

3. The volume of a globe is 46656 cubic inches: what would be the side of a cube of equal solidity?

4. A person wished to make a cubical cistern, which should hold 150 barrels of water; what must be its depth?

5. A farmer constructed a bin that would contain 1500 bushels of grain; its length and breadth were equal, and each half the height; what were its dimensions?

6. What is the difference between half a cubic yard, and a cube whose edge is half a yard?

7. A merchant paid $911,25 for some pieces of muslin. He paid as many cents a yard as there were yards in each piece, and there were as many pieces as there were yards in one piece: how many yards were there, and how much did he pay a yard?

NOTES.—1. Bodies are said to be similar when they have the same form and have their like parts proportional.

2. It is proved in Geometry, that the volumes or weights of similar bodies are to each other as the cubes of their like dimensions.

3. Those bodies which are named in the same example are supposed to be similar.

8. If a sphere 3 feet in diameter contains 14.1372 cubic feet, what are the contents of a sphere 6 feet in diameter?

$3^3 : 6^3 :: 14.1372 : 113.0976$. *Ans.*

9. If a ball $2\frac{1}{2}$ inches in diameter weighs 8 pounds, how much will one of the same kind weigh, that is 5 inches in diameter?

10. What must be the size of a cubical bin, that will contain 8 times as much as one that is 4 feet on a side?

11. How many globes, 6 inches in diameter, will it require to make one 12 inches in diameter?

12. If a ball of silver, 1 unit in diameter, be worth $8, what will be the value of one $5\frac{1}{2}$ units in diameter?

13. If a plate of silver, 6 inches long, 3 inches wide, and $\frac{1}{4}$ inch thick, be worth $100, what will be the dimensions of a similar plate of the same metal worth $800?

14. If one man can dig a cellar 12 feet long, 10 feet wide, and $4\frac{1}{2}$ feet deep, in 3 days, what will be the dimensions of a similar cellar that requires him 24 days to dig it, working at the same rate, and the ground being of the same degree of hardness?

15. If I put 2 tons of hay in a stack 10 feet high, how high must a similar stack be to contain 16 tons?

16. Four women bought a ball of yarn 6 inches in diameter, and agreed that each should take her share separately from the surface of the ball: how much of the diameter must each wind off?

ARITHMETICAL PROGRESSION.

317. If we take any number, as 2, we can, by the continued addition of any other number, as 3, form a *series* of numbers: thus,

2, 5, 8, 11, 14, 17, 20, 23, &c.,

in which each number is formed by the addition of 3 to the preceding number.

---

317. What is an arithmetical progression? What is the number added or subtracted called?

This series of numbers may also be formed by subtracting 3 continually from a larger number: thus,

23, 20, 17, 14, 11, 8, 5, 2.

An ARITHMETICAL PROGRESSION is a series of numbers in which each is derived from the preceding by the addition or subtraction of the same number.

The number which is added or subtracted is called the *common difference.*

318. When the series is formed by the continued addition of the common difference, it is called an *increasing* series; and when it is formed by the subtraction of the common difference, it is called a *decreasing* series: thus,

2, 5, 8, 11, 14, 17, 20, 23, is an increasing series.
23, 20, 17, 14, 11, 8, 5, 2, is a decreasing series.

The several numbers are called *terms* of the progression: the first and last terms are called the *extremes*, and the intermediate terms are called the *means*.

319. In every arithmetical progression there are five parts, any three of which being given or known, the remaining two can be determined. They are,

1st: The first term;
2d: The last term;
3d: The common difference;
4th: The number of terms;
5th: The sum of all the terms.

---

318. When the common difference is added, what is the series called? What is it called when the common difference is subtracted? What are the several numbers called? What are the first and last called? What are the intermediate ones called?

319. How many parts are there in every arithmetical progression? What are they? How many parts must be given before the remaining ones can be found?

CASE I.

320. *Having given the first term, the common difference, and the number of terms, to find the last term.*

1. The first term of an increasing progression is 4, the common difference 3, and the number of terms 10: what is the last term?

ANALYSIS.—By considering the manner in which the increasing progression is formed, we see that the 2d term is obtained by adding the common difference to the 1st term; the 3d, by adding the common difference to the 2d; the 4th, by adding the common difference to the 3d, and so on; *the number of additions, in every case, being* 1 *less than the number of terms found.* Instead of making the additions, we may multiply the common difference by the number of additions, that is, by 1 less than the number of terms, and *add the first term* to the product.

OPERATION.

```
 9 no. less 1
 3 com. diff.
――
27
 4 1st term.
――
31 last term.
```

RULE.—*Multiply the common difference by* 1 *less than the number of terms: if the progression is increasing, add the product to the first term, and the sum will be the last term; if it is decreasing, subtract the product from the first term and the difference will be the last term.*

EXAMPLES.

1. What is the 18th term of an arithmetical progression, of which the first term is 4, and the common difference 5?

2. A man is to receive a certain sum of money in 12 payments: the first payment is $300, and each succeeding payment is less than the previous one by $20: what will be the last payment?

3. What will $200 amount to in 15 years, at 7 per cent simple interest: the first year it increases $14, the second, $28, and so on?

---

320. When you know the first term, the common difference and the number of terms, how do you find the last term?

4. A man has a piece of land 35$\frac{3}{4}$ rods in length, which tapers to a point, and is found to increase $\frac{1}{4}$ rod in width, for every rod in length: what is the width of the wide end?

5. James and John have 100 marbles. It is agreed between them that John shall have them all, if he will place them in a straight line half a foot apart, and so that he shall be obliged to travel 300 feet to get and bring back the farthest marble; and also, if he will tell, without measuring, how far he must travel to bring back the nearest.

CASE II.

321. *Knowing the two extremes of an arithmetical progression and the number of terms, to find the common difference.*

1. The two extremes of a progression are 4 and 68, and the number of terms 17: what is the common difference?

ANALYSIS.—Since the common difference multiplied by 1 less than the number of terms gives a product equal to the difference of the extremes, if we *divide* the difference of the extremes by 1 less than the number of terms, the quotient will be the *common difference:* hence,

OPERATION.

$$\begin{array}{r} 68 \\ \underline{4} \\ 17-1=16)\overline{64}(4 \end{array}$$

RULE.—*Subtract the less extreme from the greater, and divide the remainder by* 1 *less than the number of terms: the quotient will be the common difference.*

EXAMPLES.

1. A man started from Chicago and travelled 15 days; each day's journey was increased by the distance which he travelled the first day: what was his daily increase?

2. A merchant sold 14 yards of cloth, in pieces of 1 yard each; for the first yard he received $\frac{1}{2}$, and for the last \$26$\frac{1}{2}$: what was the difference in the price per yard?

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321. When you know the extremes and number of terms, how do you find the common difference?

3. A board is 17 feet long: it is $2\frac{1}{2}$ inches wide at one end, and $14\frac{1}{2}$ at the other: what is the average increase in width per foot in length?

CASE III.

322. *To find the sum of the terms of an arithmetical progression.*

1. What is the sum of the series whose first term is 2, common difference 3, and number of terms 15?

Given series,	2	5	8	11	14	17	20	23
Same, order inverted.	23	20	17	14	11	8	5	2
Sum of both series.	25	25	25	25	25	15	25	25

ANALYSIS.—The two series are the same; hence, their sum is equal to twice the given series. But their sum is equal to the sum of the two extremes, 2 and 23, taken as many times as there are terms; and the given series is equal to half this sum, or to the sum of the extremes multiplied by half the number of terms.

RULE.—*Add the extremes together and multiply their sum by half the number of terms; the product will be the sum of all the terms.*

EXAMPLES.

1. What debt could be discharged in a year, by weekly payments in arithmetical progression, the first payment being $5, and the last $100?

2. A person agreed to build 56 rods of fence; for the first rod he was to receive 6 cents, for the second, 10 cents, and so on: what did he receive for the last rod, and how much for the whole?

3. If a person travel 30 miles the first day, and a quarter of a mile less each succeeding day, how far will he travel in 30 days?

4. If 120 stones be laid in a straight line, each at a distance of a yard and a quarter from the one next to it, how far must a person travel who picks them up singly and places them in a

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322. How do you find the sum of the series?

heap, at the distance of 6 yards from the end of the line and in its continuation?

CASE IV.

323. *Having given the first and last terms, and the common difference, to find the number of terms.*

1. The first term of an arithmetical progression is 5, the common difference 4, and the last term 41: what is the number of terms?

ANALYSIS.—Since the last term is equal to the first term added to the product of the common difference, by 1 less than the number of terms (Art. 320), it follows that, if the first term be taken from the last term, the difference will be equal to the product of the common difference by 1 less than the number of terms: if this be divided by the common difference, the quotient will be 1 less than the number of terms.

OPERATION.

$41 - 5 = 36$

$4)36(= 9$

$9 + 1 = 10$ No. terms.

RULE.—*Divide the difference of the two extremes by the common difference, and add* 1 *to the quotient: the sum will be the number of terms.*

EXAMPLES.

1. A farmer sold a number of bushels of wheat; it was agreed that for the first bushel he should receive 50 cents, and an increase of 9 cents for each succeeding bushel, and for the last he received $500: how many bushels did he sell?

2. A person proposes to make a journey, and to travel 15 miles the first day, and 33 miles the last, with a daily increase of $1\frac{1}{2}$ miles: in how many days did he make the journey, and what was the whole distance travelled?

3. I owe a debt of $2325, and wish to pay it in equal installments, the first payment to be $575, the second $500, and decreasing by a common difference, until the last payment which is $200: what will be the number of installments?

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323. Having given the first and last terms and the common difference, how do you find the number of terms?

GEOMETRICAL PROGRESSION.

324. A GEOMETRICAL PROGRESSION is a series of terms, each of which is derived from the preceding one, by multiplying it by a constant number. The constant multiplier is called the *ratio* of the progression.

325. If the ratio is *greater* than 1, each term is greater than the preceding one, and the series is said to be *increasing.*

If the ratio is *less* than 1, each term is less than the preceding one, and the series is said to be *decreasing;* thus,

1, 2, 4, 8, 16, 32, &c.—ratio 2—increasing series:
32, 16, 8, 4, 2 1, &c.—ratio $\frac{1}{2}$—decreasing series.

The several numbers resulting from the multiplication are called *terms* of the progression. The first and last are called the *extremes*, and the intermediate terms are called *means.*

326. In every Geometrical, as well as in every Arithmetical Progression, there are five parts:

1st: The first term;
2d: The last term;
3d: The common ratio;
4th: The number of terms;
5th: The sum of all the terms.

If any three of these parts are known, or given, the remain ing ones can be determined.

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324. What is a geometrical progression? What is the constant multiplier called?

325. If the ratio is greater than 1, how do the terms compare with each other? What is the series then called? If the ratio is less than 1, how do they compare? What is the series then called? What are the several numbers called? What are the first and last terms called? What are the intermediate ones called?

326. How many parts are there in every geometrical progression? What are they? How many must be known before the others can be found?

CASE I.

327. *Having given the first term, the ratio, and the number of terms, to find the last term.*

1. The first term is 4, and the common ratio 3: what is the 5th term?

ANALYSIS.—The second term is formed by multiplying the first term by the ratio; the third term by multiplying the second term by the ratio, and so on; the number of multiplicators *being* 1 *less than the number of terms:* thus,

OPERATION.

$$3 \times 3 \times 3 \times 3 = 81$$
$$\underline{\quad 4}$$
Ans. 324

$$4 = 4, \text{ 1st term,}$$
$$3 \times 4 = 12, \text{ 2d term,}$$
$$3 \times 3 \times 4 = 36, \text{ 3d term,}$$
$$3 \times 3 \times 3 \times 4 = 108, \text{ 4th term,}$$
$$3 \times 3 \times 3 \times 3 \times 4 = 324, \text{ 5th term.}$$

Therefore, *the last term is equal to the first term multiplied by the ratio raised to a power* 1 *less than the number of terms.*

RULE.—*Raise the ratio to a power whose exponent is* 1 *less than the number of terms, and then multiply this power by the first term.*

EXAMPLES.

1. The first term of a decreasing progression is 2187; the ratio is $\frac{1}{3}$, and the number of terms 8: what is the last term?

NOTE.—The 7th power of $\frac{1}{3}$ is $\frac{1}{2187}$; this multiplied by the first term, 2187, gives 1, the last term.

OPERATION.

$$(\tfrac{1}{3})^7 = \tfrac{1}{2187}$$
$$(\tfrac{1}{2187}) \times 2187 = 1.$$

2. The first term of an increasing geometrical series is 8, the ratio 5: what is the 9th term?

3. The first term of a decreasing geometrical series is 729, the ratio $\frac{1}{3}$: what is the 10th term?

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327. Knowing the first term, the ratio, and the number of terms, how do you find the last term?

4. If a farmer should sell 15 bushels of wheat, at 1 mill for the first bushel, 1 cent for the second, 1 dime for the third, and so on; what would he receive for the last bushel?

5. A man dying left 5 sons, and bequeathed his estate in the following manner; to his executors \$100; his youngest son was to have twice as much as the executors, and each son to have double the amount of the next younger brother: what was the eldest son's portion?

6. A merchant engaging in business, trebled his capital once in 4 years; if he commenced with \$2000, what will his capital amount to at the end of the 12th year?

7. A farmer wishing to buy 16 oxen of a drover, finally agreed to give him for the whole the cost of the last ox only. He was to pay 1 cent for the first, 2 cents for the second, and doubling on each one to the last: how much would they cost him?

CASE II.

328. *Knowing the two extremes and the ratio, to find the sum of the terms.*

1. What is the sum of the terms of the progression 2, 6, 18, 54, 162?

OPERATION.

$$\begin{array}{rl} 6 + 18 + 54 + 162 + 486 & = 3 \text{ times.} \\ \underline{2 + 6 + 18 + 54 + 162 \qquad\quad} & \underline{= 1 \text{ time.}} \\ 486 - 2 & = 2 \text{ times.} \end{array}$$

$$\frac{486 - 2}{2} = \frac{484}{2} = 242 \text{ sum.}$$

ANALYSIS.—If we multiply the terms of the progression by the ratio 3, we have a second progression, 6, 18, 54, 162, 486, which is 3 *times as great* as the first. If from this we subtract the first, the remainder, 486 − 2, will be 2 *times* as great as the first; and if this remainder be divided by 2, the quotient will be the sum of the terms of the first progression.

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328. Knowing the two extremes and the ratio, how do you find the sum of the terms?

But 486 is the product of the last term of the given progression multiplied by the ratio, 2 is the first term, and the divisor 2, 1 less than the ratio : hence,

RULE.—*Multiply the last term by the ratio; take the difference between this product and the first term and divide the remainder by the difference between* 1 *and the ratio.*

NOTE.—When the progression is *increasing*, the first term is subtracted from the product of the last term by the ratio, and the divisor is found by subtracting 1 from the ratio. When the progression is *decreasing*, the product of the last term by the ratio is subtracted from the first term, and the ratio is subtracted from 1.

EXAMPLES.

1. The first term of a progression is 4, the ratio 3, and the last term 78722: what is the sum of the terms?

2. The first term of a progression is 1024, the ratio ¼, and the last term 4: what is the sum of the series?

3. What debt can be discharged in one year by monthly payments, the first being \$2, the second \$8, and so on to the end of the year, and what will be the last payment?

4. A gentleman being importuned to sell a fine horse, said that he would sell him on the condition of receiving 1 cent for the first nail in his shoes, 2 cents for the second, and so on, doubling the price of every nail: the number of nails in each shoe being 8, how much would he receive for his horse?

5. A laborer agreed to thresh 64 days for a farmer on the condition that he should give him 1 grain of wheat for the first day's labor, 2 grains for the second, and double each succeeding day: what number of bushels would he receive, supposing a pint to contain 7680 grains, and what number of ships, each carrying 1000 tons burden, might be loaded, allowing 40 bushels to a ton?

ANALYSIS.

1. If 12 yards of cloth cost $48,36, what will 7 yards cost?

ANALYSIS.—One yard of cloth will cost $\frac{1}{12}$ as much as 12 yards: since 12 yards cost $48,36, one yard will cost $\frac{1}{12}$ of $48,36 = $4,03: 7 yards will cost 7 times as much as 1 yard, or 7 times $\frac{1}{12}$ of $48,36 = $28,21; therefore, if 12 yards of cloth cost $48,36, 7 yards will cost $28,21.

OPERATION.

$$\frac{\overset{4,03}{\cancel{\$48,36}}}{1} \times \frac{1}{\cancel{12}} \times \frac{7}{1} = \$28,21; \text{ or } \quad \begin{array}{r|l} & 4,03 \\ & \cancel{48,36} \\ \cancel{12} & \\ & 7 \\ \hline & \$28,21 \ \textit{Ans.} \end{array}$$

2. If 27 pounds of butter will buy 45 pounds of sugar, how much butter will buy 36 pounds of sugar?

ANALYSIS.—One pound of sugar will buy $\frac{1}{45}$ as much butter as 45 pounds, or $\frac{1}{45}$ of 27*lbs.* of butter: 36 pounds of sugar will buy 36 times as much butter as 1 pound of sugar, or 36 times $\frac{1}{45}$ of 27*lbs.*, which is $\frac{108}{5} = 21\frac{3}{5}$*lbs.*

OPERATION.

$$\frac{\overset{3}{\cancel{27}}}{1} \times \frac{1}{\underset{5}{\cancel{45}}} \times \frac{36}{1} = \frac{108}{5} = 21\tfrac{3}{5}lbs.; \text{ or } \quad \begin{array}{r|l} {}^{5}\cancel{45} & 27 \\ & \cancel{36}\ {}^{4} \\ \hline 5 & 108 \\ \hline & 21\tfrac{3}{5}lbs.\ bu. \end{array}$$

3. What will $6\frac{3}{4}$ cords of wood cost, if $2\frac{3}{8}$ cords cost $\$7\frac{1}{8}$?

ANALYSIS.—Since $2\frac{3}{8}$ cords $= \frac{19}{8}$ cords of wood costs $\$7\frac{1}{8} = \$\frac{57}{8}$ one cord will cost as many dollars as $\frac{19}{8}$ is contained times in $\frac{57}{8}$, or $3: $6\frac{3}{4}$ cords $= \frac{27}{4}$ cords, will cost $\frac{27}{4}$ times as much as 1 cord, that is, $\$3 \times \frac{27}{4} = \$\frac{81}{4} = \$20,25$.

OPERATION.

$$\frac{\overset{3}{\cancel{\$57}}}{\cancel{8}} \times \frac{\cancel{8}}{\cancel{19}} \times \frac{27}{4} = \frac{81}{4} = \$20\tfrac{1}{4}; \text{ or, } \quad \begin{array}{r|l} \cancel{8} & \cancel{57}\ {}^{3} \\ \cancel{19} & \cancel{8} \\ 4 & 27 \\ \hline 4 & 81,00 \\ \hline & \$20,25 \ \textit{Ans.} \end{array}$$

NOTE.—The fractional part of a dollar may always be reduced to cents by annexing two ciphers, and to mills by annexing three.

4. A farmer sold a number of cows, and had 12 left, which was $\frac{1}{3}$ of the number sold; if the number sold be divided by $\frac{3}{4}$ of $9\frac{1}{3}$, the quotient will be $\frac{1}{5}$ the number of dollars he received per head: how much did he receive per head for his cows?

ANALYSIS.—12 is $\frac{1}{3}$ of 3 times $12 = 36$, the number of cows sold; 36 divided by $\frac{3}{4}$ of $9\frac{1}{3} = 7$, the quotient, $\frac{36}{7}$, is $\frac{1}{5}$ of 5 times $\frac{36}{7} = \frac{180}{7}$ $= \$25\frac{5}{7}$.

OPERATION.

$$\frac{12}{1} \times \frac{3}{1} \times \frac{4 \times 3}{3 \times 28} \times \frac{5}{1} = \frac{180}{7} = \$25\frac{5}{7}; \text{ or } \begin{array}{r|l} & 12 \\ & 3 \\ \cancel{3} & \cancel{4} \\ 7\ \cancel{28} & \cancel{3} \\ & 5 \\ \hline 7 & 180,00 \\ \hline & \$25,71\frac{3}{7} \end{array} \textit{ Ans.}$$

5. What will 20 bushels of barley cost, in dollars and cents, at 7 shillings a bushel, New York currency?

NOTES.—1. Although United States money is expressed in dollars, cents, and mills, still in most of the States the dollar (always valued at 100 cents) is sometimes reckoned in pounds shillings and pence; thus.

2. In the New England States, in Indiana, Illinois, Missouri, Virginia, Kentucky, Tennesee, Mississippi, and Texas, the dollar is reckoned at 6 shillings; in New York, Ohio, and Michigan, at 8 shillings; in New Jersey, Pennsylvania, Delaware, and Maryland, at 7*s.* 6*d.*; in South Carolina and Georgia, at 4*s.* 8*d.*; in Canada and Nova Scotia, at 5 shillings.

3. It often occurs that the retail price is given in shillings and pence, and the result or cost is required in dollars and cents.

ANALYSIS.—Since 1 bushel of barley costs 7 shillings, 20 bushels will cost 20 times 7 shillings, or 140 shillings; and as 8 shillings make 1 dollar, New York currency, there will be as many dollars as 8 is contained times $140 = \$17,50$.

OPERATION.

$$20 \times 7 \div 8 = \$17\tfrac{1}{2}; \quad \text{or} \quad \begin{array}{r|l} & 7 \\ & \cancel{20}\ ^{5} \\ ^{2}\ \cancel{8} & \\ \hline 2 & 35,00 \\ \hline & \$17,50\ \textit{Ans.} \end{array}$$

6. **What will be the cost of 72 bushels of potatoes, at 3*s.* 3*d.* per bushel, New York currency?**

OPERATION.

$$\begin{array}{r|l} & \cancel{72}\ ^{9} \\ 4 & 13 \\ \cancel{8} & \\ \hline 4 & 117,00 \\ \hline & \$29,75\ \textit{Ans.} \end{array} \qquad \text{Or,} \qquad \begin{array}{r|l} & \quad ^{3} \\ & \cancel{72} \\ ^{4}\ \cancel{96} & 39 \\ \hline 4 & 117,00 \\ \hline & \$29,75\ \textit{Ans.} \end{array}$$

NOTE.—When the pence is an aliquot part of a shilling, the price may be reduced to an improper fraction, which will be the multiplier in the denomination of shillings; thus, 3*s.* 3*d.* = $3\frac{1}{4}$*s.* = $\frac{13}{4}$*s.*: or, the shillings and pence may be reduced to pence; thus, 3*s.* 3*d.* = 39*d.*, in which case the product will be pence, and must be divided by 96, the number of pence in $1.

7. **What will $12\frac{1}{2}$ pounds of tea cost at 6*s.* 8*d.* a pound, Pennsylvania currency?**

OPERATION.

$$\begin{array}{r|l} \cancel{2} & \cancel{25}\ ^{5} \\ 3 & 20 \\ ^{3}\ \cancel{15} & \cancel{2} \\ \hline 9 & 100 \\ \hline & \$11\tfrac{1}{9}\ \textit{Ans.} \end{array} \qquad \text{Or,} \qquad \begin{array}{r|l} & 25 \\ \cancel{2} & \\ ^{9}\ \cancel{90} & \cancel{80}\ ^{4} \\ \hline 9 & 100,00 \\ \hline & \$11,11\tfrac{1}{9}\ \textit{Ans.} \end{array}$$

NOTE.—In the last example the multiplier is 6*s.* 8*d.* = $6\frac{2}{3}$*s.* = $\frac{20}{3}$*s.* or 80*d.* The divisor is 7*s.* 6*d.* = $7\frac{1}{2}$*s.* = $\frac{15}{2}$*s.*, or 90*d.* Hence, to find the cost of articles in dollars and cents, when the price is in shillings and pence,

Multiply the commodity by the price, and divide the product by the value of a dollar, expressed in the unit of the price.

8. How many days work at 10*s.* 6*d.* a day, must be given for 18 bushels of corn at 5*s.* 10*d.* a bushel?

OPERATION.

$$\begin{array}{r|l} & \cancel{18} \\ \cancel{6} & \cancel{35}\ 5 \\ \cancel{21} & 2 \\ \hline & 10 \text{ days } Ans. \end{array} \qquad \text{Or,} \qquad \begin{array}{r|l} & \cancel{18} \\ \cancel{126} & \cancel{70}\ 10 \\ \hline & 10 \text{ days } Ans. \end{array}$$

NOTE.—The same rule applies in this as in the preceding examples, except that the divisor is the price of the articles received in payment, reduced to the same unit as the price of the article bought.

9. What will 5*cwt.* of coffee cost, at 1*s.* 4*d.* per pound, New York currency?

OPERATION.

$$\begin{array}{r|l} & 5 \\ & \cancel{4}\ 2 \\ \cancel{2}\ \cancel{8} & 25 \\ 3 & \cancel{4} \\ \hline 3 & 250 \\ \hline & \$83{,}333\tfrac{1}{3}\ Ans. \end{array} \qquad \text{Or,} \qquad \begin{array}{r|l} & 5 \\ & \cancel{4}\ 2 \\ 3\ \cancel{96} & 25 \\ & \cancel{16} \\ \hline 3 & 250{,}000 \\ \hline & \$83{,}333\tfrac{1}{3}\ Ans. \end{array}$$

NOTE.—Reduce the *cwts.* to *lbs.* by multiplying by 4, and then by 25, after which proceed as in the preceding examples.

10. A merchant bought a number of bales of cloth, each containg $133\frac{1}{3}$ yards, at the rate of 12 yards for \$11, and sold it at the rate of 8 yards for \$7, by which he lost \$100 in the trade: how many bales were there?

ANALYSIS.—Since he paid \$11 for 12 yards, for 1 yard he paid $\frac{1}{12}$ of \$11, or $\frac{11}{12}$ of \$1; and since he received \$7 for 8 yards, for 1 yard he received $\frac{1}{8}$ of \$7, or $\frac{7}{8}$ of \$1. He lost on 1 yard the difference between $\frac{11}{12}$ and $\frac{7}{8} = \frac{1}{24}$ of a dollar. Since his whole loss was \$100, he had as many yards as $\frac{1}{24}$ is contained times in $100 = 2400$ yards; and there were as many bales as $133\frac{1}{3}$ (the number of yards in 1 bale) is contained times in the whole number of yards = 18 bales.

OPERATION.

$$\begin{array}{l} \frac{11}{12} - \frac{7}{8} = \$\frac{1}{24} \\ (100 \div \frac{1}{24}) \div 133\frac{1}{3} = 18\ Ans. \end{array} \qquad \text{Or,} \qquad \begin{array}{r|l} & \cancel{100}\ 6 \\ 1 & \cancel{24} \\ \cancel{400} & 3 \\ \hline & 18 \text{ bales } Ans. \end{array}$$

11. A can mow an acre of grass in $4\frac{1}{2}$ hours; B, in $6\frac{1}{4}$ hours; and C, in $7\frac{1}{3}$ hours: how many days, working $6\frac{3}{4}$ hours, would they require to mow $13\frac{4}{5}$ acres?

ANALYSIS.—Since A can mow an acre of grass in $4\frac{1}{2}$ hours, B, in $6\frac{1}{4}$ hours, and C in $7\frac{1}{3}$ hours, A can mow $\frac{2}{9}$, B, $\frac{4}{25}$, and C $\frac{3}{22}$ of an acre in 1 hour. Together they can mow $\frac{2}{9}+\frac{4}{25}+\frac{3}{22}=\frac{23}{45}$ of an acre in 1 hour; and to mow 1 acre, they will require as many hours as $\frac{23}{45}$ is contained times in $1=\frac{45}{23}$ hours: to mow $13\frac{4}{5}$ acres, they will require $13\frac{4}{5}$ times $\frac{45}{23}=27$ hours, and working $6\frac{3}{4}$ hours each day, will require 4 days.

OPERATION.

$$\frac{2}{9}+\frac{4}{25}+\frac{3}{22}=\frac{23}{45}\text{ hours.}$$

$$\frac{\cancel{45}^{9}}{\cancel{23}}\times\frac{\cancel{69}^{3}}{\cancel{5}}\times\frac{4}{\cancel{27}}=4\text{ days.}$$

Or,

$$\begin{array}{r|l} \cancel{23} & \cancel{45}\ 9 \\ \cancel{5} & \cancel{69}\ 3 \\ \cancel{27} & 4 \\ \hline & 4 \text{ days } \textit{Ans.} \end{array}$$

12. A person employed three men, A, B, and C, to do a piece of work for \$132,66. A can do the work alone in $23\frac{1}{3}$ days, working 12 hours a day; B can do it in 25 days, working 8 hours a day; and C can do it in 16 days, working $11\frac{1}{4}$ hours a day. In what time can the three do it, working together, 10 hours a day, and what share of the money should each receive?

ANALYSIS.—Since A can do the work in $23\frac{1}{3}$ days, working 12 hours each day; B, in 25 days, working 8 hours each day; and C, in 16 days, working $11\frac{1}{4}$ hours each day, A can do the same work in 280 days, B, in 200 days, and C, in 180 days, working 1 hour each day: then A, B, and C, can do $\frac{1}{280}+\frac{1}{200}+\frac{1}{180}=\frac{856}{25200}$ of the work in 1 day, working 1 hour; by working 10 hours, they will do 10 times as much; or, the work done by each in 1 day of 10 hours, will be denoted by $\frac{10}{280}$, $\frac{10}{200}$, and $\frac{10}{180}$; and the whole work done in 1 day by $\frac{856}{2520}$; hence, the number of days will be denoted by the number of times which 1 contains $\frac{856}{2520}=\frac{2520}{856}=7\frac{7}{89}$ days.

If the part which each does in 1 day be multiplied by the number of days, viz., $7\frac{7}{89}$, the product will be the part done by each; viz., A, $\frac{10}{280}\times7\frac{7}{89}=\frac{45}{178}$; B, $\frac{10}{200}\times7\frac{7}{89}=\frac{63}{178}$: and C, $\frac{10}{180}\times7\frac{7}{89}=\frac{70}{178}$; therefore, A must have $\frac{45}{178}$, B, $\frac{63}{178}$ and C, $\frac{70}{178}$ of \$132,66.

OPERATION.

First: $1 \div \frac{356}{2520} = \frac{2520}{356} = 7\frac{7}{89}$ days. *Ans.*

Second: $\$132,66 \times \frac{45}{178} = \$33,53\frac{68}{89} =$ A's share.
$\$132,66 \times \frac{63}{178} = \$46,95\frac{24}{89} =$ B's share.
$\$132,76 \times \frac{70}{178} = \$52,16\frac{88}{89} =$ C's share.
Total paid $\$132\ 66$

13. If 336 men, in 5 days, working 10 hours each day, can dig a trench of 5 degrees of hardness, 70 yards long, 3 yards wide, and 2 yards deep; how many days of 12 hours each, will 240 men require to dig a trench 36 yards long, 5 yards wide, and 3 yards deep, of 6 degrees of hardness?

ANALYSIS.—Since 336 men require 5 days of 10 hours each, to dig a trench, it will take 1 man 336 times 5 days of 10 hours each, and 10 times (336 × 5) days of 1 hour each, to dig the same trench. To dig a trench 1 yard long, will require $\frac{1}{70}$ as much time as to dig one 70 yards long; to dig one 1 yard wide, $\frac{1}{3}$ as much as 3 yards wide; to dig one 1 yard deep, $\frac{1}{2}$ as much as 2 yards deep; and to dig one of 1 degree of hardness $\frac{1}{5}$ as much as to dig one of 5 degrees of hardness. 240 men can dig a trench 1 yard long, 1 yard wide, 1 yard deep, and of 1 degree of hardness in $\frac{1}{240}$ of the time that 1 man can dig the same, and in $\frac{1}{12}$ as many days of 12 hours each, as of 1 hour each; but to dig one 36 yards long, will require 36 times as much time as to dig one 1 yard long; to dig one 5 yards wide, 5 times as much as 1 yard wide; to dig one 3 yards deep, 3 times as much as 1 yard deep; and to dig one of 6 degrees of hardness will require 6 times as much time as to dig one of 1 degree of hardness.

OPERATION.

$$\frac{\cancel{336}\times\cancel{5}\times\cancel{10}}{1}\times\frac{1}{\cancel{70}}\times\frac{1}{\cancel{3}}\times\frac{1}{\cancel{2}}\times\frac{1}{\cancel{5}}\times\frac{1}{\cancel{240}}\times\frac{1}{\cancel{12}}\times\frac{\cancel{36}}{1}\times\frac{\cancel{5}}{1}\times\frac{\cancel{3}}{1}\times\frac{6}{1}=9\ da.$$

Or,

$$\begin{array}{r|ll}
\cancel{240} & \cancel{336} & \\
\cancel{1} & \cancel{5} & \\
\cancel{12} & \cancel{10} & 3 \\
\cancel{70} & \cancel{36} & \\
\cancel{3} & \cancel{5} & \\
\cancel{2} & \cancel{3} & 3 \\
\cancel{5} & \cancel{6} & \\
\hline
 & 9 \text{ days } Ans. &
\end{array}$$

NOTE.—The principle of the above analysis is this: 1st. Find how many hours it will take 1 man to dig 1 cubic yard of trench; this is done in the first part of the analysis. 2d. Find how long it will take 240 men, working 12 hours a day, to dig the required trench, working at the same rate; this is done in the second part of the analysis.

14. If 20 cords of wood are equal in value to 6 tons of hay, and 5 tons of hay to 36 bushels of wheat, and 12 bushels of wheat to 25 bushels of corn, and 14 bushels of corn to 56 pounds of butter, and 72 pounds of butter to 8 days of labor; how many cords of wood will be equal to 16 days of labor?

ANALYSIS.—Since 20 cords of wood are equal to 6 tons of hay, 1 ton of hay is equal to $\frac{1}{6}$ of 20 cords of wood, or $\frac{20}{6}$ cords; 5 tons are equal to 5 times $\frac{20}{6}$, or $\frac{50}{3}$ cords; since 5 tons of hay, or $\frac{50}{3}$ of a cord of wood are equal to 36 bushels of wheat, 1 bushel of wheat is equal to $\frac{1}{36}$ of $\frac{50}{3} = \frac{25}{54}$ cords, and 12 bushels of wheat are equal to 12 times $\frac{25}{54} = \frac{50}{9}$ cords; and since 12 bushels of wheat, or $\frac{50}{9}$ cords of wood are equal to 25 bushels of corn, 1 bushel of corn is equal to $\frac{1}{25}$ of $\frac{50}{9} = \frac{2}{9}$ of a cord of wood, and 14 bushels of corn are equal to 14 times $\frac{2}{9} = \frac{28}{9}$ cords; and since 14 bushels of corn, or $\frac{28}{9}$ cords of wood are equal to 56 pounds of butter, 1 pound of butter is equal $\frac{1}{56}$ of $\frac{28}{9} = \frac{1}{18}$ of a cord, and 72 pounds of butter are equal to 72 times $\frac{1}{18}$ = 4 cords of wood; and since 72 pounds of butter or 4 cords of wood are equal to 8 days' labor, 1 day's labor is equal to $\frac{1}{8}$ of $4 = \frac{1}{2}$ cord of wood, and 16 days labor are equal to 16 times $\frac{1}{2}$ of a cord, or 8 cords of wood.

OPERATION.

$$\frac{20}{1}\times\frac{1}{6}\times\frac{5}{1}\times\frac{1}{36}\times\frac{12}{1}\times\frac{1}{25}\times\frac{14}{1}\times\frac{1}{56}\times\frac{72}{1}\times\frac{1}{8}\times\frac{16}{1} = 8 \text{ cords; or,}$$

$$\begin{array}{r|l}
\cancel{6} & \cancel{20}\ ^{4} \\
\cancel{36} & \cancel{5} \\
\cancel{25} & \cancel{12} \\
\cancel{56} & \cancel{14} \\
\cancel{8} & \cancel{72}\ _{2} \\
 & \cancel{16} \\
\hline
 & 8 \text{ cords. } Ans.
\end{array}$$

NOTE.—This and similar examples fall under what is called the *Chain Rule*. In analyzing, then, always commence with the term *which is of the same name or kind as the required* answer.

15. A, B, and C, put in trade $5626: A's stock was in 5 months, B's, 7 months, and C's, 9 months. They gained $1260, which was so divided that A received $4 as often as B had $5, and as often as C had $3. After receiving $2164,50, B absconded. What was each one's stock in trade, and how much did A and C gain or lose by B's withdrawal?

ANALYSIS.—Since A received $4 as often as B had $5, and as often as C had $3, if the whole gain were divided into 12 equal parts, A would have $\frac{4}{12}$, B, $\frac{5}{12}$, and C, $\frac{3}{12}$, of $1260, or A would have $420, B, $525, and C, $315. Now, if their respective gains be divided by the number of months each one's stock continued in trade, the quotients will represent their monthly gains, viz., A's will be $420 ÷ 5 = $84; B's, $525 ÷ 7 = $75; and C's, $315 ÷ 9 = $35, which gives $194 as their whole gain for 1 month.

But since each one's share of the gain for a given time will be to the whole gain for the same time, as each one's stock to the whole stock; it follows that, A will have $\frac{84}{194}$, B, $\frac{75}{194}$, and C, $\frac{35}{194}$, of the whole stock, or A will have $2436, B, $2175, and C, $1015. When B ran away he was entitled to his original stock $2175, and his share of the gain for 7 months, that is, to $2175 + $525 = $2700; but as he took away only $2164,50, A and C gained $535,50 by his withdrawal, which must be divided between them in the ratio of their investments, or as 4 to 3; therefore, A will have $\frac{4}{7}$, and C $\frac{3}{7}$ of B's unclaimed portion, or A will have $306, and C $229.50.

OPERATION.

$4 + 5 + 3 = 12.$

A's whole gain	= $\frac{4}{12}$	of	$1260	=	$420
B's " "	= $\frac{5}{12}$	"	"	=	$525
C's " "	= $\frac{3}{12}$	"	"	=	$315

A's monthly gain	= $420 ÷ 5	=	$84
B's " "	= $525 ÷ 7	=	$75
C's " "	= $315 ÷ 9	=	$35
			$194

A's stock	= $\frac{84}{194}$	of	$5626	=	$2436
B's "	= $\frac{75}{194}$	"	"	=	$2175
C's "	= $\frac{35}{194}$	"	"	=	$1015

$\$2175 + \$525 - \$2164{,}50 = \$535{,}50$, what B left.
$\frac{4}{7}$ of $\$535{,}50 = \$306$ A's share of it.
$\frac{3}{7}$ " " $= \$229{,}50$ B's share of it.

16. Mr. Johnson bought goods to the amount of \$2400, $\frac{1}{3}$ to be paid in 3 months, $\frac{1}{4}$ in 4 months, $\frac{1}{4}$ in 6 months, and the remainder in 8 months: what is the equated time for the payment of the whole?

ANALYSIS.—\$800 to be paid in 3 months, is the same as \$1, to be paid in 2400 months; \$600, in 4 months, the same as \$1 in 2400 months; \$600, in 6 months, the same as \$1, in 3600 months; and \$400 in 8 months, the same as \$1, in 3200 months. Then \$1, payable in $2400 + 2400 + 3600 + 3200 = 11600$ months, is the same as \$2400 in $\frac{1}{2400}$ of 11600 months, which is $4\frac{5}{6}$ months = 4 months 25 days, the equated time of payment.

OPERATION.

$$\begin{array}{r} 800 \times 3 = 2400 \\ 600 \times 4 = 2400 \\ 600 \times 6 = 3600 \\ 400 \times 8 = 3200 \\ \hline 2400 \qquad 11600 \end{array}$$

$11600 \div 2400 = 4\frac{5}{6}mo. = 4mo.\ 25da.$ *Ans.*

17. What will be the interest on \$60,48 for 1 year 3 months, at 7 per cent?

ANALYSIS.—Since the interest on \$1 for 1 year is 7 cents, or seven hundredths of \$1, the interest on \$60,48 for 1 year, will be $\$60{,}48 \times .07 = \$4{,}2336$. The interest for 1 month will be $\frac{1}{12}$ as much as for 1 year or $\frac{1}{12}$ of $\$4.2236 = \$0.3528$, and for 1*yr.* 3*mo.* = 15 months, it will be 15 times as much as for 1 month, or $\$0{,}3528 \times 15 = \$5{,}292$.

OPERATION.

$(\$60{,}48 \times .07 \div 12) \times 15 = \$5.292$ *Ans.* Or,

$$\begin{array}{r|l} & 5{,}04 \\ & \cancel{60{,}48} \\ \cancel{12} & .7 \\ & 15 \\ \hline & \$5.292\ Ans. \end{array}$$

18. What will be the interest on \$88,92, for 8*mo.* 20*da.*, at 7 per cent?

ANALYSIS.—Since the interest on \$1 is 7 cents for 1 year, the interest on \$88,92 for 1 year will be \$88,92 × .07 = \$6,224; the interest for 1 month will be $\frac{1}{12}$ of \$6,224 = \$0,5187; and since the number of days divided by 30 will give the value of those days in decimals of a month (Art. 222) 20*da.*=.6$\frac{2}{3}$ months. The interest for 8*mo.* 20*da.* = 8.6$\frac{2}{3}$ months, will be 8.6$\frac{2}{3}$ times as much as for 1 month = 0.5187 × 8.6$\frac{2}{3}$ = \$4.4954.

OPERATION.

$$(\$88,92 \times .07 \div 12) \times 8.6\tfrac{2}{3} = \$4,4954 \ \textit{Ans.}$$

$\cancel{12}$	$\cancel{88,92}$ 2,47	Or,	$\cancel{10}$ $\cancel{12}$	$\cancel{88,92}$ 2,47
$\cancel{3}$	.07		$\cancel{30}$	.07
	26			$\cancel{260}$ 26
	\$4.4954			\$4.4954 *Ans.*

19. A liquor merchant mixed together 25 gallons of brandy at \$1,60 a gallon, 25 gallons at \$1,80, 10 gallons at \$2,50, and 20 gallons of water; what was the value of 1 gallon of the mixture, and what was the gain on a gallon if he sold it at the average price of the liquor?

ANALYSIS.—The value of 18 gallons of water would be 0; of 25 gallons of brandy at \$1,60 a gallon would be \$1,60 × 25 = \$40; of 25 gallons at \$1,80, would be \$1,80 × 25 = \$45; of 10 gallons at \$2,50, would be \$2,50 × 10 = \$25. 25 + 25 + 10 gallons of brandy + 18 gallons of water = 80 gallons, the amount of the mixture; and \$40 + \$45 + 25 = \$110, the value of the mixture; hence, if 80 gallons are worth \$110, one gallon is worth $\frac{1}{80}$ of \$110 = \$1,37$\frac{1}{2}$. But 25 + 25 + 10 = 60 gallons of brandy, are worth \$110, and \$110 ÷ 60 = \$1,83$\frac{1}{3}$, the average price per gallon of the brandy; therefore \$1,83$\frac{1}{3}$ − \$1,37$\frac{1}{2}$ = 45$\frac{5}{6}$ cents, the gain on 1 gallon.

OPERATION.

$$\begin{array}{rcl} 18 \times & 0 & = 00 \\ 25 \times & 1.60 & = 40 \\ 25 \times & 1.80 & = 45 \\ 10 \times & 2.50 & = 25 \\ \hline 80 & & 110 \end{array}$$

$$\$110 \div 80 = \$1,37\tfrac{1}{2} \text{ value: } \$110 \div 60 = \$1,83\tfrac{1}{3} \text{ average price,}$$
$$\$1,83\tfrac{1}{3} - \$1,37\tfrac{1}{2} = \$0,45\tfrac{5}{6}.$$

20. A merchant has three kinds of cloth, worth $\$1\frac{2}{3}$, $\$2\frac{1}{4}$, $\$3\frac{3}{4}$ a yard: what is the least number of whole yards he can sell, to receive an average price of $\$2\frac{1}{2}$ a yard?

ANALYSIS.—If he sells 1 yard worth $\$1\frac{2}{3}$, for $\$2\frac{1}{2}$, he will gain $\frac{5}{6}$ of a dollar; to gain 1 dollar he must sell as many yards as $\frac{5}{6}$ is contained times in 1, or $\frac{6}{5}$ yards. But since he is neither to gain or lose by the operation, if he gains on one kind, he must lose an equal sum on some other; hence, he must sell some that is worth more than the average price. If he sell 1 yard worth $\$3\frac{3}{4}$ for $\$2\frac{1}{2}$, he will lose $\frac{5}{4}$ of a dollar, and to lose $\$1$, he must sell $\frac{4}{5}$ of a yard. Therefore, to make the loss equal to the gain, he must sell $\frac{4}{5}$ of a yard at $\$3\frac{3}{4}$ a yard, as often as he sells $\frac{6}{5}$ of a yard at $\$1\frac{2}{3}$ a yard.

If he sells 1 yard worth $\$2\frac{1}{4}$, for $\$2\frac{1}{2}$, he gains $\frac{1}{4}$ of a dollar, and to gain $\$1$ he must sell 4 yards; hence, to keep the average price, he must lose as much on some other, and as he can only lose on that at $\$3\frac{3}{4}$ a yard, he must sell enough of that to lose $\$1$, which would be $\frac{4}{5}$ of a yard; therefore, as often as he sells $\frac{6}{5}$ yards at $\$1\frac{2}{3}$ a yard, he must sell $\frac{4}{5}$ yards at $\$3\frac{3}{4}$ a yard; and as often as he sells 4 yards at $\$2\frac{1}{4}$ a yard, he must sell $\frac{4}{5}$ yards at $\$3\frac{3}{4}$ a yard.

But since it is desirable to have the proportional parts expressed in the least whole numbers, we may multiply the numbers by the least common multiple of their denominators, and divide the products by their greatest common factor; this being done, we obtain in the above example, 3 yards at $\$1\frac{2}{3}$ a yard, 10 yards at $\$2\frac{1}{4}$ a yard, and 4 yards at $\$3\frac{3}{4}$ a yard.

OPERATION.

$2\frac{1}{2}$	$1\frac{2}{3}$	$\frac{6}{5}$		6		6	3
	$2\frac{1}{4}$		4		20	20	10
	$3\frac{3}{4}$	$\frac{4}{5}$	$\frac{4}{5}$	4	4	8	4

21. The hour and minute hands of a clock are together at 12 o'clock: when are they next together?

ANALYSIS.—Since the minute hand passes over 60 minute spaces while the hour-hand passes over 5, the minute-hand passes over 12 minute spaces while the hour-hand passes over 1, gaining 11 minute spaces on the hour-hand in 12 minutes of time; the minute-hand requiring one minute of time to pass over 1 minute of space. Hence, in 1 minute of time, the minute-hand gains on the hour-hand, $\frac{11}{12}$ of a minute space.

When the minute-hand returns to 12, the hour-hand will be at 1, and will require the minute-hand to gain 5 minute spaces. As the minute-hand passes over $\frac{12}{11}$ the space gained, to gain 5 minute spaces it must pass over $\frac{12}{11}$ of $5 = \frac{60}{11} = 5\frac{5}{11}$ minute spaces, requiring $5\frac{5}{11}$ minutes of time = $5mi.\ 27\frac{3}{11}sec.$, which added to 1 o'clock, gives $1hr.\ 5mi.\ 27\frac{3}{11}sec.$

SECOND ANALYSIS.—In 12 hours the minute-hand passes the hour-hand 11 times, consequently, if both are at 12, the minute-hand will pass the hour-hand the first time in $\frac{1}{11}$ of 12 hours, or $1hr.\ 5mi.\ 27\frac{3}{11}sec.$ It will pass it the second time in $\frac{2}{11}$ of 12 hours, and so on.

OPERATION.

$$5 \times \frac{12}{11} = \frac{60}{11} = 5\frac{5}{11}mi. = 5mi.\ 27\frac{3}{11}sec., \text{ which added to } 1hr. = 1hr.\ 5mi.\ 27\frac{3}{11}sec. \ \textit{Ans.}$$

21. An apple boy bought a certain number of apples at the rate of 3 for 1 cent, and as many more at 4 for 1 cent, and selling them again at 2 for 1 cent, he found that he had gained 15 cents: how many apples had he?

ANALYSIS.—Since he bought a number of apples at 3 for a cent, and as many more at 4 for a cent, he paid $\frac{1}{3}$ of a cent apiece for the first, and $\frac{1}{4}$ of a cent apiece for the second lot: then, $\frac{1}{3} + \frac{1}{4} = \frac{7}{12}$ of a cent, what he paid for one of each, and $\frac{7}{12} \div 2 = \frac{7}{24}$ of a cent, the average price for all he bought. Since he sold at 2 for a cent, or $\frac{1}{2}$ a cent a piece, he must have gained on each apple the difference between $\frac{1}{2}$ and $\frac{7}{24} = \frac{5}{24}$ of a cent; hence, to gain 1 cent he must sell as many apples as $\frac{5}{24}$ is contained times in $1 = 4\frac{4}{5}$ apples, and to gain 15 cents he must sell 15 times as many, or $4\frac{4}{5} \times 15 = 72$ apples.

OPERATION.

$$\frac{1}{3} + \frac{1}{4} = \frac{7}{12}, \quad \frac{7}{12} \div 2 = \frac{7}{24}, \quad \frac{1}{2} - \frac{7}{24} = \frac{5}{24},$$
$$1 \div \frac{5}{24} = 4\frac{4}{5}, \quad 4\frac{4}{5} \times 15 = 72 \text{ apples. } \textit{Ans.}$$

22. A gentleman left to his three sons, whose ages were 13, 15 and 17 years, $15000, to be divided in such a manner, that each share being put at interest, at 7 per cent, should give to each son the same amount when he attained the age of 21 years: what was the share of each?

ANALYSIS.—By the question their respective shares would be at interest 8, 6 and 4 years.

Find the present worth of \$1 for 8, 6 and 4 years, respectively: They are \$0,6410256 +, \$0,7042253+, and \$0,78125. These sums being put at interest at 7 per cent, will each amount to \$1 at the expiration of their respective times; and the sum of these numbers, \$0,6410256 + \$0,7042253+\$0,78125=\$2,1265009+ is the amount, which being so distributed among them, will produce \$1 to each. If each number be divided by the sum, \$2,1265009, the quotients will denote the parts of \$1, which according to the conditions of the question, each person should receive, and which put at interest will produce equal amounts at the end of their respective times; therefore, each person will receive for his entire share 15000 like parts of one dollar.

OPERATION.

\$1 ÷ 1.56 =	\$0,6410256 +	present worth of \$1 for 8 years.
\$1 ÷ 1.42 =	\$0,7042253 +	" " " 6 "
\$1 ÷ 1.28 =	\$0,78125	" " " 4 "
	\$2,1265009	

\$0,6410256 ÷ 2.1265 × 15000 = \$4521,694
\$0,7042253 ÷ 2.1265 × 15000 = \$4967,494
\$0,78125 ÷ 2.1265 × 15000 = \$5510,815

23. A, B, C, and D, agree to do a piece of work for \$312. A, B, and C, can do it in 10 days; B, C, and D, in $7\frac{1}{2}$ days; C, D, and A, in 8 days; and D, A, and B, in $8\frac{4}{7}$ days. In how many days can all do it, working together; in how many days can each do it working alone; and what part of the pay ought each to receive?

ANALYSIS.—Since A, B, C, can do the work in 10 days, they can do $\frac{1}{10}=\frac{12}{120}$ of it in 1 day: since B, C, D, can do it in $7\frac{1}{2}$ days, they can do $\frac{2}{15}=\frac{16}{120}$ of it in 1 day; since C, D, A, can do it in 8 days, they can do $\frac{1}{8}=\frac{15}{120}$ of it in 1 day; and since D, A, B, can do it in $8\frac{4}{7}$ days, they can do $\frac{7}{60}=\frac{14}{120}$ of it in 1 day; hence, A, B, C, and D, by working 3 days each, will do $\frac{12}{120}+\frac{16}{120}+\frac{15}{120}+\frac{14}{120}=\frac{57}{120}$ of the work, and in 1 day they will do $\frac{1}{3}$ of $\frac{57}{120}=\frac{19}{120}$. It will then take them as many days to do the whole as $\frac{19}{120}$ is contained times in $1 = 6\frac{6}{19}$ days.

By subtracting, in succession, what the three can do in 1 day, when they work together, from what the four can do in 1 day, we shall have what each one will do in 1 day; viz., $\frac{19}{120}-\frac{12}{120}=\frac{7}{120}$, what D will do in 1 day; $\frac{19}{120}-\frac{16}{120}=\frac{3}{120}$, what A can do in 1 day; $\frac{19}{120}-\frac{15}{120}=\frac{4}{120}$, what B can do in 1 day; $\frac{19}{120}-\frac{14}{120}=\frac{5}{120}$, what C can do in 1 day. It will take each as many days to do the whole work as the part which he can do in 1 day is contained times in 1; viz., $1\div\frac{3}{120}=40$ days, A's time to do it; $1\div\frac{4}{120}=30$ days, B's; $1\div\frac{5}{120}=24$ days, C's; $1\div\frac{7}{120}=17\frac{1}{7}$ days, D's.

Now, each should receive such a part of the whole amount paid, viz., \$312, as he did of the whole work. This part will be denoted by what he did in 1 day multiplied by the number of days he worked; viz., A, $\frac{3}{120}\times6\frac{6}{19}=\frac{3}{19}$; B, $\frac{4}{120}\times6\frac{6}{19}=\frac{4}{19}$; C, $\frac{5}{120}\times6\frac{6}{19}=\frac{5}{19}$; D, $\frac{7}{120}\times6\frac{6}{19}=\frac{7}{19}$.

OPERATION.

$\frac{1}{10}=\frac{12}{120}$, what A, B, C, does in 1 day.
$\frac{2}{15}=\frac{16}{120}$, " B, C, D, " "
$\frac{1}{8}=\frac{15}{120}$, " C, D, A, " "
$\frac{7}{60}=\frac{14}{120}$, " D, A, B, " "

$\frac{12}{120}+\frac{16}{120}+\frac{15}{120}+\frac{14}{120}=\frac{57}{120}$, what A, B, C, and D, can do in 3 days.

$\frac{57}{120}\div3=\frac{19}{120}$, what A, B, C, and D, can do in 1 day.

$\frac{19}{120}-\frac{16}{120}=\frac{3}{120}$ what A can do in 1 day; $1\div\frac{3}{120}=40da.$
$\frac{19}{120}-\frac{15}{120}=\frac{4}{120}$ " B, " " $1\div\frac{4}{120}=30da.$
$\frac{19}{120}-\frac{14}{120}=\frac{5}{120}$ " C, " " $1\div\frac{5}{120}=24da.$
$\frac{19}{120}-\frac{12}{120}=\frac{7}{120}$ " D, " " $1\div\frac{7}{120}=17\frac{1}{7}d.$

Hence, the share of each will be,

$\$312\times\frac{3}{19}=\$\ 49{,}26\frac{6}{19}$, A's share.
$\$312\times\frac{4}{19}=\$\ 65{,}68\frac{8}{19}$, B's share.
$\$312\times\frac{5}{19}=\$\ 82{,}10\frac{10}{19}$, C's share.
$\$312\times\frac{7}{19}=\$114{,}94\frac{14}{19}$, D's share.

\$312,00 amount paid to A, B, C, and D.

24. A person owning $\frac{2}{3}$ of a vessel, sells $\frac{5}{8}$ of his share for \$1736: what was the value of the whole vessel?

25. If a man performs a journey in $7\frac{1}{3}$ days, travelling $14\frac{2}{3}$ hours a day, in how many days will he perform the same journey, by travelling $10\frac{6}{7}$ hours a day?

26. If $\frac{1}{9}$ of a pole stands in the mud, 2 feet in the water, and $\frac{5}{6}$ above the water, what is the length of the pole?

27. After spending $\frac{1}{4}$ of my money, and $\frac{1}{5}$ of the remainder, I had \$1062 left: how much had I at first?

28. Suppose a cistern has two pipes, and that one can fill it in $7\frac{1}{2}$ hours, and the other in $4\frac{1}{6}$ hours: in what time can both fill it running together?

29. If 54 yards of ribbon cost \$9, what will 26 yards cost?

30. If 2 acres of land cost $\frac{1}{4}$ of $\frac{6}{7}$ of $\frac{7}{9}$ of \$300, what will $\frac{1}{2}$ of $\frac{3}{4}$ of $10\frac{2}{3}$ acres cost?

31. There is a regiment of soldiers to be clothed: each suit is to contain $3\frac{1}{2}$ yards of cloth $1\frac{3}{8}$ yards wide: how much shalloon that is $\frac{7}{8}$ yards wide is necessary for lining?

32. How much tea at 7*s.* 6*d.* a pound must be given for 234 bushels of oats, at 3*s.* 9*d.* a bushel, New York currency?

33. What will 3 pipes of wine cost at 2*s.* 9*d.* per quart, New England currency?

34. A gives B 165 yards of cotton cloth, at 2*s.* 6*d.* per yard, Missouri currency, for 625 pounds of lump sugar: how much was the sugar worth a pound?

35. If the expense of keeping 1 horse 1 day is 3*s.* 4*d.* Canada currency, what will be the expense of keeping 4 horses 3 weeks at the same rate?

36. Bought 10 bales of cloth, each bale containing 14 pieces, and each piece $22\frac{1}{2}$ yards, at 10*s.* 8*d.* per yard, Illinois currency: what was the cost of the cloth?

37. A has $7\frac{1}{2}$*cwt.* of sugar, worth 12 cents a pound, for which B gave him $12\frac{1}{2}$*cwt.* of flour: what was the flour worth a pound?

38. What is the value of 2*hhd.* of molasses, at 1*s.* 2*d.* per quart, Georgia currency?

39. What will be the value of 3 pieces of cloth, each piece

containing $24\frac{1}{2}$ yards, at 4*s.* 6*d.* per yard, Pennsylvania currency?

40. Bought 120 yards of cloth, at 6*s.* 8*d.* a yard, New York currency, and gave in payment 76 bushels of rye, at 4*s.* 6*d.* a bushel, New England currency, and the balance in money: how many dollars will pay the balance?

41. A merchant bought 21 pieces of cloth, each piece containing 41 yards, for which he paid \$1260; he sold the cloth at \$1,75 per yard: did he gain or lose, and how much?

42. The hour and minute hands of a watch are together at 12: at what moment will they be together between 5 and 6?

43. How many yards of carpeting $\frac{3}{4}$ of a yard wide will cover the floor of a room 18 feet long and 15 feet wide?

44. If 9 men can build a house in 5 months, by working 12 hours a day, how many hours a day must the same men work to do it in 6 months?

45. B and C can do a piece of work in 12 days: with the assistance of A they can do it in 9 days: in what time can A do it alone?

46. A can mow a certain field of grass in 3 days, B can do it in 4 days, and C can do it in 5 days: in what time can they do it, working together?

47. Divide the number 480 into 4 such parts that they shall be to each other as the numbers 3, 5, 7 and 9?

48. What length of a board that is $8\frac{4}{7}$ inches broad, will make a square foot?

49. The provisions in a garrison were sufficient for 1800 men, for 12 months; but at the end of 3 months, it was reinforced by 600 men, and 4 months afterwards, a second reinforcement of 400 was sent in: how long would the provisions last after the last reinforcement arrived?

50. A merchant bought a quantity of broadcloth and baize for \$488,80; there was $117\frac{1}{2}$ yards of broadcloth, at $\$3\frac{1}{2}$ per yard; for every 5 yards of broadcloth he had $1\frac{1}{2}$ yards of baize: how many yards of baize did he buy, and what did it cost him per yard?

51. If the freight of 40 tierces of sugar, each weighing $8\frac{1}{2}$ *cwt.*, for 150 miles, costs $42, what must be paid for the freight of 10*hhd.* each weighing 12*cwt.*, for 50 miles?

52. If 1 pound of tea be equal in value to 50 oranges, and 70 oranges be worth 84 lemons, what is the value of a pound of tea, when a lemon is worth 2 cents?

53. What amount must be discounted, at 7 per cent, to make a present payment of a note of $500, due 2 years 8 months hence?

54. If the interest on $225 for $4\frac{1}{2}$ years is $91,12$\frac{1}{2}$, what would be the interest on $640 at the same rate for $2\frac{1}{4}$ years?

55. A farmer having 1000 bushels of wheat to sell, can have $1,75 a bushel cash, or $1,80 a bushel in 90 days: which would be most advantageous to him, money being worth 7 per cent?

56. A merchant bought goods to the amount of $1575 on 9 months credit; he sells the same for $1800 in cash: money being worth 6 per cent, what did he gain?

57. Three persons in partnership gain $482,62; A put in $\frac{3}{4}$ as much capital as B, and B put in $\frac{5}{6}$ as much as C: what was each one's share of the gain?

58. A father divided his estate, worth $9268,60, among his 4 children, giving A, $\frac{1}{4}$ of it, B, $\frac{1}{5}$, and C, $5 as often as he gave D $6: how much did each receive?

59. A tax of $475,50 was laid upon 4 villages, A, B, C, and D; it was so distributed, that as often as A and B each paid $5, C paid $7, and D, $8: what part of the whole tax did each village pay?

60. There are 1000 men besieged in a town, with provisions for 5 weeks, allowing each man 16 ounces a day. If they are reinforced by 400 men, and no relief can be afforded till the end of 8 weeks, what must be the daily allowance to each man?

61. A reservoir has 3 pipes, the first can fill it in 10 days, the second, in 16 days, and the third can empty it in 20 days: in what time will the cistern be filled if they are all allowed to run at the same time?

62. Two persons, A and B, are on opposite sides of a wood, which is 536 yards in circumference; they begin to travel in

the same direction at the same time; A goes at the rate of 11 yards a minute, and B, at the rate of 34 yards in 3 minutes: how many times will B go round the wood before he overtakes A?

63. Two men and a boy were engaged to do a piece of work. one of the men could do it in 10 days, the other in 16 days, and and the boy could do it in 20 days: how long would it take them to do it together?

64. A owes B $500, of which $150 is to be paid in 3 months, $175 in 6 months, and the remainder in 8 months: what would be the equated time for the payment of the whole?

65. If 42 men, in 270 days, working $8\frac{1}{2}$ hours a day, can build a wall $98\frac{3}{4}$ feet long, $7\frac{1}{2}$ feet high, and $2\frac{1}{2}$ feet thick; in how many days can 63 men build a wall $45\frac{1}{3}$ feet long, $6\frac{7}{12}$ feet high, and $3\frac{1}{8}$ feet thick, working $11\frac{1}{3}$ hours a day?

66. After one-third part of a cask of wine had leaked away, 21 gallons were drawn, when it was found to be half full: how much did the cask hold?

67. A man had a bond and mortgage for $2500, dated July 1st, 1854. He is not satisfied with 7 per cent annual interest, and on the first day of September, 1854, he purchased 10 shares, of $100 each, of railroad stock, at 115. Nov. 1st, he bought 8 shares more of the same stock, at 98; and on April 1st, 1855, he bought 5 shares more at the same rate. On the first days of August and February, in each year, he received a regular semi-annual dividend of 4 per cent, and at the end of the year (January 1st, 1856,) sold his whole stock at 99: which was the more profitable investment, and how much?

68. A landlord being asked how much he received for the rent of his property, answered, that after deducting 9 cents from each dollar, for taxes and repairs, there remained $3014,30: what was the amount of his rents?

69. If 165 pounds of soap cost $16,50, for how much will it be necessary to sell 390 pounds, in order to gain the cost of 36 pounds?

70. What is the height of a wall which is $14\frac{1}{2}$ yards in length, and $\frac{7}{10}$ of a yard in thickness, and which has cost \$406, it having been paid for at the rate of \$10 per cubic yard?

71. A thief escaping from an officer, has 40 miles the start, and travels at the rate of 5 miles an hour; the officer in pursuit travels at the rate of 7 miles an hour: how far must he travel before he overtakes the thief?

72. Two families bought a barrel of flour together, for which they paid \$8, and agreed that each child should count half as much as a grown person. In one family there were 3 grown persons and 8 children, and in the other, 4 grown persons and 10 children; the first family used from the flour 2 weeks, and the second 3 weeks: how much ought each to pay?

73. At \$42 a thousand, how much lumber should be given for a farm containing 33*A.*, 2*R.*, 16*P.*, valued at \$125 an acre?

74. How many building lots, each 50 feet by 100 feet, can be made out of $2\frac{1}{2}$ acres of ground?

75. A person pays \$150 for an insurance on goods, at $3\frac{3}{4}$ per cent, and finds that in case the goods are lost, he will receive the value of the goods, the premium of insurance, and \$25 besides: what was the value of the goods?

76. A distiller purchased 5000 bushels of rye, which he can have at 96 cents a bushel, ready money, or \$1, with 2 months' credit; which would be the more advantageous to him, to buy it on credit, or to borrow the money at 7 per cent, and pay the cash?

77. A stockholder bought $\frac{3}{8}$ of the capital of a company at par; he sold $\frac{1}{8}$ of his purchase at par, and the remainder for \$25000, and by the latter sale made \$5000: what was the value of the whole capital?

78. How many bushels of grain will a bin contain, that is 3*ft.* 5*in.* wide, 2*ft.* 6*in.* long, and 6*ft.* deep?

79. If the two sides of a triangle are 75 feet and 90 feet, and the perpendicular to the third side 45 feet, what is the length of the third side?

80. Three travellers have 2160 miles to go before they reach

the end of their journey; the first goes 30 miles a day, the second 27, and the third 24: how many days should one set out after another that they may arrive together?

81. A house which was sold a second time for $7180, would have given a profit of $420 if the second proprietor had purchased it $130 cheaper than he did: at what price did he purchase it?

82. A piece of land of 188 acres was cleared by two companies of men, working together; the first numbered 25 men, and the second 22: the first company received $84 more than the second: how many acres did each company clear, and what did the clearing cost per acre?

83. I have three notes payable as follows: one for $100, due Feb. 12th, 1856, the second for $400, due March 12th, and the third for $300, due April 1st: what is the average time of payment?

84. How many marble slabs, 15*in.* square, will it take to pave a floor 32 feet long, and 25 feet wide? What will be the cost at $3 a square yard for the marble, and 40 cents a square yard for labor?

85. A man, in his will, bequeathed $500 to A, $425 to B, $300 to C, $250 to D, and $175 to E; but after settling up the estate and paying expenses, there was but $1155 left: what is each one's share?

86. If 3*lbs.* of tea are worth 7*lbs.* of coffee, and 14*lbs.* of coffee are worth 48*lbs.* of sugar, and 18*lbs.* of sugar are worth 27*lbs.* of soap; how many pounds of soap are 6*lbs.* of tea worth?

87. What is the hour, when the time past noon is $\frac{4}{5}$ the time to midnight?

88. If $\frac{3}{4}$ of a yard of cloth cost $\$\frac{2}{5}$, being $\frac{7}{8}$ of a yard wide, what is the value of $\frac{5}{8}$ of a yard $1\frac{3}{4}$ yards wide, of the same quality?

89. A farmer sold 60 fowls, a part turkeys, and a part chickens; for the turkeys he received $1,10 apiece, and for the chickens 50 cents apiece, and for the whole he received $51,60: how many were there of each?

90. A person hired a man and two boys; to the man he gave 6 shillings a day, to one boy 4 shillings and to the other 3 shillings a day, and at the end of the time he paid them 104 shillings: how long did they work?

91. Divide $6471 among 3 persons, so that as often as the first gets $5, the second will get $6, and the third $7.

92. Two partners have invested in trade $1600, by which they have gained $300; the gain and stock of the second amount to $1140: what is the stock and the gain of each?

93. What is the height of a tower that casts a shadow 75.75 yards long, at the same time that a perpendicular staff 3 feet high, gives a shade of 4.55 feet in length?

94. A can do a certain piece of work in 3 weeks; B can do 3 times as much in 8 weeks; and C can do 5 times as much in 12 weeks: in what time can they all together do the first piece of work?

• 95. Two persons pass a certain point at an interval of 4 hours; the first travelling at the rate of $11\frac{1}{2}$, and the second $17\frac{1}{2}$ miles an hour: how far and how long must the first travel before he is overtaken by the second?

96. Three persons engage in trade, and the sum of their stock is $1600. A's stock was in trade 6 months, B's 12 months, and C's 15 months; at the time of settlement, A receives $120 of the gain, B $400, and C $100: what was each person's stock?

97. A, B and C, start at the same time, from the same point, and travel in the same direction, around an island 73 miles in circumference. A goes at the rate of 6 miles, B 10 miles, and C 16 miles per day: in what time will they all be together again?

98. What length of wire, $\frac{1}{8}$ of an inch in diameter, can be drawn from a cube of copper, of 2 feet on a side, allowing 10 per cent for waste?

99. A person having $10000 invested in 6 per cent. stocks, sells out at 65, and invests the proceeds in 5 per cents at $82\frac{1}{2}$: what will be the difference in his income?

100. In order to take a boat through a lock from a certain

river into a canal, as well as to descend from the canal into the river, a volume of water is necessary $46\frac{1}{2}$ yards long, 8 yards wide and $2\frac{3}{5}$ yards deep. How many cubic yards of water will this canal throw into the river in a year, if 40 boats ascend and 40 descend each day except Sundays and eight holidays?

101. A company numbering sixty-six shareholders have constructed a bridge which cost $200000: what will be the gain of each partner at the end of 22 years, supposing that 6400 persons pass each day, and that each pays one cent toll, the expense for repairs, &c., being $5 per year for each shareholder?

102. Five merchants were in partnership for four years, the first put in $60, then, 5 months after, $800; the second put in first $600, and 6 months after $1800; the third put in $400; and every six months after he added $500; the fourth did not contribute till 8 months after the commencement of the partnership; he then put in $900, and repeated this sum every 6 months; the fifth put in no capital, but kept the accounts, for which the others agreed to allow him $800 a year, to be paid in advance and put in as capital. What is each one's share of the gain, which was $20,000?

103. A general arranging his army in the form of a square, finds that he has 44 men remaining, but by increasing each side by another man, he wants 49 to fill up the square: how many men had he?

104. A, B and C, are to share $100 in the proportion of $\frac{1}{3}$, $\frac{1}{4}$ and $\frac{1}{5}$, respectively; but by the death of C, it is required to divide the whole sum proportionally between the other two: what will each have?

105. A lady going out shopping spent at the first place she stopped, one-half her money, and half a dollar more; at the next place, half the remainder and half a dollar more; and at the next place half the remainder and half a dollar more, when she found that she had but three dollars left: how much had she when she started?

106. If a pipe of 6 inches discharge a certain quantity of

fluid in 4 hours, in what time will 4 pipes, each of 3 inches bore, discharge twice that quantity?

107. A man bought 12 horses, agreeing to pay $40 for the first, and in an increasing arithmetical progression for the rest, paying $370 for the last: what was the difference in the cost, and what did he pay for them all?

108. A, B, C and D, engaging in speculation, lost a sum of money, of which A, B and C, paid $297,60; B, C and D, $321,92; C, D and A, $375,83; and D, A and B, $402,50: what did each one pay?

109. If for £3000 exchange we pay $7\frac{1}{2}$ per cent premium, giving in payment notes at 4 months, 12 per cent discount, what rate ought we to make the premium, giving notes at 6 months, 10 per cent discount?

110. A purchase of $15000 worth of goods is to be paid for in three equal payments without interest; the first in 4 months, the second in 6 months, and the third in 9 months: money being worth 7 per cent, how much ready money ought to pay the debt?

111. If an iron bar 5 feet long, $2\frac{1}{2}$ inches broad, and $1\frac{3}{4}$ inches thick, weigh 45 pounds, how much will a bar of the same metal weigh, that is 7 feet long, 3 inches broad, and $2\frac{1}{4}$ inches thick?

112. A market woman bought a certain number of eggs at the rate of 4 for 3 cents, and sold them at the rate of 5 for 4 cents, by which she made 4 cents: what did she pay apiece for the eggs? What did she make on each egg sold? How many did she sell to gain 4 cents?

113. A person passed $\frac{1}{6}$ of his life in childhood, $\frac{1}{12}$ of it in youth, 5 years more than $\frac{1}{7}$ of it in matrimony: he then had a son, whom he survived 4 years, and who reached only $\frac{1}{2}$ the age of his father. At what age did he die?

114. A well is to be stoned, of which the diameter is 6 feet 6 inches, the thickness of the wall is to be 1 foot 6 inches, leaving the diameter of the well within the wall 3 feet 6 inches. If

the well is 40 feet deep, how many cubic feet of stone will be required?

115. A surveyor measured a piece of ground in the form of a rectangle, and found one side to be 37 chains, and the other 42 chains 16 links: how many acres did it contain?

116. A, B and C, can build a barn in 10 days; after 4 days, A leaves, and B and C go on with the work for 5 days longer, when B leaves, $\frac{3}{10}$ of the work being yet unfinished: C proceeds with the work and finishes it in $11\frac{1}{4}$ days after B left: how long would it take each to build the barn?

117. A farmer bought a piece of land for $1500, and agreed to pay principal and interest in 5 equal annual instalments: how much was the annual payment?

118. A fountain has 4 receiving pipes, A, B, C and D; A, B and C will fill it in 6 hours; B, C and D in 8 hours; C, D and A in 10 hours; and D, A and B in 12 hours: it has also 4 discharging pipes, E, F, G and H; E, F and G will empty it in 6 hours; F, G and H in 5 hours; G, H and E in 4 hours; H, E and F in 3 hours. Suppose the fountain full of water, and all the pipes open, in what time would it be emptied?

119. How many planks 15 feet long, and 15 inches wide, will floor a barn $60\frac{1}{2}$ feet long, and $33\frac{1}{2}$ feet wide?

120. If a ball 2 inches in diameter weigh 5 pounds, what will be the diameter of another ball of the same material that weighs 78.125 pounds?

121. A gives B his bond for $5000, dated April 1st, 1851, payable in 10 equal annual instalments of $500 each, on and after the first day of April, 1852. A afterwards agreed to take up his bond on the first day of April, 1853, deducting semi-annual discount, at the rate of 7 per cent. per annum, on the several payments, which fell due after the first day of April, 1852: what sum, on the first day of April, 1853, will cancel the bond?

APPLICATIONS OF ARITHMETIC.

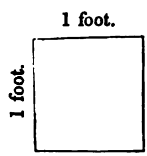
MENSURATION.

329. MENSURATION is the process of determining the contents of geometrical figures. It is divided into two parts, the mensuration of Surfaces and the mensuration of Volumes.

MENSURATION OF SURFACES.

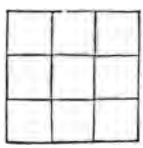
330. Surfaces have length and breadth. They are measured by means of a square, which is called the *unit of surface.*

A square is the space included between four equal lines, drawn perpendicular to each other. Each line is called a side of the square. If each side be one foot, the figure is called a *square foot.*



If the sides of a square be each four feet, the square will contain sixteen square feet. For, in the large square there are sixteen small squares, the sides of which are each one foot. Therefore, the square whose side is four feet, contains sixteen square feet.

The number of small squares that is contained in any large square is always equal to the product of two of the sides of the large square. As in the figure, $3 \times 3 = 9$ square feet. The number of square inches contained in a square foot is equal to $12 \times 12 = 144$.



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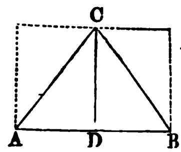
329. What is mensuration?

330. What is a *surface?* What is a square? What is the number of *small squares contained in* a large square equal to?

331. A triangle is a figure bounded by three straight lines. Thus, ACB is a triangle.



The lines BA, AC, BC, are called *sides;* and the corners, B, A and C, are called *angles.* The side AB is the *base.*

When a line like CD is drawn, making the angle CDA equal to the angle CDB, then CD is said to be perpendicular to AB, and CD is called the *altitude* of the triangle. Each triangle CAD or CDB is called a right-angled triangle. The side BC, or the side AC, opposite the right angle, is called the *hypothenuse.*

The area or contents of a triangle is equal to half the product of its base by its altitude (Bk. IV., Prop. VI).*

EXAMPLES.

1. The base, AB, of a triangle is 50 yards, and the perpendicular, CD, 30 yards: what is the area?

OPERATION.

$$\begin{array}{r} 50 \\ 30 \\ \hline 2)1500 \\ \hline 750 \end{array}$$

Ans. 750 *square yards.*

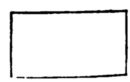
ANALYSIS.—We first multiply the base by the altitude, and the product is square yards, which we divide by 2 for the area.

2. In a triangular field the base is 60 chains, and the perpendicular 12 chains: how much does it contain?

3. There is a triangular field, of which the base is 45 rods, and the perpendicular 38 rods: what are its contents?

4. What are the contents of a triangle whose base is 75 chains, and perpendicular 36 chains?

332. A rectangle is a four-sided figure like a square, in which the sides are perpendicular to each other, but the adjacent sides are not equal.

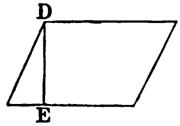


* All the references are to Davies' Legendre.

331. What is a triangle? What is the base of a triangle? What the altitude? What is a right-angled triangle? Which side is the hypothenuse? What is the area of a triangle equal to?

332. What is a rectangle?

333. A parallelogram is a four-sided figure which has its opposite sides equal and parallel, but its angles not right-angles. The line DE, perpendicular to the base, is called the altitude.



334. To find the area of a square, rectangle, or parallelogram.

Multiply the base by the perpendicular height, and the product will be the area (Bk. IV., Prop. V.)

EXAMPLES.

1. What is the area of a square field, of which the sides are each 66.16 chains?

2. What is the area of a square piece of land, of which the sides are 54 chains?

3. What is the area of a square piece of land, of which the sides are 75 rods each?

4. What are the contents of a rectangular field, the length of which is 80 rods, and the breadth 40 rods?

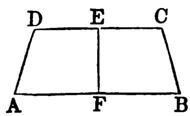
5. What are the contents of a field 80 rods square?

6. What are the contents of a rectangular field, 30 chains long and 5 chains broad?

7. What are the contents of a field, 54 chains long and 18 rods broad?

8. The base of a parallelogram is 542 yards, and the perpendicular height 720 feet: what is the area?

335. A trapezoid is a four-sided figure ABCD, having two of its sides, AB, DC, parallel. The perpendicular EF is called the altitude.



336. To find the area of a trapezoid.

Multiply the sum of the two parallel sides by the altitude,

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333. What is a parallelogram?

334. How do you find the area of a square, rectangle, or parallelogram?

335. What is a trapezoid?

336. How do you find the area of a trapezoid?

divide the product by 2, *and the quotient will be the area* (Bk. IV. Prop. VII).

EXAMPLES.

1. Required the area or contents of the trapezoid ABCD, having given AB = 643.02 feet, DC = 428.48 feet, and EF = 342.32 feet.

ANALYSIS.—We first find the sum of the sides, and then multiply it by the perpendicular height, after which we divide the product by 2, for the area.

OPERATION.

643.02 + 428.48 = 1071.50 = sum of parallel sides. Then, 1071.50 × 342.32 = 366795.88; and $\frac{366795.88}{2}$ = 183397.94 = the area.

2. What is the area of a trapezoid, the parallel sides of which are 24.82 and 16.44 chains, and the perpendicular distance between them 10.30 chains?

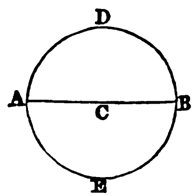
3. Required the area of a trapezoid, whose parallel sides are 51 feet and 37 feet 6 inches, and the perpendicular distance between them 20 feet and 10 inches.

4. Required the area of a trapezoid, whose parallel sides are 41 and 24.5, and the perpendicular distance between them 21.5 yards.

5. What is the area of a trapezoid, whose parallel sides are 15 chains, and 24.5 chains, and the perpendicular height 30.80 chains?

6. What are the contents of a trapezoid, when the parallel sides are 40 and 64 chains, and the perpendicular distance between them 52 chains?

337. A circle is a portion of a plane bounded by a curved line, every point of which is equally distant from a certain point within, called the centre.



The curved line AEBD is called the *circumference;* the point C the *centre;* the line AB passing through the centre a *diameter;* and CB the *radius.*

The circumference AEBD is 3.1416 *times* as great as the diameter AB.

Hence, if the diameter is 1, the circumference will be 3.1416. Therefore, if the diameter is known, the circumference is found by multiplying 3.1416 by the diameter (Bk. V. Prop. XIV).

EXAMPLES.

1. The diameter of a circle is 8 : what is the circumference?

OPERATION.

ANALYSIS.—The circumference is found by simply multiplying 3.1416 by the diameter.

$$\begin{array}{r} 3.1416 \\ 8 \\ \hline \textit{Ans. } 25.1328 \end{array}$$

2. The diameter of a circle is 186 : what is the circumference?

3. The diameter of a circle is 40 : what is the circumference?

4. What is the circumference of a circle whose diameter is 57?

338. Since the circumference of a circle is 3.1416 times as great as the diameter, it follows, that if the circumference is known, we may find the diameter by dividing it by 3.1416.

EXAMPLES.

1. What is the diameter of a circle whose circumference is 157.08?

OPERATION.

ANALYSIS.—We divide the circumference by 3.1416, the quotient 50 is the diameter.

$$\begin{array}{l} 3.1416)157.080(50 \\ \underline{157.080} \end{array}$$

2. What is the diameter of a circle whose circumference is 23304.3888?

3. What is the diameter of a circle whose circumference is 13700?

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337. What is a circle? What is the centre? What is the circumference? What is the diameter? What the radius? How many times greater is the circumference than the diameter? How do you find the circumference when the diameter is known?

338. How do you find the diameter when the circumference is known?

339. To find the area or contents of a circle.

Multiply the square of the diameter by the decimal .7854 (Bk. V. Prop. XII. Cor. 2).

EXAMPLES.

1. What is the area of a circle whose diameter is 12?

ANALYSIS.—We first square the diameter, giving 144, which we then multiply by the decimal .7854: the product is the area of the circle.

OPERATION.

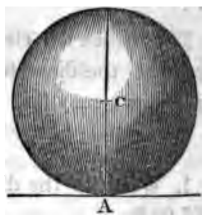
$\overline{12}^2 = 144$

$144 \times .7854 = 113.0976$

Ans. 113.0976

2. What is the area of a circle whose diameter is 5?
3. What is the area of a circle whose diameter is 14?
4. How many square yards in a circle whose diameter is 3½ feet?

340. A sphere is a figure terminated by a curved surface, all the points of which are equally distant from a certain point within, called the centre. The line AD, passing through its centre C, is called the diameter of the sphere, and AC its radius.



341. To find the surface of a sphere,

Multiply the square of the diameter by 3.1416 (Bk. VIII. Prop. X. Cor.)

EXAMPLES.

1. What is the surface of a sphere whose diameter is 6?

ANALYSIS.—We simply multiply the number 3.1416 by the square of the diameter: the product is the surface.

OPERATION.

3.1416

$6^2 =$ 36

Ans. 113.076

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339. How do you find the area of a circle?
340. What is a sphere? What is a diameter? What is a radius?
341. How do you find the surface of a sphere?

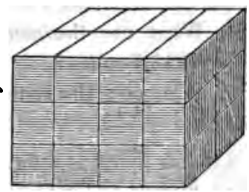
2. What is the surface of a sphere whose diameter is 14?

3. Required the number of square inches in the surface of a sphere whose diameter is 3 feet or 36 inches.

4. Required the area of the surface of the earth, its mean diameter being 7918.7 miles.

MENSURATION OF VOLUMES.

342. A SOLID or VOLUME is a figure having three dimensions; length, breadth, and thickness. It is measured by a *cube* called the *cubic unit* or *unit of volume*.



3 feet = 1 yard.

A CUBE is a figure having six equal faces, which are squares. If the sides of the cube be each one foot long, the figure is called a cubic foot. But when the sides of the cube are one yard, as in the figure, it is called a cubic yard. The base of the cube, which is the face on which it stands, contains $3 \times 3 = 9$ square feet. Therefore, 9 cubes, of one foot each, can be placed on the base. If the figure were one foot high it would contain 9 cubic feet; if it were 2 feet high it would contain two tiers of cubes, or 18 cubic feet; and if it were 3 feet high, it would contain three tiers, or 27 cubic feet. Hence, *the contents of such a figure are equal to the product of its length, breadth, and height.*

343. To find the contents of a sphere,

Multiply the surface by the diameter, and divide the product by 6, *the quotient will be the contents* (Bk. VIII. Prop. XIV. Sch. 3).

EXAMPLES.

1. What are the contents of a sphere whose diameter is 12?

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342. What is a volume? What is a cube? What is a cubic foot? What is a cubic yard? How many cubic feet in a cubic yard? What are the contents of a figure of three dimensions equal to?

343. *How do you find* the contents of a sphere?

ANALYSIS.—We first find the surface by multiplying the square of the diameter by 3.1416. We then multiply the surface by the diameter, and divide the product by 6.

OPERATION.

	$\overline{12}^2 = 144$
multiply by	3.1416
surface	$= 452.3904$
diameter	12
	6)5428.6848
solidity	$= 904.7808$

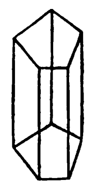
2. What are the contents of a sphere whose diameter is 8?

3. What are the contents of a sphere whose diameter is 16 inches?

4. What are the contents of the earth, its mean diameter being 7918.7 miles?

5. What are the contents of a sphere whose diameter is 12 feet?

344. A prism is a figure whose ends are equal plane figures and whose faces are parallelograms. The sum of the sides which bound the base is called the perimeter of the base, and the sum of the parallelograms which bound the figure is called the convex surface.



345. To find the convex surface of a right prism.

Multiply the perimeter of the base by the perpendicular height, and the product will be the convex surface (Bk. VII. Prop. I).

EXAMPLES.

1. What is the convex surface of a prism whose base is bounded by five equal sides, each of which is 35 feet, the altitude being 52 feet?

2. What is the convex surface when there are eight equal sides, each 15 feet in length, and the altitude is 12 feet?

---

344. What is a prism? What is the perimeter of the base? What is the convex surface?

345. How do you find the convex surface of a prism?

346. How do you find the contents of a prism?

346. To find the contents of a prism,

Multiply the area of the base by the altitude, and the product will be the contents (Bk. VII. Prop. XIV).

EXAMPLES.

1. What are the contents of a square prism, each side of the square which forms the base being 16, and the altitude of the prism 30 feet?

OPERATION.

ANALYSIS.—We first find the area of the square which forms the base, and then multiply by the altitude.

$$\overline{16}^2 = 256$$
$$\underline{30}$$
Ans. $\overline{7680}$

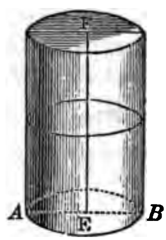
2. What are the contents of a cube, each side of which is 48 inches?

3. How many cubic feet in a block of marble, of which the length is 3 feet 2 inches, breadth 2 feet 8 inches, and height or thickness 5 feet?

4. How many gallons of water will a cistern contain, whose dimensions are the same as in the last example?

5. Required the solidity of a triangular prism, whose height is 20 feet, and area of the base 691.

347. A CYLINDER is a round body with circular ends. The line EF is called the axis or altitude, and the circular surface the *convex surface* of the cylinder.



348. To find the convex surface of a cylinder,

Multiply the circumference of the base by the altitude, and the product will be the convex surface (Bk. VIII. Prop. I).

347. What is a cylinder? What is the axis or altitude? What is the convex surface?

348. How do you find the convex surface?

EXAMPLES.

1. What is the convex surface of a cylinder, the diameter of whose base is 20 and the altitude 40?

ANALYSIS.—We first multiply 3.1416 by the diameter, which gives the circumference of the base. Then, multiplying by the altitude, we obtain the convex surface.

OPERATION.

	3.1416
	20
	62.8320
	40
Ans.	2513.2800

2. What is the convex surface of a cylinder whose altitude is 28 feet and the circumference of its base 8 feet 4 inches?

3. What is the convex surface of a cylinder, the diameter of whose base is 15 inches and altitude 5 feet?

4. What is the convex surface of a cylinder, the diameter of whose base is 40 and altitude 50 feet?

349. To find the volume of a cylinder,

Multiply the area of the base by the altitude: the product will be the contents or volume (Bk. VIII. Prop. II).

EXAMPLES.

1. Required the contents of a cylinder of which the altitude is 11 feet, and the diameter of the base 16 feet.

ANALYSIS.—We first find the area of the base, and then multiply by the altitude: the product is the solidity.

OPERATION.

	$16^2 = 256$
	.7854
area base,	201·0624
	11
	2111.6864

2. What are the contents of a cylinder, the diameter of whose base is 40, and the altitude 29?

3. What are the contents of a cylinder, the diameter of whose base is 24, and the altitude 30?

4. What are the contents of a cylinder, the diameter of whose base is 32, and altitude 12?

5. What are the contents of a cylinder, the diameter of whose base is 25 feet, and altitude 15?

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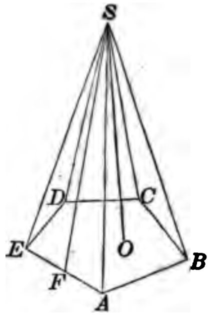
349. How do you find the contents of a cylinder?



350. A Pyramid is a figure formed by several triangular planes united at the same point S, and terminating in the different sides of a plane figure, as ABCDE. The altitude of the pyramid is the line SO, drawn perpendicular to the base.

351. To find the contents of a pyramid.

Multiply the area of the base by the altitude, and divide the product by 3 (Bk. VII., Prop. XVII).

EXAMPLES.

1. Required the contents of a pyramid, the area of whose base is 86, and the altitude 24.

OPERATION.
86
24
3)2064
Ans. 688

Analysis.—We simply multiply the area of the base 86, by the altitude 24, and then divide the product by 3.

2. What are the contents of a pyramid, the area of whose base is 365, and the altitude 36?

3. What are the contents of a pyramid, the area of whose base is 207, and altitude 36?

4. What are the contents of a pyramid, the area of whose base is 562, and altitude 30?

5. What are the contents of a pyramid, the area of whose base is 540, and altitude 32?

6. A pyramid has a rectangular base, the sides of which are 50 and 24; the altitude of the pyramid is 36: what are its contents?

7. A pyramid with a square base, of which each side is 15, has an altitude of 24: what are its contents?

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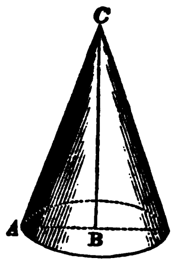
350. What is a pyramid? What is the altitude of a pyramid?

351. How do you find the contents of a pyramid?



352. A Cone is a round body with a circular base, and tapering to a point called the *vertex*. The point C is the vertex, and the line CB is called the axis or *altitude*.

353. To find the contents of a cone.

Multiply the area of the base by the altitude, and divide the product by 3; *or, multiply the area of the base by one-third of the altitude* (Bk. VIII., Prop. V.)

EXAMPLES.

1. Required the contents of a cone, the diameter of whose base is 6, and the altitude 11.

OPERATION.

$$6^2 = 36$$
$$36 \times .7854 = 28.2744$$
$$\underline{\quad\quad 11}$$
$$3)\overline{311.0184}$$
$$Ans.\ \ 103.6728$$

Analysis.—We first square the diameter, and multiply it by .7854, which gives the area of the base. We next multiply by the altitude, and then divide the product by 3.

2. What are the contents of a cone, the diameter of whose base is 36, and the altitude 27?

3. What are the contents of a cone, the diameter of whose base is 35, and the altitude 27?

4. What are the contents of a cone, whose altitude is 27 feet, and the diameter of the base 20 feet?

GAUGING.

354. Cask-Guaging is the method of finding the number of gallons which a cask contains, by measuring the external dimensions of the cask.

---

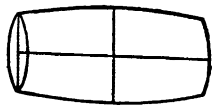
352. What is a cone? What is the vertex? What is the axis?

353. How do you find the contents of a cone?

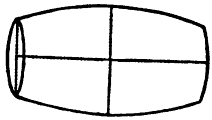
354. What is cask-gauging?

355. Casks are divided into four varieties, according to the curvature of their sides. To which of the varieties any cask belongs, must be judged of by inspection.

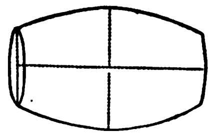
1. Of the least curvature.



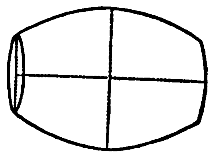
2d Variety.



3d Variety.



4th Variety.



356. The first thing to be done is to find the mean diameter. To do this,

Divide the head diameter by the bung diameter, and find the quotient in the first column of the following table, marked Qu. *Then if the bung diameter be multiplied by the number on the same line with it, and in the column answering to the proper variety, the product will be the true mean diameter, or the diameter of a cylinder having the same altitude and the same contents with the cask proposed.*

---

355. Into how many varieties are casks divided?

356. How do you find the mean diameter?

Qu.	1stVar.	2dVar.	3dVar.	4thVar.	Qu.	1st Var.	2d Var.	3d Var.	4th Var.
50	8660	8465	7905	7637	76	9270	9227	8881	8827
51	8680	8493	7937	7681	77	9296	9258	8944	8874
52	8700	8520	7970	7725	78	9324	9290	8967	8922
53	8720	8548	8002	7769	79	9352	9320	9011	8970
54	8740	8576	8036	7813	80	9380	9352	9055	9018
55	8760	8605	8070	7858	81	9409	9383	9100	9066
56	8781	8633	8104	7902	82	9438	9415	9144	9114
57	8802	8662	8140	7947	83	9467	9446	9189	9163
58	8824	8690	8174	7992	84	9496	9478	9234	9211
59	8846	8720	8210	8037	85	9526	9510	9280	9260
60	8869	8748	8246	8082	86	9556	9542	9326	9308
61	8892	8777	8282	8128	87	9586	9574	9372	9357
62	8915	8806	8320	8173	88	9616	9606	9419	9406
63	8938	8835	8357	8220	89	9647	9638	9466	9455
64	8962	8865	8395	8265	90	9678	9671	9513	9504
65	8986	8894	8433	8311	91	9710	9703	9560	9553
66	9010	8924	8472	8357	92	9740	9736	9608	9602
67	9034	8954	8511	8404	93	9772	9768	9656	9652
68	9060	8983	8551	8450	94	9804	9801	9704	9701
69	9084	9013	8590	8497	95	9836	9834	9753	9751
70	9110	9044	8631	8544	96	9868	9867	9802	9800
71	9136	9074	8672	8590	97	9901	9900	9851	9850
72	9162	9104	8713	8637	98	9933	9933	9900	9900
73	9188	9135	8754	8685	99	9966	9966	9950	9950
74	9215	9166	8796	8732	100	10000	10000	10000	10000
75	9242	9196	8838	8780					

EXAMPLES.

1. Supposing the diameters to be 32 and 24, it is required to find the mean diameter for each variety.

Dividing 24 by 32, we obtain .75; which being found in the column of quotients, opposite thereto stand the numbers,

$$\left.\begin{matrix} .9242 \\ .9196 \\ .8838 \\ .8780 \end{matrix}\right\}$$ which being each multiplied by 82, produce respectively, $$\left\{\begin{matrix} 29.5744 \\ 29.4272 \\ 28.2816 \\ 28.0960 \end{matrix}\right\}$$ for the corresponding mean diameters required.

2. The head diameter of a cask is 26 inches, and the bung diameter 3 feet 2 inches: what is the mean diameter, the cask being of the third variety?

3. The head diameter is 22 inches, the bung diameter 34 *inches:* what is the mean diameter of a cask of the fourth variety?

357. Having found the mean diameter, we multiply the square of the mean diameter by the decimal .7854, and the product by the length; this will give the contents in cubic inches. Then, if we divide by 231, we have the contents in wine gallons (see Art. 414), or if we divide by 282, we have the contents in beer gallons (Art. 415).

ANALYSIS.—For wine measure, we multiply the length by the square of the mean diameter, then by the decimal .7854, and divide by 231.

OPERATION.

$l \times d^2 \times \frac{.7854}{231} =$
$l \times d^2 \times .0034.$

If then, we divide the decimal .7854 by 231, the quotient carried to four places of decimals is .0034, and this decimal multiplied by the square of the mean diameter and by the length of the cask, will give the contents in wine gallons.

For similar reasons, the content is found in beer gallons by multiplying together the length, the square of the mean diameter, and the decimal .0028.

OPERATION.

$l \times d^2 \times \frac{.7854}{282} =$
$l \times d^2 \times .0028.$

Hence, for gauging or measuring casks,

Multiply the length by the square of the mean diameter; then multiply by 34 *for wine, and by* 28 *for beer measure, and point off in the product four decimal places. The product will then express gallons and the decimals of a gallon.*

1. How many wine gallons in a cask, whose bung diameter is 36 inches, head diameter 30 inches, and length 50 inches; the cask being of the first variety?

2. What is the number of beer gallons in the last example?

3. How many wine, and how many beer gallons in a cask whose length is 36 inches, bung diameter 35 inches, and head diameter 30 inches, it being of the first variety?

4. How many wine gallons in a cask of which the head diameter is 24 inches, bung diameter 36 inches, and length 3 feet 6 inches, the cask being of the second variety?

---

357. How do you find the contents in cubic inches? How do you find the contents in wine gallons? In beer gallons?

OF THE MECHANICAL POWERS.

358. There are six simple machines, which are called *Mechanical powers.* They are, the *Lever*, the *Pulley*, the *Wheel* and *Axle*, the *Inclined Plane*, the *Wedge*, and the *Screw.*

359. To understand the nature of a machine, four things must be considered.

1st. The power or force which acts. This consists in the efforts of men or horses, of weights, springs, steam, &c.

2d. The resistance which is to be overcome by the power. This generally is a weight to be moved.

3d. We are to consider the centre of motion, or *fulcrum*, which means a prop. The prop or fulcrum is the point about which all the parts of the machine move.

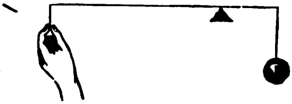
4th. We are to consider the respective velocities of the power and resistance.

360. A machine is said to be in equilibrium when the resistance exactly balances the power, in which case all the parts of the machine are at rest, or in uniform motion.

We shall first examine the lever.

361. The *Lever*, is a bar of wood or metal, which moves around a fixed point, called the fulcrum. There are three kinds of levers.

1st. When the fulcrum is between the weight and the power.

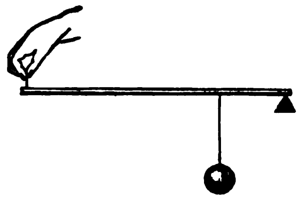


358. How many simple machines are there? What are they called?

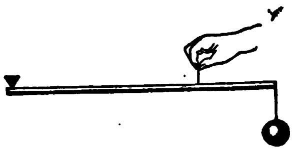
359. What things must be considered, in order to understand the power of a machine?

360. When is a machine said to be in equilibrium?

361. What is a lever? How many kinds of levers are there? Describe the first kind? Where is the weight placed in the second kind? Where is the power placed in the third kind?



2d. When the weight is between the power and the fulcrum.



3d. When the power is between the fulcrum and the weight.

The perpendicular distance from the fulcrum to the directions of the weight and power, are called the *arms* of the lever.

362. An equilibrium is produced in all the levers, when the weight multiplied by its distance from the fulcrum is equal to the product of the power multiplied by its distance from the fulcrum. That is,

The weight is to the power, as the distance from the power to the fulcrum, is to the distance from the weight to the fulcrum.

EXAMPLES.

1. In a lever of the first kind, the fulcrum is placed at the middle point: what power will be necessary to balance a weight of 40 pounds?

2. In a lever of the second kind, the weight is placed at the middle point: what power will be necessary to sustain a weight of 50*lbs.*?

3. In a lever of the third kind, the power is placed at the middle point: what power will be necessary to sustain a weight of 25*lbs.*?

4. A lever of the first kind is 8 feet long, and a weight of 60*lbs.* is at a distance of 2 feet from the fulcrum: what power will be necessary to balance it?

---

362. When is an equilibrium produced in all the levers? What is then the *proportion* between the weight and power?

5. In a lever of the first kind, that is 6 feet long, a weight of 200*lbs.* is placed at 1 foot from the fulcrum: what power will balance it?

6. In a lever of the first kind, like the common steelyard, the distance from the weight to the fulcrum is one inch: at what distance from the fulcrum must the poise of 1*lb.* be placed, to balance a weight of 1*lb.*? A weight of $1\frac{1}{2}$*lbs.*? Of 2*lbs.*? Of 4*lbs.*?

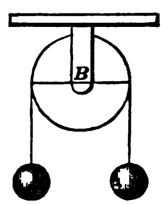
7. In a lever of the third kind, the distance from the fulcrum to the power is 5 feet, and from the fulcrum to the weight 8 feet: what power is necessary to sustain a weight of 40*lbs.*?

8. In a lever of the third kind, the distance from the fulcrum to the weight is 12 feet, and to the power 8 feet: what power will be necessary to sustain a weight of 100*lbs.*?

363. Remarks.—In determining the equilibrium of the lever, we have not considered its weight. In levers of the first kind, the weight of the lever generally adds to the power, but in the second and third kinds, the weight goes to diminish the effect of the power.

In the previous examples, we have stated the circumstances under which the power will exactly sustain the weight. In order that the power may overcome the resistance, it must of course be somewhat increased. The lever is a very important mechanical power, being much used, and entering, indeed, into most other machines.

OF THE PULLEY.

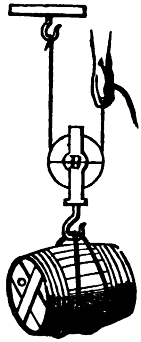


364. The pulley is a wheel, having a groove cut in its circumference, for the purpose of receiving a cord which passes over it. When motion is imparted to the cord, the pulley turns around its axis, which is generally supported by being attached to a beam above.

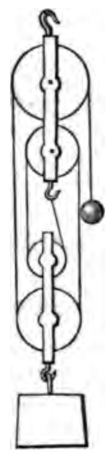
363. Has the weight been considered in determining the equilibrium of the lever? In a lever of the first kind, will the weight increase or diminish the power? How will it be in the two other kinds?

364. What is a pulley?

365. Pulleys are divided into two kinds, fixed pulleys and movable pulleys. When the pulley is fixed, it does not increase the power which is applied to raise the weight, but merely changes the direction in which it acts.



366. A movable pulley gives a mechanical advantage. Thus, in the movable pulley, the hand which sustains the cask does not actually support but one-half the weight of it; the other half is supported by the hook to which the other end of the cord is attached.



367. If we have several movable pulleys, the advantage gained is still greater, and a very heavy weight may be raised by a small power. A longer time, however, will be required, than with the single pulley. It is, indeed, a general principle in machines, that *what is gained in power, is lost in time;* and this is true for all machines. There is also an actual loss of power, viz., the resistance of the machine to motion, arising from the rubbing of the parts against each other, which is called the *friction* of the machine. This varies in the different machines, but must always be allowed for, in calculating the power necessary to do a given work. It would be wrong, however, to suppose that

365. How many kinds of pulleys are there? Does a fixed pulley give any increase of power?

366. Does a movable pulley give any mechanical advantage? In a *single movable pulley,* how much less is the power than the weight?

367. Will an advantage be gained by several movable pulleys? State

the loss was equivalent to the gain, and that no advantage is derived from the mechanical powers. We are unable to augment our strength, but, by the aid of science we so divide the resistance, that by a continued exertion of power, we accomplish that which it would be impossible to effect by a single effort.

If in attaining this result, we sacrifice time, we cannot but see that it is most advantageously exchanged for power.

368. It is plain, that in the movable pulley, all the parts of the cord will be equally stretched, and hence, each cord running from pulley to pulley, will bear an equal part of the weight; consequently,

The power will always be equal to the weight divided by the number of cords which reach from pulley to pulley.

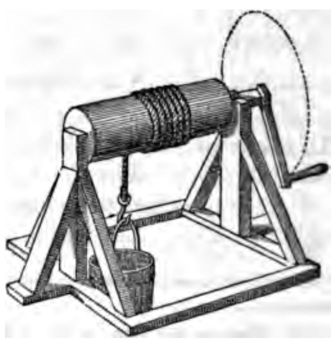
EXAMPLES.

1. In a single immovable pulley, what power will support a weight of 60*lbs.*?

2. In a single movable pulley, what power will support a weight of 80*lbs.*?

3. In two movable pulleys with 4 cords, (see last fig.,) what power will support a weight of 100*lbs.*?

WHEEL AND AXLE.



369. This machine is composed of a wheel or crank—firmly attached to a cylindrical axle. The axle is supported at its ends by two pivots, which are of less diameter than the axle around which the rope is coiled, and which turn freely about the points of support. In order to balance the weight, we must have,

The power to the weight, as the radius of the axle, to the length of the crank, or radius of the wheel.

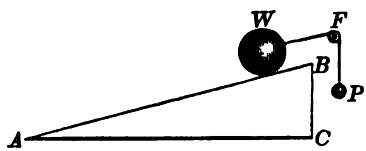
EXAMPLES.

1. What must be the length of a crank or radius of a wheel, in order that a power of 40*lbs.* may balance a weight of 600*lbs.* suspended from an axle of 6 inches radius?

2. What must be the diameter of an axle, that a power of 100*lbs.* applied at the circumference of a wheel of 6 feet diameter may balance 400*lbs.*?

INCLINED PLANE.

380. The inclined plane is nothing more than a slope or declivity, which is used for the purpose of raising weights. It is not difficult to see that a weight can be forced up an inclined plane, more easily than it can be raised in a vertical line. But in this, as in the other machines, the advantage is obtained by a partial loss of power.



Thus, if a weight W, be supported on the inclined plane ABC, by a cord passing over a pulley at F, and the cord from the pulley to the weight be parallel to the length of the plane AB, the power P, will balance the weight W, when

P : W :: height BC : length AB.

---

the general principle in machines. What does the actual loss of power arise from? What is this rubbing called? Does this vary in different machines?

368. In the movable pulley, what proportion exists between the cord and the weight?

369. Of what is the machine called the wheel and axle, composed? How is the axle supported? Give the proportion between the power and the weight.

370. What is an inclined plane? What proportion exists between the power and weight when they are in equilibrium?

It is evident, that the power ought to be less than the weight, since a part of the weight is supported by the plane: hence,

The power is to the weight as the height of the plane is to its length.

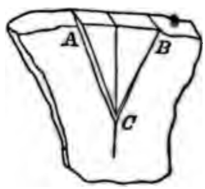
EXAMPLES.

1. The length of a plane is 30 feet, and its height 6 feet: what power will be necessary to balance a weight of 200*lbs.*?

2. The height of a plane is 10 feet, and the length 20 feet: what weight will a power of 50*lbs.* support?

3. The height of a plane is 15 feet, and length 45 feet: what power will sustain a weight of 180*lbs.*?

THE WEDGE.



381. The wedge is composed of two inclined planes, united together along their bases, and forming a solid ACB. It is used to cleave masses of wood or stone. The resistance which it overcomes is the attraction of cohesion of the body which it is employed to separate. The wedge acts principally by being struck with a hammer, or mallet, on its head, and very little effect can be produced with it, by mere pressure.

All cutting instruments are constructed on the principle of the inclined plane or wedge. Such as have but one sloping edge, like the chisel, may be referred to the inclined plane, and such as have two, like the axe and the knife, to the wedge.

Half the thickness of the head of the wedge is to the length of one of its sides, as the power which acts against its head to the effect produced at its side.

EXAMPLES.

1. If the head of a wedge is 4 inches thick, and the length

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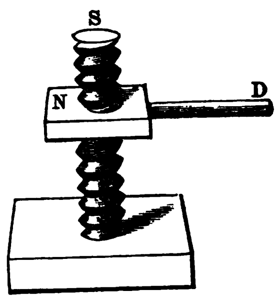
371 What is the wedge? What is it used for? What resistance is it used to overcome?

of one of its sides 12 inches, what will measure the effect of a force denoted by 96 pounds?

2. If the head of a wedge is 6 inches thick, the length of the side 27 inches, and the force applied measure by 250 pounds, what will be the measure of the effect?

THE SCREW.

381. The screw is composed of two parts—the screw S, and the nut N.



The screw S, is a cylinder with a spiral projection winding around it. The nut N is perforated to admit the screw, and within it is a groove into which the thread of the screw fits closely.

The handle D, which projects from the nut, is a lever which works the nut upon the screw. The power of the screw depends on the distance between the threads. The closer the threads of the screw, the greater will be the power; but then the number of revolutions made by the handle D, will also be proportionably increased; so that we return to the general principle—what is gained in power is lost in time. The power of the screw may also be increased by lengthening the lever attached to the nut.

The screw is used for compression, and to raise heavy weights. It is used in cider and wine-presses, in coining, and for a variety of other purposes.

As the distance between the threads of a screw is to the circumference of the circle described by the power, so is the power employed to the weight raised.

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381. Of how many parts is the screw composed? Describe the screw. What is the thread? What is the nut? What is the handle used for? To what uses is the screw applied? What is the power of the screw?

EXAMPLES.

1. If the distance between the threads of a screw is half an inch, and the circumference described by the handle 15 feet, what weight can be raised by a power denoted by 720 pounds?

2. If the threads of a screw are one-third of an inch apart, and the handle is 12 feet long, what power must be applied to sustain 2 tons?

3. What force applied to the handle of a screw 10 feet long, with threads 1 inch apart, working on a wedge whose head is 5 inches, and length of side 30 inches, will produce an effect measured by 10000*lbs.*?

4. If a power of 300 pounds applied at the end of a lever 15 feet long will sustain a weight of 282744*lbs.*, what is the distance between the threads of the screw?

QUESTIONS IN NATURAL PHILOSOPHY.

UNIFORM MOTION.

382. If a moving body passes over equal spaces in equal successive small portions of time, it is said to move with uniform motion, or uniformly.

383. The velocity of a moving body is measured by the space passed over in a second of time.

384. The space passed over in any time is equal to the product of the velocity multiplied by the number of seconds in the time.

If we denote the velocity by V, the space passed over by S, and the time by T, we have

$$S = V \times T.$$

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382. What is a uniform motion?

383. What is the velocity of a moving body?

384. To what is the space passed over in a unit of time equal? What is the space passed over equal to, in uniform motion?

EXAMPLES.

1. A steamboat moves with a velocity of 23 feet: what space does it pass over in $1\frac{1}{2}$ hours?

2. A locomotive is moving with a velocity of 32 feet: what distance will it travel in 3 minutes?

3. A horse travels uniformly a distance of 12 miles with a velocity of 6 feet: what time does he require to perform the journey?

4. A carriage performs a journey of 15 miles in $2\frac{3}{4}$ hours: with what velocity does it move?

5. The hammer of a pile-driver is moved upward a distance of 35 feet with a velocity of $1\frac{1}{2}$ feet: what time is required to raise it?

6. A ton of coal is raised from a mine 1000 feet deep in $3\frac{3}{4}$ minutes: with what velocity does it move?

7. A vessel containing a criminal, after leaving a port, sailed with a daily speed of 170 miles; four days after, a clipper was dispatched in pursuit, and sailed at a daily rate of 275 miles: in what time did the clipper overtake the vessel?

8. A bird flew a distance of 100 miles in 11 hours: with what velocity did it travel?

9. Sound moves with a velocity of 1127 feet. If the report of a gun was heard 31.3 seconds after the flash was seen, what distance was the gun from the observer?

10. A hurricane moves with a velocity of 95 feet: what time does it take to move through 3 degrees of latitude, the degree being estimated at $69\frac{1}{6}$ miles?

11. The velocity of light has been found, by astronomical observations, and by experiments made in France, to be 191,300 miles: what time will it occupy to traverse the mean distance of the earth from the sun, or 95000000 of miles?

12. If a message sent by electro-magnetic telegraph 2300 miles requires 14 seconds for its transmission, what is the velocity of the magnetic current in this telegraph line?

LAWS OF FALLING BODIES.

385. A body falling vertically downward in a vacuum, falls through $16\frac{1}{12}$*ft.* during the first second after leaving its place of rest, $48\frac{1}{4}$*ft.* during the second second, $80\frac{5}{12}$*ft.* the third second, and so on: the spaces forming an arithmetical progression of which the common difference is $32\frac{1}{6}$*ft.*, or double the space fallen through during the first second. This number is called the measure of the force of gravity, and is denoted by g.

386. It is seen from the above that the velocity of a body is continually increasing. If H denote the height fallen through, T, the time, V, the velocity acquired, and g, the force of gravity, the following formulas have been found to express the relations between these quantities:

$$V = g \times T \quad . \quad . \quad . \quad (1).$$
$$V^2 = 2g \times H \quad . \quad . \quad . \quad (2).$$
$$H = \tfrac{1}{2}V \times T \quad . \quad . \quad . \quad (3).$$
$$H = \tfrac{1}{2}g \times T^2 \quad . \quad . \quad . \quad (4).$$

From which we see,

1st. *That the velocity acquired at the end of any time, is equal to the force of gravity* ($32\frac{1}{6}$) *multiplied by the time.*

2d. *That the square of the velocity is equal to twice the force of gravity multiplied by the height; or, the velocity is equal to the square root of that quantity.*

3d. *That the space fallen through is equal to one-half the velocity multiplied by the time.*

4th. *That the space fallen through is equal to one-half the force of gravity multiplied by the square of the time.*

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385. If a body falls vertically, in a vacuum, how far will it fall in the first second of time? How far on the second second second? In the third? What is the common difference of the spaces? What is the measure of the force of gravity?

386. How does the velocity of a falling body change? What is the velocity acquired at the end of any time equal to? What is the space fallen through equal to?

387. If a body is thrown vertically upwards in a vacuum, its motion will be continually retarded by the action of gravitation. It will finally reach the highest point of its ascent, and then begin to descend. The height to which it will rise may be found by the second formula in the preceding paragraph, when the velocity with which it is projected upward is known; for the times of ascent and descent will be equal.

388. The above laws are only approximately true for bodies falling through the air, in consequence to its resistance. We may measure the depths of wells or mines and the heights of elevated objects approximately by using dense bodies, as leaden bullets or stones, which present small surface to the air.

EXAMPLES.

1. A body has been falling 12 seconds: what space has it described in the last second, and what in the whole time?

2. A body has been falling 15 seconds: find the space described and the velocity acquired.

3. How far must a body fall to acquire a velocity of 120 feet?

4. How many seconds will it take a body to fall through a space of 100 feet?

5. Find the space through which a heavy body falls in 10 seconds, and the velocity acquired.

6. How far must a body fall to acquire a velocity of 1000 feet?

7. A stone is dropped into a well and strikes the water in 32 seconds: what is the depth of the well?

8. A stone is dropped from the top of a bridge and strikes the water in 2.5 seconds: what is the height of the bridge?

9. A body is thrown vertically upward with a velocity of 160 feet: what height will it reach, and what will be the time of ascent?

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387. How far will a body ascend when projected upwards?

388. *Are the above laws perfectly or only approximately true?*

10. An arrow shot perpendicularly upwards returned again in 10 seconds. Required the velocity with which it was shot, and the height to which it rose.

11. If a body falls freely in vacuum, what will be its velocity after 45 seconds of fall?

12. During how many seconds must a body fall in a vacuum to acquire a velocity of 1970 feet, which is that of a cannon ball?

13. What time is required for a body to fall in a vacuum, from an elevation of 3280 feet?

14. From what height must a body fall to acquire a velocity of 984 feet?

15. A rocket is projected vertically upward with a velocity of 386 feet: after what time will it begin to fall, and to what height will it rise?

SPECIFIC GRAVITY.

389. The SPECIFIC GRAVITY of a body is the weight of a unit of volume. Distilled rain water is the standard for measuring the specific gravity of bodies. Thus, 1 cubic foot of distilled rain water weighs 1000 ounces avoirdupois. If a piece of stone, *of the same volume*, weighs 2500 ounces, its specific gravity is 2.5; that is, the stone is 2.5 times as heavy as water.

If, then, we denote the standard by 1, the specific gravity of all other bodies will be expressed in terms of this standard: and if we multiply the number denoting the specific gravity of any body by 1000, the product will be the weight in ounces of 1 cubic foot of that body.

If any body be weighed in air and then in water, it will weigh less in water than in air. The difference of the weights will be equal to the sustaining force of the water, which is found to be equal to the weight of an equal volume of water: hence,

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389. What is the specific gravity of a body? What is the standard for measuring the specific gravity of a body? What is the numerical value of the cubic foot of a body? How do you find the specific gravity of a body?

If we know or can find the weight of a body in air and in water, the difference of these weights will be equal to that of an equal volume of water; and the weight of the body in air divided by this difference will be the measure of the specific gravity of the body, compared with water as a standard.

TABLE

OF SPECIFIC GRAVITIES.—WATER 1.

NAMES OF BODIES.	SPEC. GRAV.	NAMES OF BODIES.	SPEC. GRAV.
METALS.		Porphyry, . . .	2.60
Platinum, ...	21.000	Sandstone, . .	2.50
Gold, ...	19.500	Brick, ...	1.86
Quicksilver, . . .	13.500		
Lead, ...	11.350	WOODS.	
Silver, ...	10.51	Oak, fresh felled,	1.049
Copper, ...	8.800	White Willow, .	0.9859
Bronze, ...	8.758	Box, ...	0.9822
Brass, ...	8.000	Elm, ...	0.9476
Steel, ...	7.800	Hanbeam, . .	0.9452
Iron, ...	7.500	Larch, ...	0.9206
Tin, ...	7.291	Pine, ...	0.9121
Zinc, ...	7.215	Maple, ...	0.9036
		Ash, ...	0.9036
BUILDING STONES.		Birch, ...	0.9012
Hornblende, . . .	3.10	Fir, ...	0.8941
Basalt, ...	3.10	Horse Chestnut, .	0.8614
Alabaster, . . .	3.00		
Syenite, ...	3.00	SOLID BODIES.	
Dolerite, ...	2.93	Common earth, .	1.480
Gniess, ...	2.90	Moist sand, . .	2.050
Quartz, ...	2.75	Clay, ...	2.150
Limestone, . . .	2.72	Flint, ...	2.542
Phonolite, ...	2.69	Ice, ...	0.926
Granite, ...	2.66	Lime, ...	1.842
Stone for building, .	2.62	Tallow, . . .	0.942
Trachytæ, . . .	2.60	Wax, ...	0.969

By inspecting this Table, we see the weight of each body compared with an equal volume of water. Thus, platina is 21 times as heavy as water; gold, 19 times as heavy; iron, 7½ times as heavy, &c.

EXAMPLES ILLUSTRATING SPECIFIC GRAVITY.

1. A piece of copper weighs 93 grains in air, and $82\frac{1}{2}$ grains in water: what is its specific gravity?

2. How many cubic feet are there in 2240 pounds of dry oak, of which the specific gravity is .925, a cubic foot of standard water weighing 1000 ounces?

3. A piece of pumice stone weighs in air 50 ounces, and when it is connected with a piece of copper which weighs 390 ounces in air, and 345 ounces in water, the compound weighs 344 ounces in water: what is the specific gravity of the stone?

4. A prism of ice having 6 rectangular faces, and of which the height is 20.45 yards, the breadth 15.75 yards, and the height 10.5 yards, floats on the sea; the specific gravity of the ice is .930, and that of the sea water 1.026: what is the height of the prism above the surface of the water?

5. A vessel in a dock was found to displace 6043 cubic feet of water: what was the weight of the vessel, each cubic foot of the water weighing 63 pounds?

6. A piece of glass was found to weigh in the air 33 ounces, and in the water 21 ounces: what was its specific gravity?

7. A piece of zinc weighed in the air 17 pounds, and lost when weighed in water 2.35 pounds: what was its specific gravity?

8. If a piece of glass weighed in water loses 318 ounces of its weight, and weighed in alcohol loses 250 ounces, what is the specific gravity of the alcohol?

9. A flask filled with distilled water weighed 14 ounces; filled with brandy, it weighed 13.25 ounces; the flask itself weighed 8 ounces: what was the specific gravity of the brandy?

10. What is the weight of a cubic foot of statuary marble, of which the specific gravity is 2.837, the cubic foot of water weighing 1000 ounces?

11. A jar containing air weighed 24 ounces 33 grains; the air was then excluded, and the jar weighed 24 ounces; the jar being then filled with oxygen gas weighed 24 ounces 36.4

grains: what was the specific gravity of the oxygen, the air being taken as the standard?

12. A cylindrical vase having a base whose interior diameter is 4 inches, stands upon a horizontal plane: 26.2 pounds of mercury is poured into the vase. Required the height to which the liquid will rise, the specific gravity of mercury being 13.596.

13. A piece of alabaster weighs in the air 7.55 grains, in the water 5.17 grains, and in another liquid 6.35 grains: what is the specific gravity of the alabaster and of the liquid?

14. What effort will be required to prevent a cubic inch of platinum, immersed in mercury, from sinking, the specific gravity of the platinum being 21.5, and that of the mercury 13.6?

15. What weight of mercury will a conical vase contain of which the radius of the base is 9 inches and the altitude 34 inches, the specific gravity of the mercury being 13.596?

MARIOTTE'S LAW.

390. This law, which relates to air and all other gases, steam, and all other vapors, was discovered by the abbé Mariotte, a French philosopher, who died in 1684. It will be easily understood from a particular example.

Suppose an upright cylindrical vessel in a vacuum contains a gas which is confined in the vessel by a piston at the upper end. Suppose the gas or vapor fills the whole vessel, and the piston is loaded with a weight of 5 pounds. If now, the piston be loaded with a weight of 10 pounds, the gas will be compressed and occupy only half its former space. If the weight be increased to 15 pounds, the gas will have only one-third of its original volume, and so on. At the same time, the density of the gas or vapor will be doubled, made three times as great, and so on. The law, therefore, may be thus stated:

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390. To what is the volume of a vapor or gas proportional? To what is its density proportional?

The temperature remaining the same, the volume of a gas or vapor is inversely proportional to the pressure which it sustains. Also, the density of a *gas or vapor is directly proportional to the pressure.*

EXAMPLES.

1. A vase contains 4.3 quarts of air, the pressure being 10 pounds: what will be the volume of the air when the pressure is 12.3 pounds, the temperature remaining the same?

2. Under a pressure of 15 pounds to the square inch, a certain quantity of gas occupies a volume of 20 quarts: what pressure must be applied to reduce the volume to 8 quarts?

3. A quart of air weighs 2.6 grains under a pressure of 15 pounds: what will be the weight of a quart if the pressure be reduced to 14.2 pounds?

4. The pressure upon the steam contained in a cylinder is increased from 25 pounds upon the square inch to 47 pounds: what part of the original volume will be occupied?

5. How will the density of the steam in the last example, at the second pressure, compare with that at the first?

6. Eight quarts of hydrogen gas are contained in a vessel and submitted to a pressure of 22 pounds: how many quarts of gas will there be if the pressure is changed 9½ pounds?

APPENDIX.

DIFFERENT KINDS OF UNITS.

391. There are eight kinds of units:

1st. Abstract Units;
2d. Units of Currency or Coin;
3d. Linear Units, or Units of Length;
4th. Units of Surface, or Superficial Units;
5th. Units of Volume, including Cubic Units and Gallons;
6th. Units of Weight;
7th. Units of Time; and
8th. Units of Circular or Angular Measure.

ABSTRACT UNITS.

392. The abstract unit 1 is the base of all numbers, and is called a unit of the first order. The unit 1 ten is a unit of the second order; the unit 1 hundred is a unit of the third order; and so for units of the higher orders. These are abstract numbers formed from the unit 1, according to the scale of tens. All abstract numbers are formed from collections of these units.

UNITS OF CURRENCY.

393. In all civilized and commercial countries, great care is taken to fix a standard value for money, which standard is called the *Unit of Currency.*

In the United States, the unit of currency is 1 dollar; in Great Britain it is 1 pound sterling, equal to $4,84; in France it is 1 franc, equal to $18\frac{3}{4}$ cents. All sums of money are expressed in the unit of currency or in units derived from the unit of currency, and having fixed proportions to it.

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391. How many kinds of units are there in Arithmetic? Name them.

392. What is said of the abstract unit 1? What is a unit of the 2d order? What of the 3d? 4th? 5th? &c. How are these numbers formed from 1?

UNITS OF LENGTH.

394. One of the most important units of measure is that for distances, or for the measurement of length. A practical want has ever been felt of some fixed and invariable standard with which all distances may be compared: such fixed standard has been sought for in nature.

There are two natural standards, either of which affords this desired natural element. Upon one of them, the English have founded their system of measures, from which ours is taken, and upon the other, the French have based their system. These two systems, being the only ones of importance, will be alone considered.

395. FIRST.—The English system of measures, to which ours conforms, is based upon the law of nature, that the *force of gravity is constant at the same point of the earth's surface*, and consequently, that the length of a pendulum which oscillates a certain number of times, in a given period, is also constant. Had this unit been known *before* the adoption and use of a system of measures, it would have formed the natural unit for division, and been the natural base of the system of linear measure. But the foot and inch had long been used as units of linear measure; and hence, the length of the pendulum, the new and invariable standard, was expressed in terms of the known units, and found to be equal to 39.1393 inches. The new unit was therefore declared invariable—to contain 39.1393 equal parts, each of which was called an *inch*; 12 of these parts were declared by act of Parliament to be a *standard foot*, and 36 of them, an *Imperial yard*. The Imperial yard and the standard foot are marked upon a brass bar, at the temperature of $62\frac{1}{2}°$, and these are the linear measures from which

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393. What is a unit of currency? What is the unit of currency in the United States? What in Great Britain? What in France?

394. For what is an invariable standard of length used?

395. What is the standard unit of length in the English system? What in ours?

ours are taken. The comparison has been made by means of a brass scale 82 inches long, manufactured by Troughton in London, and now in the possession of the Treasury Department.

396. SECOND.—The French system of measures is founded upon the principle of the invariability of the length of an arc of the same meridian between two fixed points. By a very minute survey of the length of an arc of the meridian from Dunkirk to Barcelona, the length of a quadrant of the meridian was computed, and it has been decreed by the French law that the *ten-millionth* part of this length shall be regarded as a standard French *mètre*, and from this, by multiplication and division, the entire system of linear measures has been established.

On comparing two scales, very accurately, it has been found that the French mètre is equal to 39.37079 English inches—differing very little from the English yard. This relation enables us to convert all measures in either system into the corresponding measures of the other.

UNITS OF SURFACE.

397. The linear unit having been established, the most convenient UNIT OF SURFACE is the area of a square, one of whose sides is the unit of length. Thus, the units of surface in common use, are

A square inch = a square on 1 inch.
A square foot = 144 square inches.
A square yard = 9 square feet.
A square rod = $30\frac{1}{4}$ square yards.
&c. &c.

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396. What is the standard unit of length in the French system? How was it found? How does the French mètre compare with the Imperial yard?

397. What is the most convenient unit of surface? What are those in common use?

UNITS OF VOLUME.

398. The *Unit of Volume*, for the measurement of solids, is taken equal to the volume of a cube one of whose edges is equal to the linear unit. The units of volume in common use are

A cubic inch = a cube whose edge is 1 inch;
A cubic foot = a cube whose edge is 1 foot = 1728 cubic *in.*
A cubic yard = a cube whose edge is 1 yard = 27 cubic feet.
A perch of stone = $24\frac{3}{4}$ cubic feet;
or a block of stone 1 rood long, 1 foot thick, and $1\frac{1}{2}$ feet wide.

The standard unit of volume for the measurement of liquids is the wine gallon, which contains 231 cubic inches.

The *standard unit of dry measure* is the Winchester bushel, which contains 2150.4 cubic inches, nearly.

UNITS OF WEIGHT.

399. Having fixed an invariable unit of length, we passed easily to an invariable unit of surface, and then, to an invariable unit of volume. We wish now to define an invariable unit of weight.

It has been found that distilled rain water is the most invariable substance; hence, this, at a given temperature, has been adopted as the standard.

We have two units of weight, the avoirdupois pound, and the pound troy.

The standard avoirdupois pound is the weight of 27.701554 cubic inches of distilled water.

The *standard Troy pound* is the weight of 22.794422 cubic

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398. What is the unit of volume for the measurement of solids? What are those in common use? What is the standard unit for the measurement of liquids? What for dry measure?

399. What is used as a standard in fixing the units of weight? How many units of weight have we? How is the standard avoirdupois pound determined? How the Troy pound? Which is represented by a standard at the mint?

inches of distilled rain water. This standard is at present kept in the United States Mint at Philadelphia, and is the *standard unit of weight.*

UNITS OF TIME.

400. Time can only be measured by motion. The diurnal revolution of the earth affords the only invariable motion; hence, the time in which it revolves once on its axis, is the natural unit, and is called a day. From the day, by addition, we form the weeks, months and years; and by division, the hours, minutes and seconds.

UNITS OF CIRCULAR OR ANGULAR MEASURE.

401. This measure is used for the measurement of angles, and the natural unit is the right angle. But this is not the most convenient unit. The unit chiefly used is the 360 part of the circumference of a circle, called a *degree*, which is divided into 60 equal parts called *minutes*, and these again into 60 equal parts called *seconds.*

REMARKS.

402. It is seen that all the units, determined by the pendulum, depend on *time* as the ultimate base; that is, the length of a pendulum which will vibrate seconds determines all the *units of measure and weight.*

Now, time is measured by motion, and the motion of the earth on its axis is the only invariable motion. Hence, we refer to this to fix the unit of time, on which the unit of length depends, and from which all the other units are derived.

403. No class of pupils can rightly and clearly apprehend the nature of numbers and the operations performed upon them,

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400. How is time measured? What motion is uniform? What is the natural unit?

401. For what is circular or angular measure used? What is the unit?

402. On what do the units determined by the pendulum depend? How is time measured? To what then are all these units referred?

403. How are the ideas of the absolute and relative values of the units to be communicated to a class? What apparatus is necessary?

without distinct and fixed notions of the units; hence, every teacher should labor to point out their absolute and relative values: this can only be done by means of sensible objects.

Every school room, therefore, should be provided with a complete set of all the denominate units. The inch, the foot, the yard, the rod, should be accurately marked off on a conspicuous part of the room, together with the principal units of surface, the square inch, square foot, square yard, &c.

The units of volume should also be exhibited. The cubic inch and the cubic foot will serve as illustrations for one class of the units of volume; and the pint, quart, gallon and bushel, should be exhibited to illustrate the others.

The unit of weight should also be seen and handled. A child even can apprehend what is meant by an *ounce* or a *pound* when it takes one of these weights in its hand; and mature years can acquire the idea in no other way.

Let, therefore, every school room be furnished with a complete set of models to illustrate and explain the *absolute* and *relative* values of the different units.

UNITED STATES MONEY.

404. UNITED STATES MONEY is the currency established by Congress, A. D. 1786. The names or denominations of its units are, Eagles, Dollars, Dimes, Cents, and Mills.

The coins of the United States are of gold, silver, and copper, and are of the following denominations:

1. Gold: Eagle, half-eagle, three-dollars, quarter-eagle, dollar.
2. Silver: Dollar, half-dollar, quarter-dollar, dime, half-dime, and three-cent piece.
3. Copper: Cent, half cent.

TABLE.

10 Mills	make	1 Cent, marked	*ct.*
10 Cents	- -	1 Dime, - -	*d.*
10 Dimes	- -	1 Dollar, - -	$.
10 Dollars	- -	1 Eagle, - -	*E.*

Mills.	Cents.	Dimes.	Dollars.	Eagles.
10	= 1			
100	= 10	= 1		
1000	= 100	= 10	= 1	
10000	= 1000	= 100	= 10	= 1

405. It is seen, from the above table, that in United States money, the *primary unit* is 1 mill; that the units of the *scale*, in passing from mills to cents, are 10. The second unit is 1 cent, and the units of the scale, in passing to dimes, are 10. The third unit is 1 dime, and the units of the scale, in passing to dollars, are 10. The fourth unit is 1 dollar, and the units of the scale, in passing to eagles, are 10. This scale is the *same as in simple numbers;* therefore,

The units of United States money may be added, subtracted, multiplied, and divided by the same rules as are applicable to simple numbers.

NOTES.—The present *standard* or degree of purity of the coins was fixed by Act of Congress in 1837. It is this:

1. Nine hundred equal parts of pure gold, are mixed with 100 parts of alloy, of copper and silver, (of which not more than one-half must be silver) thus forming 1000 parts, equal to each other in weight. The silver coins contain 900 parts of pure silver, and 100 parts of pure copper. The copper coins are of pure copper.

2. The eagle contains 258 grains of standard gold, and the other gold coins in the same proportion. The dollar contains 412½ grains of standard silver, and the others in the same proportion The cent, 168 grains of pure copper.

3. If a given quantity of gold or silver be divided into 24 equal parts, each part is called a *carat.* If any number of carats be mixed with so many equal carats of a less valuable metal, that there be 24

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404. What is United States money? What are the names of its units? What are the coins of the United States? Which gold? Which silver? Which copper?

405. What is the primary unit in United States money? What are the units of the scale in passing from one denomination to another?—How does this compare with the scale in simple numbers?

carats in the mixture, then the compound is said to be as many carats fine as it contains carats of the more precious metal, and to contain as much alloy as it contains carats of the baser.

For example, if 20 carats of gold be mixed with 4 of silver, the mixture is called gold of 20 carats fine, and 4 parts alloy.

4. Although the currency of the United States is in dollars, cents and mills, yet in some of the States the old currency of pounds, shillings and pence, is still nominally preserved.

In all the States the shilling is reckoned at 12 pence, the pound at 20 shillings, and the dollar at 100 cents.

The following table shows the number of shillings in a dollar, the value of £1 in dollars, and the value of $1 in the fraction of a pound:

In English currency,	4*s.* 6*d.*	- £1 = $4,84,	and $1 = £$\frac{1}{4.84}$.
In N. E., Va., Ky., Tenn.,	6*s.*	- £1 = 3\frac{1}{3}$,	and $1 = £ $\frac{3}{10}$.
In N. Y., Ohio, N. Carolina.	8*s.*	- £1 = 2\frac{1}{2}$,	and $1 = £ $\frac{2}{5}$.
In N. J., Pa., Del, Md.,	7*s.* 6*d.*	- £1 = 2\frac{2}{3}$,	and $1 = £ $\frac{3}{8}$.
In S. Carolina and Ga.	4*s.* 8*d.*	- £1 = 4\frac{2}{7}$,	and $1 = £ $\frac{7}{30}$.
In Canada & Nova Scotia,	5*s.*	- £1 = $4,	and $1 = £ $\frac{1}{4}$.

ENGLISH MONEY.

406. The units or denominations of English money are guineas, pounds, shillings, pence, and farthings.

NOTES.—1. What is the degree of purity of the gold coins? Of the silver coins? Of the copper?

2. How much pure gold in the eagle? How much pure silver in the dollar?

3. What is a carat? How are metals mixed by carats?

4. In what denominations is money sometimes reckoned in the different states?

406. What are the denominations of English money?

TABLE.

4 farthings, marked *far.*, make	1 penny,	marked	*d.*
12 pence - - - -	1 shilling,	-	*s.*
20 shillings - - -	1 pound, or sovereign,		£
21 shillings - - -	1 guinea.		

far.	*d.*	*s.*	£.
4	= 1		
48	= 12	= 1	
960	= 240	= 20	= 1

TABLE OF FOREIGN COINS WHOSE VALUES ARE FIXED BY LAW.

	$	*cts.*
Franc of France and Belgian,	0	$18\frac{6}{16}$
Florin of the Netherlands,		40
Guilder of do.		40
Livre Tournois of France,		$18\frac{1}{2}$
Milrea of Portugal,	1	12
Milrea of Madeira,	1	00
Milrea of the Azores,		$83\frac{1}{3}$
Marc Banco of Hamburg,		35
Pound Sterling of Great Britain,	4	84
Pagoda of India,	1	84
Real Vellon of Spain,		05
Real Plate of do.		10
Rupee Company,		$44\frac{1}{2}$
Rupee of British India,		$44\frac{1}{2}$
Rix Dollar of Denmark,	1	00
Rix Dollar of Prussia,		$68\frac{1}{2}$
Rix dollar of Bremen,		$78\frac{3}{4}$
Rouble, silver, of Russia,		75
Tale of China,	1	48
Dollar of Sweden and Norway,	1	06
Specie Dollar of Denmark,	1	05
Dollar of Prussia and Northern States of Germany,		69
Florin of Southern States of Germany,		40
Florin of Austria and city of Augsburg,		$48\frac{1}{2}$
Lira of the Lombardo Venetian Kingdom,		16
Lira of Tuscany,		16
Lira of Sardinia,		$18\frac{6}{10}$
Ducat of Naples,		80
Ounce of Sicily,	2	40
Pound of Nova Scotia, New Brunswick, Newfoundland, and Canada,	4	04

TABLE OF FOREIGN COINS WHOSE VALUES ARE FIXED BY USAGE.

	$	cts.
Berlin Rix Dollar,		$69\frac{1}{2}$
Current Marc,		28
Crown of Tuscany,	1	05
Elberfeldt Rix Dollar,		$69\frac{3}{4}$
Florin of Saxony,		48
" Bohemia,		48
" Elberfeldt,		40
" Prussia,		$22\frac{3}{4}$
" Trieste,		48
" Nuremburg,		40
" Frankfort,		40
" Basil,		41
" St. Gaul,		$40\frac{86}{100}$
" Creveld,		40
Florence Livre,		15
Genoa do.,		$18\frac{3}{4}$
Geneva do.,		21
Jamaica Pound,	5	00
Leghorn Dollar,		90
Leghorn Livre ($6\frac{1}{2}$ to the dollar),		$15\frac{5}{13}$
Livre of Catalonia,		$53\frac{1}{3}$
Neufchatel Livre,		$26\frac{1}{2}$
Pezza of Leghorn,		90
Rhenish Rix Dollar,		$60\frac{3}{4}$
Swiss Livre,		27
Scuda of Malta,		40
Turkish Piastre,		05

[The above Tables are taken from a work on the Tariff, by E. D. Ogden, Esq., of the New York Custom House].

NOTES.—1. The primary unit in English money is 1 farthing. The units of the scale, in passing from farthings to pence, are 4; in passing from pence to shillings, the units of the scale are 12; in passing from shillings to pounds, they are 20.

2. Farthings are generally expressed in fractions of a penny. Thus, $1 far. = \frac{1}{4}d.$; $2 far. = \frac{1}{2}d.$; $3 far. = \frac{3}{4}d.$

3. The standard of the gold coin is 22 parts of pure gold and 2 parts of copper.

NOTE.—1. What are the primary units of the English currency? Name the units of the scale.

The standard of silver coin is 37 parts of pure silver, and 3 parts of copper.

A pound of gold is worth 14.2878 times as much as a pound of silver. In the copper coin 24 pence make 1 pound avoirdupois.

By reading the second table from right to left, we can see the value of any unit expressed in each of the lower denominations. Thus, 1*d.* = 4*far.*; 1*s.* = 12*d.* = 48*far.*; £1 = 20*s.* = 240*d.* = 960*far.*

LINEAR MEASURE.

407. This measure is used to measure distances, lengths, breadths, heights and depths.

TABLE.

12 inches	make	1 foot,	marked *ft.*
3 feet	-	1 yard,	*yd.*
$5\frac{1}{2}$ yards or $16\frac{1}{2}$ feet	-	1 rod,	*rd.*
40 rods	-	1 furlong,	*fur.*
8 furlongs or 320 rods	-	1 mile,	*mi.*
3 miles	-	1 league,	*L.*
$69\frac{1}{2}$ statute miles, or 60 geographical miles,	-	1 degree on the equator,	*deg.* or °.
360 degrees	-	a circumference of the earth.	

in.	*ft.*	*yd.*	*rd.*	*fur.*	*mi.*
12	= 1				
36	= 3	= 1			
198	= $16\frac{1}{2}$	= $5\frac{1}{2}$	= 1		
7920	= 660	= 220	= 40	= 1	
63360	= 5280	= 1760	= 320	= 8	= 1

NOTES.—1. A fathom is a length of six feet, and is generally used to measure the depth of water.

2. A hand is 4 inches, and is used to measure the height of horses.

3. The units of the scale, in passing from inches to feet, are 12; in passing from feet to yards, 3; from yards to rods, $5\frac{1}{2}$; from rods to furlongs, 40; and from furlongs to miles, 8.

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407. For what is linear measure used? What are its denominations? Repeat the table? What is a fathom? What is a hand? What are the units of the scale in linear measure?

FOREIGN MEASURES OF LENGTH.

408. The Imperial yard of Great Britain is the one from which ours is taken. Hence, the units of measure are identical.

FRENCH SYSTEM.

409. The base of the new French system of measures is the measure of the meridian of the earth, a quadrant of which is 10,000,000 *metres*, measured at the temperature of 32° Fahr. The multiples and divisions of it are decimals, viz.: 1 metre = 10 decimetres = 100 centimetres = 1000 millimetres = 3.280899 United States feet, or 39.37079 inches.

This relation enables us to convert all measures in either system into the corresponding measures of the other.

Austrian,	1 foot = 12.448 U. S. inches	= 1.03737 foot.	
Prussian, *Rhineland,*	1 foot = 12.361 " "	= 1.0300 "	
Swedish,	1 foot = 11.690 " "	= 0.974145 "	
Spanish,	1 foot = 11.034 " "	= 0.9195 "	
	league (royal) = 25000 Span. ft.	= $4\frac{1}{3}$ miles	nearly.
	" (common) = 19800 "	= $3\frac{1}{2}$ "	nearly.

CLOTH MEASURE

410. Cloth measure is used for measuring all kinds of cloth, ribbons, and other things sold by the yard.

TABLE.

$2\frac{1}{4}$ inches, *in.*	make	1 nail, marked	*na.*
4 nails	-	1 quarter of a yard,	*qr.*
3 quarters -	-	1 Ell Flemish,	*E. Fl.*
4 quarters -	-	1 yard, - -	*yd.*
5 quarters -	-	1 Ell English, -	*E. E.*

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408. How does the Imperial yard compare with the standard in the United States?

409. What is the unit of the French system of measures? How does the metre compare with our standard yard?

410. For what is cloth measure used? What are its denominations? Repeat the table. What are the units of the scales?

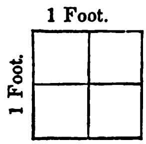
in.	*na.*	*qr.*	*E. Fl.*	*yd.*	*E. E.*
$2\frac{1}{4}$	$= 1$				
9	$= 4$	$= 1$			
27	$= 12$	$= 3$	$= 1$		
36	$= 16$	$= 4$	$= 1\frac{1}{3}$	$= 1$	
45	$= 20$	$= 5$	$= 1\frac{2}{3}$	$= 1\frac{1}{4}$	$= 1$

NOTE.—The units of the scale, in this measure, are $2\frac{1}{4}$, 4, 3, $\frac{4}{3}$, and $\frac{5}{4}$.

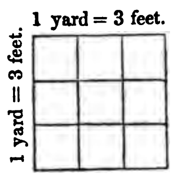
SQUARE MEASURE.

411. Square measure is used in measuring land, or anything in which length and breadth are both considered.

A square is a figure bounded by four equal lines at right angles to each other. Each line is called a side of the square. If each side be one foot, the figure is called a *square foot.*



If the sides of the square be each one yard, the square is called a *square yard.* In the large square there are nine small squares, the sides of which are each one foot. Therefore, the square yard contains 9 square feet.



The number of small squares that is contained in any large square is always equal to the product of two of the sides of the large square. As in the figure, $3 \times 3 = 9$ square feet. The number of square inches contained in a square foot is equal to $12 \times 12 = 144$.

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411. For what is square measure used? What is a square? If each side be one foot, what is it called? If each side be a yard, what is it called? How many square feet does the square yard contain? How is the number of small squares contained in a large square found? Repeat the table. What are the units of the scale?

TABLE.

144 square inches, *sq. in.*	make	1 square foot,	*Sq. ft.*
9 square feet	- -	1 square yard,	*Sq. yd.*
$30\frac{1}{4}$ square yards	- -	1 square rod or perch,	*P.*
40 square rods or perches		1 rood, - - -	*R.*
4 roods	- - -	1 acre, - - -	*A.*
640 acres	- - -	1 square mile, -	*M.*

Sq. in.	*Sq. ft.*	*Sq. yd.*	*P.*	*R.*	*A.*
144	= 1				
1296	= 9	= 1			
39204	= $272\frac{1}{2}$	= $30\frac{1}{4}$	= 1		
1568160	= 10890	= 1210	= 40	= 1	
6272640	= 43560	= 4840	= 160	= 4	= 1.

NOTE.—The units of the scale in this measure are 144, 9, $30\frac{1}{4}$, 40, and 4.

SURVEYORS' MEASURE.

412. The Surveyor's or Gunter's chain is generally used in surveying land. It is 4 poles or 66 feet in length, and is divided into 100 links.

TABLE.

$7\frac{92}{100}$ inches	make	1 link, marked - -	*l.*
4 rods $=66ft.=100$ links		1 chain, - - - -	*c.*
80 chains - - -		1 mile, - - - -	*mi.*
1 square chain - -		16 square rods or perches,	*P.*
10 square chains -		1 acre, - - - -	*A.*

NOTES.—1. Land is generally estimated in square miles, acres, roods, and square rods or perches.

2. The units of the scale, in this measure, are $7\frac{92}{100}$, 4, 80, 1, and 10.

CUBIC MEASURE.

413. Cubic measure is used for measuring stone, timber, earth, and such other things as have the three dimensions, length, breadth, and thickness.

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412. What chain is used in land surveying? What is its length? How is it divided? Repeat the table? In what is land generally estimated? What are the units of the scale?

TABLE.

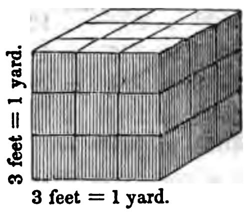
1728 cubic inches, *Cu. in.*, make	1 cubic foot, -	*Cu. ft.*
27 cubic feet - - -	1 cubic yard, -	*Cu. yd.*
40 feet of round or / 50 feet of hewn timber,	1 ton, - -	*T.*
42 solid feet - - -	1 ton of shipping,	*T.*
8 cord feet, or / 128 cubic feet - -	1 cord, - -	*C.*
$24\frac{3}{4}$ cubic feet of stone -	1 perch, - -	*P.*

NOTES.—1. A cord of wood is a pile 4 feet wide, 4 feet high, and 8 feet long.

2. A cord foot is 1 foot in length of the pile which makes a cord.

3. A CUBE is a figure bounded by six equal squares, called *faces;* the sides of the squares are called *edges.*

4. A cubic foot is a cube, each of whose faces is a square foot; its edges are each 1 foot.



5. A cubic yard is a cube, each of whose edges is 1 yard.

6. The base of a cube is the face on which it stands. If the edge of the cube is one yard, it will contain $3 \times 3 = 9$ square feet; therefore, 9 cubic feet can be placed on the base, and hence, if the solid were 1 foot thick, it would contain 9 cubic feet; if it were 2 feet thick it would contain 2 tiers of cubes, or 18 cubic feet; if it were 3 feet thick, it would contain 27 cubic feet; hence,

The contents of such a figure are found by multiplying the length, breadth, and thickness together.

7. A ton of *round timber*, when square, is supposed to produce 40 cubic feet: hence, *one-fifth is lost by squaring.*

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413. For what is cubic measure used? What are its denominations? What is a cord of wood? What is a cord foot? What is a cube? What is a cubic foot? What is a cubic yard? How many cubic feet in a cubic yard? What are the contents of a volume equal to? Repeat the table. What are the units of the scale?

WINE MEASURE.

414. Wine measure is used for measuring all liquids except ale, beer and milk.

TABLE.

4 gills, *gi.*	make	1 pint,	marked	*pt.*
2 pints	- - -	1 quart,	- -	*qt.*
4 quarts	- - -	1 gallon,	- -	*gal.*
31½ gallons	- - -	1 barrel,	- -	*bar.* or *bbl*
42 gallons	- - -	1 tierce,	- -	*tier.*
63 gallons	- - -	1 hogshead,	-	*hhd.*
2 hogsheads	- -	1 pipe,	- -	*pi.*
2 pipes or 4 hogsheads		1 tun,	- -	*tun.*

gi.	*pt.*	*qt.*	*gal.*	*bar.*	*tier.*	*hhd.*	*pi.*	*tun.*
4	= 1							
8	= 2	= 1						
32	= 8	= 4	= 1					
1008	= 252	= 126	= 31½	= 1				
1344	= 336	= 168	= 42		= 1			
2016	= 504	= 252	= 63		= 1½	= 1		
4032	= 1008	= 504	= 126		= 3	= 2	= 1	
8064	= 2016	= 1008	= 252		= 6	= 4	= 2	= 1.

NOTES.—1. The *standard unit*, or gallon of wine measure, in the United States, contains 231 cubic inches, and hence, is equal to the weight, avoirdupois, of 8.339 cubic inches of distilled water, very nearly.

2. The English Imperial wine gallon contains 277.274 cubic inches, and hence, is equal to 1.2 times the wine gallon of the United States.

BEER MEASURE.

415. Beer measure is used for measuring ale, beer, and milk.

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414. What is measured by wine measure? What are its denominations? Repeat the table. What are the units of the scale? What is a standard wine gallon?

415. For what is beer measure used? What are its denominations? Repeat the table. What are the units of the scale?

TABLE.

2 pints, *pt.*	make	1 quart,	marked	*qt.*
4 quarts	- -	1 gallon,	- -	*gal.*
36 gallons	- -	1 barrel,	- -	*bar.*
54 gallons	- -	1 hogshead,	-	*hhd.*

pt.	*qt.*	*gal.*	*bar.*	*hhd.*
2	= 1			
8	= 4	= 1		
288	= 144	= 36	= 1	
432	= 216	= 54	= $1\frac{1}{2}$	= 1.

NOTES.—1. The *standard gallon,* beer measure, contains 282 cubic inches, and hence, is equal to the weight of 10.1799 cubic inches of distilled rain water.

2. Milk in many places is sold by wine measure.

DRY MEASURE.

416. Dry measure is used in measuring all dry articles, such as grain, fruit, salt, coal, &c.

TABLE.

2 pints, *pt.*	make	1 quart,	marked	*qt.*
8 quarts	- -	1 peck,	- -	*pk.*
4 pecks	- -	1 bushel,	- -	*bu.*
36 bushels	-	1 chaldron,	-	*ch.*

pt.	*qt.*	*pk.*	*bu.*	*ch.*
2	= 1			
16	= 8	= 1		
64	= 32	= 4	= 1	
2304	= 1152	= 144	= 36	= 1.

NOTES.—1. The *standard bushel* of the United States is the *Winchester* bushel of England. It is a circular measure $18\frac{1}{2}$ inches in diameter and 8 inches deep, and contains 2150.4 cubic inches, nearly. It contains 77.627413 pounds avoirdupois of distilled water.

2. A gallon, dry measure, contains 268.8 cubic inches.

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416. What articles are measured by dry measure? What are its denominations? Repeat the table. What are the units of the scale? What is the standard bushel? What are the contents of a gallon?

3. Wine measure, Beer measure, and Dry measure, and all measures of volume, differ from the cubic measure only in the unit which is used as a standard.

AVOIRDUPOIS WEIGHT.

417. By this weight all coarse articles are weighed, such as hay, grain, chandlers' wares, and all metals except gold and silver.

TABLE.

16 drams, *dr.*	make	1 ounce,	marked	*oz.*
16 ounces -	-	1 pound,	- -	*lb.*
25 pounds	-	1 quarter,	- -	*qr.*
4 quarters	-	1 hundred weight,		*cwt.*
20 hundred weight		1 ton,	- -	*T.*

dr.	*oz.*	*lb.*	*qr.*	*cwt.*	*T.*
16	= 1				
256	= 16	= 1			
6400	= 400	= 25	= 1		
25600	= 1600	= 100	= 4	= 1	
512000	= 32000	= 2000	= 80	= 20	= 1.

NOTES.—1. The standard avoirdupois pound is the weight of 27.7015 cubic inches of distilled water; and hence, 1 cubic foot weighs 1000 ounces, *very nearly*.

2. By the old method of weighing, adopted from the English system, 112 pounds were reckoned for a hundred weight. But now, the laws of most of the States, as well as general usage, fix the hundred weight at 100 pounds.

3. The units of the scale, in passing from drams to ounces, are 16; from ounces to pounds, 16, from pounds to quarters, 25; from quarters to hundreds, 4; and from hundreds to tons, 20.

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417. For what is avoirdupois weight used? How is the table to be read? How can you determine, from the second table, the value of any unit in units of the lower denominations?

NOTES.—1. What is the standard avoirdupois pound?

2. What is a hundred weight by the English method? What is a hundred weight by the United States method?

3. Name the units of the scale in passing from one denomination to

TROY WEIGHT.

418. Gold, silver, jewels, and liquors, are weighed by Troy weight.

TABLE.

24 grains, *gr.*	make	1 pennyweight, marked	*pwt.*
20 pennyweights	-	1 ounce, - -	*oz.*
12 ounces -	-	1 pound, - -	*lb.*

gr.	*pwt.*	*oz.*	*lb.*
24	= 1		
480	= 20	= 1	
5760	= 240	= 12	= 1.

NOTES.—1. The standard Troy pound is the weight of 22.794377 cubic inches of distilled water. Hence, it is less than the pound avoirdupois

2.

7000	troy grains =	1 pound	avoirdupois.
175	troy pounds =	144 pounds	"
175	troy ounces =	192 ounces	"
$437\frac{1}{2}$	troy grains =	1 ounce.	"

3. The Troy pound being the one deposited in the mint at Philadelphia, is generally regarded as the *standard* of weight.

4. The units of the scale are 24, 20, and 12.

APOTHECARIES' WEIGHT.

419. This weight is used by apothecaries and physicians in mixing their medicines. But medicines are generally sold, in the quantity, by avoirdupois weight.

TABLE.

20 grains, *gr.*	make	1 scruple, marked	℈.
3 scruples -	-	1 dram, - -	ʒ.
8 drams -	-	1 ounce, - -	℥.
12 ounces -	-	1 pound, - -	℔.

418. What articles are weighed by Troy weight? What are its denominations? Repeat the table. What is the standard Troy pound? What are the units of the scale, in passing from one unit to another?

419. What is the use of Apothecaries' weight? What are its denominations? Repeat the table. What are the values of the pound and ounce? *What are the units* of the scale, in passing from one unit to another?

gr.	℈	ʒ	℥	℔
20	= 1			
60	= 3	= 1		
480	= 24	= 8	= 1	
5760	= 288	= 96	= 12	= 1

NOTES.—1. The pound and ounce are the same as the pound and ounce in Troy weight.

2. The units of the scale, in passing from grains to scruples, are 20; in passing from scruples to drams, 3; from drams to ounces, 8; and from ounces to pounds, 12.

NEW FRENCH SYSTEM.

420. The basis of this system of weights is the weight in vacuo of a cubic decimetre of distilled water. This weight is called a kilogramme, and is the unit of the French system. It is equal to 2.204737 pounds avoirdupois. The other denominations are as follows:

100 kilogrammes = 1 quintal; 10 quintals = 1 ton sea water; 1 gramme = 10 hectogrammes; 1 hectogramme = 10 decogrammes; 1 decogramme = 10 grammes; 1 gramme = 10 decigrammes; 1 decigramme = 10 centigrammes; 1 centigramme = 10 milligrammes.

COMPARISON OF WEIGHTS.

English,	1 pound	= 1.000936	pounds	avoirdupois.
French,	1 kilogramme	= 2.204737	"	"
Spanish,	1 pound	= 1.0152	"	"
Swedish,	1 pound	= 0.9376	"	"
Austrian,	1 pound	= 1.2351	"	"
Prussian,	1 pound	= 1.0333	"	"

MEASURE OF TIME.

421. TIME is a part of duration. The time in which the earth revolves on its axis is called a *day*. The time in which it goes round the sun is called a solar year. Time is divided into parts according to the following

TABLE.

60 seconds, *sec.*	make	1 minute,	marked	*m.*
60 minutes	- -	1 hour,	- -	*hr.*
24 hours	- - -	1 day,	- -	*da.*
7 days	- - -	1 week,	- -	*wk.*
4 weeks	- - -	1 month,	- -	*mo.*
12 calendar months, or 365*da.* 6*hr.*, nearly,	}	1 Julian or civil year,		*yr.*
100 years	- - -	1 century,	- -	*cent.*

sec.	*m.*	*hr.*	*da.*	*wk.*	*yr.*
60	= 1				
3600	= 60	= 1			
86400	= 1440	= 24	= 1		
604800	= 10080	= 168	= 7	= 1	
31557600	= 525960	= 8766	= $365\frac{1}{4}$	= 52	= 1.

The year is divided into 12 calendar months:

No.	*No. days.*	*No.*	*No. days.*
1st. January, - -	31	7th. July, - - - -	31
2d. February, - -	28	8th. August, - - -	31
3d. March, - - -	31	9th. September, - -	30
4th. April, - - -	30	10th. October, - - -	31
5th. May, - - -	31	11th. November, - - -	30
6th. June, - - -	30	12th. December, - -	31

The number of days in each month may be remembered by the following:

Thirty days hath September,
April, June, and November;
All the rest have thirty-one,
Excepting February, twenty-eight alone.

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421. What are the denominations of time? How long is a year? How many days in a common year? How many days in a Leap year? How many calendar months in a year? Name each, and its number, and the number of days in each. How many days has February in the leap year? How do you remember which of the months have 30 days, and which 31?

NOTE.—1. How are the centuries numbered? How are the years numbered? The days? The hours?

NOTES.—1. The centuries are *numbered* from the beginning of the Christian Era. The year 30, for example, at its *commencement*, was called the 30th year of the first century, though neither the century nor the year had elapsed. Thus, June 2d, 1856, is the 6th month of the 56th year of the 19th century.

2. Days are numbered in each month from the first day of the month.

3. The civil day begins and ends at 12 o'clock at night. In the civil day, the hours are reckoned from that time.

DATES.

1. The length of the solar year is 365*da.* 5*hr.* 48*m.* 48*sec.*, very nearly. It is desirable to have the periods and dates of the civil year correspond to those of the solar year; else, the summer months of the one would in time become the winter months of the other, thereby producing great confusion in dates and history.

2. The civil year is reckoned at 365*da.* 6*hr.*; but the whole days only, are numbered in each year. The 6 hours accumulate for 4 years before they are counted, when they amount to 1 day, and are added to February; and the year is called a *bissextile* or *leap year*.

3. The odd 6 hours have been so added that the leap years occur in those numbers which are divisible by 4. Thus, 1856, 1860, 1864, &c., are leap years; and when any number is not divisible by 4, the remainder denotes how many years have passed since a leap year.

4. This method of disposing of the fractional part of the year would be without error if the solar year were exactly 365*da.* 6*hr.* in length; but it is not; it is only 365*da.* 5*hr.* 48*m.* 48*sec.* long: hence, the civil year is reckoned at *too much*, and to correct this error, every *centennial* year is reckoned as a *common year*. But this makes an error again, on the other side, and every fourth centennial year the day is retained. Thus, 1800 was not, and 1900 will not be, reckoned a leap year: the error will then be on the other side, and 2000 will be a leap year. This disposition of the fractional part of the year causes the civil and solar years to correspond very nearly, and indicates the following rule for finding the leap years:

Every year which is divisible by 4 is a leap year, unless it is a centennial year, and then it is not a leap year unless the number of the century is also divisible by 4.

5. The registration of the days, by reckoning the civil year at

365*da*. 6*hr*., was established by the Roman Emperor, Julius Cæsar, and hence this period is sometimes called the Julian year.

The error, arising from the fractional part, continued to increase until 1582, when it amounted to 10 days; that is, as the year had been reckoned *too long* the number of days had been *too few*, and the *count*, in the civil year, was *behind* the count in the solar year.

In this year, (1582), Pope Gregory decreed the 4th day of October to be called the 14th, and this brought the civil and the solar years together. The new calendar is sometimes called the *Gregorian Calendar.*

6. The method of dating by the old count, is called *Old Style*; and by the new, *New Style*. The difference is now 12 days. In Russia, they still use the old style; hence, their dates are 12 days behind ours. Their 4th of January is our 16th.

CIRCULAR MEASURE.

422. Circular measure is used in estimating *latitude* and *longitude*, in measuring the motions of the heavenly bodies, and also in measuring angles.

The circumference of every circle is supposed to be divided into 360 equal parts, called *degrees*. Each degree is divided into 60 minutes, and each minute into 60 seconds.

TABLE.

60 seconds ″	make	1 minute, marked	′.
60 minutes	- -	1 degree, -	°.
30 degrees	- -	1 sign, - -	s.
12 signs or 360°	-	1 circle, - -	c.

″	′	°	s.	c.
60	= 1			
3600	= 60	= 1		
108000	= 1800	= 30	= 1	
1296000	= 21600	= 360	= 12	= 1.

---

422. For what is circular measure used? How is every circle supposed to be divided? Repeat the table.

MISCELLANEOUS TABLE.

12 units, or things	make	1 dozen.
12 dozen - - -	-	1 gross.
12 gross, or 144 dozen	-	1 great gross.
20 things - - -	-	1 score.
100 pounds - -	-	1 quintal of fish.
196 pounds - -	-	1 barrel of flour.
200 pounds - -	-	1 barrel of pork.
18 inches - - -	-	1 cubit.
22 inches, nearly -	-	1 sacred cubit.
14 pounds of iron or lead	-	1 stone.
$21\frac{1}{2}$ stones - - -	-	1 pig.
8 pigs - - -	-	1 fother.

BOOKS AND PAPER.

The terms, *folio*, *quarto*, *octavo*, *duodecimo*, &c., indicate the number of leaves into which a sheet of paper is folded.

A sheet folded in	2 leaves	is called	a folio.
A sheet folded in	4 leaves	"	a quarto, or 4to.
A sheet folded in	8 leaves	"	an octavo, or 8vo.
A sheet folded in	12 leaves	"	a 12mo.
A sheet folded in	16 leaves	"	a 16mo.
A sheet folded in	18 leaves	"	an 18mo.
A sheet folded in	24 leaves	"	a 24mo.
A sheet folded in	32 leaves	"	a 32mo.

24 sheets of paper	make	1 quire.
20 quires - - - -	-	1 ream.
2 reams - - - -	-	1 bundle.
5 bundles - - -	-	1 bale.

ANSWERS.

P.	EX.	ANS.	EX.	ANS.	EX.	ANS.
16.	1	XVI.	8	DCCLI.	15	DCCCCLVII.
16.	2	XIV.	9	MLX.	16	MCCVI.
16.	3	XVIII.	01	MMXCI.	17	CCCCXCV.
16.	4	LXIX.	11	DLXIX.	18	DCCLV.
16.	5	LXXVIII.	12	DCCXLV	19	MDCCCXLVII.
16.	6	CXV.	13	DCCCCLXI	20	MMDXX.
16.	7	CCCCIX.	14	DCXCIX.	—	———

19.	1	7	3	9000	5	961	6	7408
19.	2	80	4	93	-	——	——	——

20.	7	897,021	14	75,605,070,905,008
20.	8	86,029,430	15	904,000,800,200,720
20.	9	4,328,021,063	16	6,000.900,704,098,020
20.	10	967,040,932	17	80,510,006,040,900,040,900
20.	11	30,430,208,123	18	6,050,900,001
20.	12	360,030,702,010	19	987,654,321,012,345,678
20.	13	5,800,006,000,812	20	208,104,111,001,111,111

23.	1	621	6	40,000,241
23.	2	5,702	7	59,000,310
23.	3	8,001	8	12,111
23.	4	10,406	9	300,001,006
23.	5	65,029	10	69,003,000,211

24.	11	47,000,069,000,465,207
24.	12	800,000,000,000,429,006,009
24.	13	95,000,000,000,000,089,089,306
24.	14	6,000,000,451,065,047,104
24.	15	999,065,841,411
24.	16	470,040,000.000,000,000,000,000,004,006,204
24.	17	65,000,800,000,750,751,975,310

31.	1	2 ; 7	2	7 ; 3	3	1 ; 7

33.	6	{ 42600 ;	7	36860	9	433005	11	£1 12*s.* 8*d.* 1*far.*
33.		{ 426000	8	$8,75	10	8996	—	———

34.	12	15445*lb.*	17	30*lb.* 4 ℥ 3 ʒ 2 ℈ 7*gr.*
34.	13	7 *T.* 14*cwt.* 1*qr.* 20*lb.*	18	249*in.*
34.	14	26215*grs.*	19	{ 1600*rd.* 8800*yds.*
34.	15	122*lb.* 2*oz.* 18*pwt.* 9*gr.*		{ 26400*ft.* 316800*in*
34.	16	29362*gr.*	20	75*yd.* 2*ft.* 6*in.*

P.	EX.	ANS.	EX.	ANS.
34.	21	6*sq. yd.* 2*sq. ft.*	30	78 *E.E.* 1*qr.*
34.	22	2*A.* 0*R.* 35*P.*	31	1008*qt.*
34.	23	45*A.* 6*sq. Ch.*	32	15*hhd.*
34.	24	568*P.*	33	3024*pt.*
34.	25	967680*cu. in.*	34	129*bar.*
34.	26	3968*cu. ft.*.	35	1984*pt.*
34.	27	440*cords.*	36	0*ch.* 32*bu.* 3*pk.* 7*qt.*
34.	28	2512*na.*	37	63113856*sec.*
34.	29	144*yd.*.	38	8*mo.* 2*wk.*

38.	1	182630	7	395873	13	32921
38.	2	87539	8	30534	14	185876
38.	3	110526	9	74716	15	93684
38.	4	79165	10	29909	16	34289
38.	5	73285	11	74022	17	243972
38.	6	4148907	12	833516	—	——

39.	18	$991,546.	27	43*T.* 2*cwt.* 0*qr.* 7*lb.*
39.	19	$85,465.	28	312*yd.* 0*qr.* 2*na.*
39.	20	$770,560.	29	251 *E. E.* 1*qr.* 3*na.*
39.	21	$525,892.	30	143*L.* 2*mi.* 6 *fur.*
39.	22	$9638.495.	31	4*fur.* 4*rd.* 0*yd.* 1*ft.* 7*in.*
39.	23	£223 2*s.* 5*d.* 1*f.*	32	322*A.* 1*R.* 4*P.*
39.	24	1296*lb.* 10*oz.* 2*pwt.*	33	2224*T.* 0*hhd.* 5*gal.*
39.	25	453℔ 9℥ 3ʒ	34	339*gal.* 3*qt.*
39.	26	2*cwt.* 3*qr.* 8*lb.* 8*oz.* 5*dr.*	—	——

40.	35	230*chal.* 25*bu.* 3*pk.* 4*qt.*	40	$137915940.
40.	36	823*yr.* 10*mo.*	41	88056.
40.	37	904*da.* 18*hr.* 1*mi.*	42	121*mi.* 4*fur.* 8*rd.* 5*ft.*
40.	38	2*T.* 14*cwt.* 1*qr.* 20*lb.* 15*oz.*	43	519190.
40.	39	23592550.	44	1124749.

41.	45	$22,009.	50	29026.
41.	46	$27,740.	51	8209,75.
41.	47	4*tun.* 2*hhd.* 29*gal.* 2*qt.* 0*pt.*	52	26326424.
41.	48	$20308675.	53	29714.
41.	49	30569853.	54	50110025.

42.	55	59808512	60	4*lb.* 5*oz.* 6*pwt.*
42.	56	2*T.* 4*cwt.* 2*qr.* 1*lb.*	61	1053420
42.	57	205 *acres.*	62	4089507
42.	58	$75002,295	63	32341
42.	59	$7425	—	

P.	EX.	ANS.	EX.	ANS.
43.	64	$27131,23	69	481125
43.	65	$28,105	70	66585383
43.	66	39*yd.* 1*qr.*	71	$1019,10
43.	67	$180,825	72	$33800
43.	68	$35068,807	—	———
44.	73	380 *bushels.*	75	£57 14*s.* 2*d.* 3*far.*
44.	74	$458,342	76	5860

P.	EX.	ANS.	EX.	ANS.	EX.	ANS.
47.	1	363296	5	41923288 *rods.*	9	62786*bu.*
47.	2	56579	6	$7838180	10	198621115*in.*
47.	3	733071	7	106026 *mills.*	11	3591757651*m.*
47.	4	1711927	8	4391*bar.*	—	———

P.	EX.	ANS.	EX.	ANS.
48.	12	4199675 *cords.*	26	25 *E. E.* 1*qr.* 3*na.*
48.	13	8878778*gal.*	27	79℔ 10℥ 6ʒ
48.	14	99999977*lb.*	28	12℥ 4ʒ 2℈
48.	15	$8443,641	29	124 *E. E.* 3*qr.* 3*na.*
48.	16	$806,384	30	96*E. F.* 1*qr.* 1*na.*
48.	17	$4853673,758	31	12*T.* 17*cwt.* 3*qr.*
48.	18	£14 18*s.* 3*d.* 1*far.*	32	2*cwt.* 2*qr.* 22*lb.*
48.	19	3*T.* 8*cwt.* 2*qr.* 7*lb.*	33	69*qr.* 2*lb.* 14*oz.*
48.	20	117*yds.* 2*qr.* 1*na.*	34	134*lb.* 14*oz.* 13*dr.*
48.	21	59*L.* 1*mi.* 3*fur.* 28*rd.*	35	10*A.* 2*R.* 18*P.*
48.	22	8*tun* 1*hhd.* 53*gal.* 3*qt.*	36	37*A.* 2*R.* 34*P*
48.	23	89*A.* 2*R.* 37*P.*	37	147*da.* 21*hr.* 56*mi.*
48.	24	975*bu.* 1*pk.* 6*qt.*	38	52*hr.* 50*m.* 54*sec.*
48.	25	124 *cords* 58*ft.* 522*in.*	—	———
49.	39	$8759,625	46	£121 17*s.* 0*d.* 1*far.*
49.	40	183666662	47	6*yr.* 0*mo.* 0*wk.* 6*da.* 9*hr.* 2*mi.*
49.	41	6*yr.* 9*mo.* 3*wk.* 11*da.*	48	6353870
49.	42	88℔ 0℥ 6ʒ	49	5747
49.	43	$8,20	50	$6020
49.	44	$39,868	51	25712808,91
49.	45	$10,626	52	$36190
50.	53	683021	59	$199,625 *lost.*
50.	54	107445034	60	$175,875
50.	55	6274	61	$3,25
50.	56	4*T.* 3*cwt.* 2*qr.* 23*lb.*	62	19987563
50.	57	£19 19*s.* 2*d.* 3*far.*	63	2899248
50.	58	2299*mi.* 2*fur.* 4*rd.*	64	$73675
51.	65	22815	67	80*yr.* 8*mo.* 0*da.* 3*hr.* 30*m.*
51.	66	$198,625	68	$655,125

P.	EX.	ANS.	EX.	ANS.
51.	69	249*yr.* 1*mo.* 11*da.*	73	$7398
51.	70	17877	74	$2360
51.	71	$7310756	75	526
51.	72	$62727794	76	6274
52.	77	$356,35 *gained.*	83	2*yr.* 8*mo.* 19*da.*
52.	78	3*A.* 2*R.* 39*P.*	84	30*gal.* 2*qt.* 1*pt.*
52.	79	41 *cords* 5 *cord feet.*	85	50062
52.	80	$3280,105	86	15550
52.	81	$44161,987	87	12° 23′ 53″
52.	82	$14352,50	—	———
53.	88	$161,175 *gained.*	90	32*yd.* 0*qr.* 2*na.*
53.	89	2271707	91	£950 2*s.* 8*d.*
60.	1	6776368	14	301144560000
60.	2	68653214	15	610071000
60.	3	3422454	16	14783518400
60.	4	1952883	17	£81 6*s.* 8*d.*
60.	5	4354224	18	24*T.* 7*cwt.* 3*qr.*
60.	6	1028540646	19	118*yd.* 1*ft.* 3*in.*
60.	7	24668698404	20	114° 26′ 15″
60.	8	$70,84	21	56*hhd.* 7*gal.* 2*qt.* 0*pt.*
60.	9	$12517,764	22	598 *E. F.*
60.	10	$961662,960	23	865*T.* 11*cwt.* 3*qr.* 20*lb.*
60.	11	201638228149	24	320*yr.* 2*mo.* 0*wk.* 1*da.* 15*hr.* 12*m.*
60.	12	4281770760	25	4890
60.	13	174809600	—	———
61.	26	234048	39	9072040000 ; 907204000000
61.	27	4482566	40	74040900 ; 740409000
61.	28	314986464	41	67493600 ; 67493600000
61.	29	320021195962	42	129359360000
61.	30	556321146764	43	13729103000000
61.	31	1747125213301	44	664763206000000
61.	32	2324684880333	45	8799238229600000
61.	33	7110969649 2112	46	252642601790869 5000000
61.	34	9001235585 7332	47	10936893684450843787770 40
61.	35	549600	48	1671441067735958158 3737
61.	36	670460; 6704600	49	$61975
61.	37	5704900; 57049000	50	3240*mi.*
61.	38	4980496000 ;	51	$2097
61.		49804960000	52	133*yd.* 3*qr.* 2*na,*

P.	EX.	ANS.	EX.	ANS.
62.	53	£3 19*s.* 4*d.* 2*far.*	54	$1031,68
62.	55	$15	61	6*da.* 6*hr.* 37*m.*
62.	56	$506,88	62	657*dollars.*
62.	57	$6336	63	$24,375
62.	58	$5545	64	868*miles.*
62.	59	$16763832	65	7℔ 2℥ ʒ7 0℈ 12*grs.*
62.	60	488*mi.* 1*fur.* 24*rd.*	66	411*bu.* 1*pk.* 0*qt.*
63.	67	146484 *yards.*	74	$1417
63.	68	427816 *barrels.*	75	$65962788,75
63.	69	$84,26	76	750 *days.*
63.	70	$16875,60	77	$13500
63.	71	2*T.* 18*cwt.* 1*qr.* 21*lb.*	78	$243
63.	72	$971,04	—	——
63.	73	461 *barrels left.*	—	——
63.		$1315 *cost.*	—	——

P.	EX.	ANS.	EX.	ANS.	EX.	ANS.
64.	79	$4770,755	84	84*rd.* 14 *ft.*	88	216 *men.*
64.	80	$61	85	50	89	$149,20
64.	81	$672	86	24*cords.*	90	37816 *tons.*
64.	82	$11914	87	$92	91	$70,02
64.	83	286*yr.* 9*mo.*	—	——	—	——

P.	EX.	ANS.	EX.	ANS.
65.	92	4*bar.* 32*gal.* 3*qt.*	95	$202,50
65.	93	669*hhd.* 40*gal.* 2*qt.*	96	$21,475
65	94	$13650000	97	$927,35

P.	EX.	ANS.	EX.	ANS.	EX.	ANS.
66.	98	$18844,01	99	$132,935	100	£175 18*s.* 6*d.*
71.	1	6579	4	275155	6	1147187
71.	2	36842	5	7948312	7	72331642
71.	3	269368	—	——	—	——
72.	8	£15 19*s.* 9*d.*	20	$17,4512	32	$13195133\frac{1842}{3574}$
72.	9	4*A* 0*R.* 33*P.*	21	$\$3,842\frac{86}{124}$	33	$125139201\frac{13010}{45705}$
72.	10	9*yd.* 2*qr.* 1*na.*	22	$1,125	34	$269577255882\frac{5561}{17493}$
72.	11	$79,3445	23	$0,375	35	$142437577748\frac{35411}{47143}$
72.	12	$209,728	24	$0,81	36	$15395919\frac{12214}{37149}$
72.	13	$66862,18	25	$5,01	37	$30001000\frac{6347}{57143}$
72.	14	$15311409\frac{2}{48}$	26	$52,88	38	$131809655\frac{104990}{374567}$
72.	15	237132	27	9	39	$300335575\frac{273118}{571007}$
72.	16	177242	28	95	40	$9948157977\frac{218293}{678957}$
72.	17	68	29	$8	41	$59085714\frac{84}{121}$
72.	18	44670	30	763521	42	$125812\frac{15785}{57149}$
72.	19	275	31	$407294\frac{1080}{1754}$	43	$119191753\frac{90107}{123456}$

P.	EX.	ANS.	EX.	ANS.
72.	44	17*A.* 3*R.* 5*P.*	48	2*bush.* 0*pk.* 7*qt.*
72.	45	1*da.* 12*hr.* 31*m.* 30*sec.*	49	$25,25
72.	46	35*mi.* 0*fur.* 29*rd.*	50	2*s.* 4*d.*
72.	47	49*gal.* $3\frac{87}{120}$*qt.*	—	—

P.	EX.	ANS.	EX.	ANS.	EX.	ANS.
73.	51	22*mi.* 1*fur.* 8*rd.*	57	$4,75	63	47 *rings.*
73.	52	316*A.* 1*R.* 35*P.*	58	$12,50	64	1*T.* 13*cwt.* 3*qr.*
73.	53	$27, 397 +	59	$757188\frac{48}{104}$	65	45*c. ft.* 995*c. in.*
73.	54	98765	60	$1,625	66	$30\frac{7152}{9285}$*tons.*
73.	55	$11250	61	365*da.*	67	$4424\frac{9}{213}$*vols.*
73.	56	$148018\frac{248}{365}$	62	800008	—	—

P.	EX.	ANS.	EX.	ANS.	EX.	ANS.
74.	2	7175	2	2725	5	15423
74.	3	4600	3	387321	6	2674584
74.	4	168525	4	4413840	7	280082
74.	5	76850	—	—	—	—

P.	EX.	ANS.	EX.	ANS.	EX.	ANS.	EX.	ANS.
75.	1	4800	4	8380225	3	355850400	2	2199176000
75.	2	5950	1	$55975066\frac{2}{3}$	4	148072400	3	242601500
75.	3	185000	2	$49357466\frac{2}{3}$	1	7408000	4	17573500

P.	EX.	ANS.	EX.	ANS.	EX.	ANS.	EX.	ANS.
76.	1	254	5	$211488\frac{60}{100}$	9	140848	13	242172
76.	2	$853\frac{80}{100}$	6	978	10	2025	14	2250
76.	3	$26251\frac{20}{100}$	7	852	11	39252	15	48126
76.	4	291147	8	954	12	475542	16	16215

P.	EX.	ANS.	EX.	ANS.	EX.	ANS.	EX.	ANS.
77.	1	387	3	532	5	15911	7	1809
77.	2	1548	4	804	6	1935	8	3216

P.	EX.	ANS.	EX.	ANS.	EX.	ANS.
78.	1	$1322\frac{275}{315}$	4	$83253\frac{94}{105}$	6	$4073\frac{652}{1155}$
78.	2	$1740\frac{226}{462}$	5	$2459\frac{328}{385}$	7	$9507\frac{1687}{7560}$
78.	3	$218\frac{3399}{3456}$	—	—	—	—

P.	EX.	ANS.	EX.	ANS.	EX.	ANS.
79.	1	$4\frac{292654}{420000}$	3	$91\frac{271606}{801400}$	5	$253\frac{43400}{146000}$
79.	2	146	4	$158732\frac{25800}{72000}$	6	$2247\frac{53398}{63000}$

P.	EX.	ANS.	EX.	ANS.	EX.	ANS.	EX.	ANS.
80.	1	$196\frac{22}{33}$	3	$61096\frac{52}{57}$	5	$9095\frac{25}{48}$	7	$6150\frac{75}{140}$
80.	2	$3117\frac{30}{46}$	4	$727\frac{369}{409}$	6	$6992\frac{46}{89}$	8	$4079\frac{34}{122}$

P.	EX.	ANS.	EX.	ANS.	EX.	ANS.	EX.	ANS.	EX.	ANS.	EX.	ANS.	EX.	ANS.
81.	1	$142	2	$17	3	$14	4	$35	6	$172	7	$120	8	$90

P.	EX.	ANS.	EX.	ANS.	EX.	ANS.
82.	1	$121,615	2	$67,50	3	$121,9115

P.	EX.	ANS.	EX.	ANS.	EX.	ANS.	EX.	ANS.
83.	1	$3,024	2	$12,8915	3	$9,198	4	$18,22765

P.	EX.	ANS.	EX.	ANS.
85.	1	5° 13′ diff. in long.	2	1*hr.* 2*m.* 8*sec.* P.M.
85.		20*m.* 52*sec.* diff. in time	—	—

P.	EX.	ANS.	EX.	ANS.
86.	3	13° 23′	5	8*hr.* 12*m.* A. M.
86.	4	4*m.* 36*sec.*	6	10° 34′

P.	EX.	ANS.	EX.	ANS.
86.	7	35° 11′	1	$128
86.	8	84° 42′	2	2*bu.* 1*pk.*
86.		10*hr.* 17*m.* 48*sec.* P M.	3	$53,28
86.	9	120°	4	32 *barrels.*
86.	10	1*hr.* 2*m.* 30*sec.* Fast.	5	463684

87.	6	41666$\frac{4}{6}$ *gallons.*	15	$812,25
87.	7	57979$\frac{276}{400}$	16	$147,9375
87.	8	555$\frac{25}{45}$	17	£14 14*s.*
87.	9	7*mo.* 1*wk.* 4½*da.*	18	£166 2*s.* 8*d.*
87.	10	12 *years.*	19	6*d.*
87.	11	6*mo.* 0*w.* 5*d.* 14*h.* 40*m.*	20	$6,95175
87.	12	77 *barrels.*	21	$8,64
87.	13	$72	22	$93.
87.	14	$5	—	———

88.	23	$7,875	28	$19	33	7680
88.	24	18 *cents.*	29	6480*c. ft.*	34	1*lb.* 7*oz.* 12*pwt.* 11*gr.*
88.	25	36	30	$773,395	35	$10
88.	26	45*lb.* 6*oz.* 11*pwt.*	31	$4,2408	36	2*bu.* 1*pk.* 7*qt.*
88.	27	$50	32	$16,7025	37	$0,75

89.	38	104 sheep.	44	$598281
89.	39	42 days	45	31680 times.
89.	40	16 cannisters.	46	130 farms.
89.	41	52*gal.* 1*qt.*	47	119$\frac{7747}{34337}$ acres.
89.	42	4424	48	$44397293
89.	43	96 acres.	49	11*hr.* 4*mi.* 32*sec.*, A. M.

90.	50	127° ′30	57	6° 11*mi.* 1*fur.* 34*rd.* 2*yd.*
90.	51	79° 5′ A's Long.	58	1000000*ft.*
90.		9*hr.* 19*m.* P. M., B's time.	59	13824*rods.*
90.	52	10*cords*, 7*cord ft.* 15*c. ft.*	60	36100
90.	53	1*cwt.* 3*qr.* 9*lb.* 10*oz.*	61	29*mi.* 3*fur.* 2*rd.* 16*ft.*
90.	54	$164,475	62	10*acres.*
90.	55	282*yr.* 6*mo.* 8*da*	63	36*yards.*
90.	56	6*gal.* 2*qt.* 0*pt.* 2*gi.*	—	———

91.	64	3*yd.* 1*qr.* 3*na.*	69	25*yr.* 6*mo.* 16*da.* 9*hr.*
91.	65	33 of each.	70	$10591021,60
91.	66	13209 +	71	$2478 widow's share
91.	67	$11,88		$1239 each child's "
91.	68	1*yr.* 205*da.* 10*hr.* 52*m.* 30*sec.*	72	$9

92.	73	13068 shingles.	74	107° 47′ diff. in long.
92.	—	———		7*hr.* 11*m.* 8*sec.* diff. in time.

P.	EX.	ANS.	EX.	ANS.
92.	75	1*hr.* 11*m.* 8*sec.*, *P. M.*	78	$2
92.	76	4*hr.* 56*m.*, *P. M.*	79	4333⅗ *schoolhouses.*
92.		26° *from N. York.*	80	46½*lbs.*
92.	77	48 *hours.*	81	14 *days.*
93.	82	28*bar.* 6*gal.*	90	482*bu.* 1*pk.* 2*qts.* = 1st.
93.	83	24*bar.* 19*gals.*		160*bu.* 3*pk.* 0*qt.* 1⅓*pt.* = 2d.
93.	84	$85,33⅓		321*bu.* 2*pk.* 1*qt.* ⅔*pt.* = 3d.
93.	85	11⅗ *rolls.*	91	40° 50′ *East.*
93.	86	7*mi.* 6*fur.* 20*rd.*		35$\frac{7}{20}$ *days.*
93.	87	8750 *pounds.*	92	$2400 = *Captain's share.*
93.	88	$16,025		$2000 = 2 *Lieut.'s* "
93.	89	2500 *barrels.*		$3600 = 6 *Midship.* "
93.	—	—— ——		$ 200 = *each sailor's* "
94.	93	87° 30′	98	514 *eagles.*
94.	94	9*hr.* 33*m.* 14*sec.*, *A. M.*	99	2011 *bushels.*
94.	95	10*hr.* 54*m.* 16*sec.* *A. M.*	100	$7410
94.	96	19°	101	1*yr.* 338*da.* 12*hr.*
94.	97	4800*yds.*	—	—— ——
96.	1	3×3; 2×5; 2×2×3; 2×7; 2×2×2×2;		
96.		3×3×2; 2×2×2×3; 3×3×3; 2×2×7.		
96.	2	2×3×5; 2×11; 2×2×2×2×2; 3×3×2×2;		
96.		2×19; 2×2×2×5; 3×3×5; 7×7;		
96.	3	2×5×5; 2×2×2×7; 2×29; 2×2×3×5;		
96.		2×2×2×2×2×2; 2×3×11; 2×2×17;		
96.		2×5×7; 2×2×2×3×3.		
96.	4	2×2×19; 2×3×13; 2×2×2×2×5; 2×41;		
96.		2×2×3×7; 2×43; 2×2×2×11; 2×3×3×5.		
97.	5	2×2×23; 2×47; 2×2×2×2×2×3; 2×7×7;		
97.		3×3×11; 2×2×5×5; 2×3×17; 2×2×2×13.		
97.	6	3×5×7; 2×53; 2×2×3×3×3; 2×5×11;		
97.		5×23; 2×2×29; 2×2×2×3×5; 5×5×5.		
97.	7	2×151; 5×61; 2×2×151; 5×5×5×7;		
97.		3×5×5×13; 5×131.		
97.	8	5×3×2.		
97.	9	2×3×7.		
97.	10	3×5×7.		
97.	11	2×3×7.		
97.	12	2×2.		
97.	13	2×3×5×7.		

100.	2	18	3	12	4	5	5	6	6	10	7	28	8	14

P.	EX.	ANS.	EX.	ANS.	EX.	ANS.
101.	1	16	6	81	11	$22 *per head.*
101.	2	7	7	45 *bushels.*		13, *A bought.*
101.	3	22	8	25 *acres.*		21, *B* "
101.	4	124	9	12 *feet.*		29, *C* "
101.	5	62	10	3 *bushels.*	—	—— ——

P.	EX.	ANS.	EX.	ANS.	EX.	ANS.	EX.	ANS.
103.	1	1260	5	10500	9	336	12	112 *men at* $15
103.	2	7200	6	10800	10	1176		105 " $16
103.	3	1260	7	540	11	144*rods.*		80 " $21
103.	4	1008	8	420	12	$1680		70 " $24

P.	EX.	ANS.	EX.	ANS.
104.	13	210 *bushels.*	14	60 *days.*
104.		*bags* 105 *times.*		*A*, 3 *times.*
104.		*barrels* 70 "		*B*, 4 "
104.		*boxes* 30 "		*C*, 5 "
104.		*hhds.* 14 "		*D*, 6 "

P.	EX.	ANS.	EX.	ANS.	EX.	ANS.
106.	1	32	6	$4\frac{2}{3}$	11	9 *dozen.*
106.	2	$3\frac{3}{4}$	7	8	12	36 *pounds.*
106.	3	14	8	$\frac{7}{45}$	13	46 *bushels.*
106.	4	48	9	$6\frac{2}{5}$	14	4 *firkins.*
106.	5	$8\frac{8}{9}$	10	27	—	——

P.	EX.	ANS.	EX.	ANS.	EX.	ANS.
107.	15	$16\frac{2}{3}$ *days.*	18	15 *barrels.*	21	$17\frac{1}{2}$ *bushels.*
107.	16	8 *pieces.*	19	6210 *bushels.*	22	$11\frac{1}{4}$ *days.*
107.	17	$471\frac{1}{2}$ *bushels.*	20	$6\frac{3}{4}$ *bushels.*	23	$4\frac{1}{2}$ *boxes.*

P.	EX.	ANS.	EX.	ANS.
111.	2	$\frac{15}{19}$; $\frac{37}{49}$.	4	$\frac{45}{68}$; $\frac{56}{68}$; $\frac{85}{68}$; $\frac{95}{68}$; $\frac{37}{68}$.
111.	3	$\frac{27}{40}$; $\frac{95}{40}$; $\frac{106}{40}$; $\frac{87}{40}$; $\frac{41}{40}$.	5	$\frac{9}{90}$; $\frac{87}{90}$; $\frac{75}{90}$; $\frac{65}{90}$; $\frac{85}{90}$; $\frac{90}{90}$; $\frac{100}{90}$.

P.	EX.	ANS.	EX.	ANS.
114.	1	$\frac{18}{8}$; $\frac{21}{8}$	2	$8\frac{1}{2}$; $4\frac{1}{4}$; $2\frac{1}{17}$.
114.	2	$3\frac{4}{8}$; $7\frac{7}{8}$.	3	$\frac{3}{4}$; $\frac{9}{8}$; $\frac{3}{2}$; $\frac{9}{4}$; $\frac{9}{3}$.
114.	3	$1\frac{24}{31}$; $1\frac{29}{31}$.	4	$\frac{7}{5}$; $\frac{7}{6}$; $\frac{7}{3}$; $\frac{7}{2}$;
114.	4	$3\frac{9}{25}$; $3\frac{23}{25}$.	5	$\frac{17}{24}$; $\frac{17}{16}$; $\frac{17}{12}$; $\frac{17}{8}$; $\frac{17}{6}$; $\frac{17}{4}$; $\frac{17}{3}$; $\frac{17}{2}$.
114.	5	$9\frac{2}{5}$; $12\frac{8}{15}$.	6	$\frac{6}{20}$; $\frac{6}{10}$; $\frac{6}{8}$; $\frac{6}{4}$; $\frac{6}{2}$.
114.	6	$5\frac{3}{19}$; $6\frac{12}{19}$.	7	$\frac{7}{5}$; $\frac{7}{7}$.
114.	7	$8\frac{11}{28}$; $16\frac{22}{28}$.	8	$\frac{6}{2}$; $\frac{6}{7}$; $\frac{6}{6}$; $\frac{6}{14}$; $\frac{6}{21}$.
114.	8	$2\frac{23}{29}$; $10\frac{7}{29}$.	9	$\frac{19}{18}$; $\frac{19}{12}$; $\frac{19}{9}$; $\frac{19}{6}$; $\frac{19}{4}$; $\frac{19}{3}$.
114.	9	$1\frac{1}{4}$; $2\frac{1}{2}$.	—	—— ——

P.	EX.	ANS.	EX.	ANS.
115.	1	$\frac{8}{19}$; $\frac{4}{19}$; $\frac{2}{19}$; $\frac{1}{19}$.	4	$\frac{12}{26}$; $\frac{10}{26}$; $\frac{6}{26}$; $\frac{4}{26}$; $\frac{3}{26}$.
115.	2	$\frac{7}{11}$; $\frac{2}{11}$; $\frac{1}{11}$.	5	$\frac{9}{19}$; $\frac{6}{19}$; $\frac{3}{19}$; $\frac{2}{19}$.
115.	3	$\frac{10}{19}$; $\frac{4}{19}$; $\frac{5}{19}$; $\frac{2}{19}$.	6	$\frac{8}{25}$; $\frac{4}{25}$; $\frac{3}{25}$; $\frac{2}{25}$.

P.	EX.	ANS.	EX.	ANS.
115.	7	$\frac{9}{29}$; $\frac{3}{29}$; $\frac{1}{29}$.	4	$\frac{30}{282}$; $\frac{30}{276}$; $\frac{30}{517}$.
115.	8	$\frac{9}{59}$; $\frac{6}{59}$; $\frac{2}{59}$; $\frac{1}{59}$.	5	$\frac{15}{119}$; $\frac{15}{85}$; $\frac{15}{51}$.
115.	1	$\frac{3}{24}$; $\frac{3}{28}$; $\frac{3}{32}$.	6	$\frac{14}{189}$; $\frac{14}{216}$; $\frac{14}{162}$.
115.	2	$\frac{4}{45}$; $\frac{4}{36}$; $\frac{4}{81}$.	7	$\frac{25}{57}$; $\frac{25}{133}$; $\frac{25}{209}$.
115.	3	$\frac{14}{51}$; $\frac{14}{68}$; $\frac{14}{204}$.	8	$\frac{11}{120}$; $\frac{11}{60}$; $\frac{11}{150}$.
116.	1	$\frac{28}{32}$; $\frac{42}{48}$; $\frac{35}{40}$.	2	$\frac{1}{2}$.
116.	2	$\frac{40}{55}$; $\frac{64}{88}$; $\frac{72}{99}$; $\frac{88}{121}$.	3	$\frac{12}{18}$; $\frac{8}{12}$; $\frac{6}{9}$; $\frac{4}{6}$; $\frac{2}{3}$.
116.	3	$\frac{112}{133}$; $\frac{128}{152}$; $\frac{144}{171}$;	4	$\frac{24}{32}$; [illegible]; $\frac{6}{8}$; $\frac{3}{4}$.
116.	4	$\frac{70}{145}$; $\frac{112}{232}$; $\frac{84}{174}$; $\frac{168}{348}$.	5	$\frac{36}{48}$; $\frac{24}{32}$; $\frac{18}{24}$; $\frac{12}{16}$; $\frac{6}{8}$.
116.	5	$\frac{46}{50}$; $\frac{69}{75}$; $\frac{92}{100}$; $\frac{115}{125}$.	6	$\frac{18}{72}$; $\frac{12}{48}$; $\frac{9}{36}$; $\frac{6}{24}$; $\frac{1}{4}$.
116.	1	$\frac{2}{4}$; $\frac{1}{2}$.	—	—— ——

117.	1	$\frac{1}{7}$	3	$\frac{1}{3}$	5	$\frac{5}{8}$	7	$\frac{2}{13}$	9	$\frac{7}{9}$
117.	2	$\frac{1}{5}$	4	$\frac{1}{8}$	6	$\frac{117}{265}$	8	$\frac{7}{9}$	10	$\frac{13}{7}$.

118.	11	$\frac{4}{7}$	21	$\frac{1}{25}$	8	$22\frac{3}{14}$
118.	12	$\frac{187}{515}$	22	$\frac{2}{123}$	9	$640\frac{9}{160}$ *acres.*
118.	13	$\frac{41}{51}$	1	$1\frac{5}{7}$	10	$5\frac{273}{841}$
118.	14	$\frac{69}{349}$	2	12	11	$14\frac{375}{1256}$
118.	15	$\frac{309}{361}$	3	$5\frac{4}{9}$	12	225
118.	16	$\frac{183}{2381}$	4	$24\frac{7}{8}$	13	$191\frac{3}{5}$
118.	17	$\frac{12}{13}$	5	9*lb.*	14	14
118.	18	$\frac{5}{24}$	6	$56\frac{1}{7}$*da.*	15	$376\frac{317}{999}$
118.	19	$\frac{11}{37}$	7	$112\frac{3}{14}$*yd.*	16	$10731\frac{55}{349}$
118.	20	$\frac{21}{134}$	—	——	—	——

119.	1	$\frac{319}{8}$	9	$\frac{28278}{151}$	16	$\frac{1503}{4}$
119.	2	$\frac{1129}{10}$	10	$\frac{1346}{9}$	17	$\frac{29251}{63}$
119.	3	$\frac{10259}{24}$	11	$\frac{37219}{99}$	18	$\frac{61451}{640}$
119.	4	$\frac{34513}{51}$	12	$\frac{1749383049}{99999}$	19	$\frac{110249}{112}$
119.	5	$\frac{38177}{104}$	13	$\frac{459287}{95}$	20	$\frac{12882}{366}$
119.	6	$\frac{148261}{175}$	14	$\frac{16106}{9}$	21	1178
119.	7	$\frac{59267822}{879}$	15	$\frac{881}{7}$	22	78
119.	8	$\frac{135187}{200}$	—	——	23	$333\frac{3}{3}$

120.	1	$\frac{126}{7}$	4	$\frac{406}{14}$	7	$\frac{5400}{12}$	10	$\frac{14863}{89}$
120.	2	$\frac{300}{12}$	5	$\frac{2405}{37}$.	8	$\frac{11772}{36}$	11	$\frac{24375}{75}$
120.	3	$\frac{152}{8}$	6	$\frac{1305}{9}$	9	$\frac{12416}{128}$	—	——

P.	EX.	ANS.	EX.	ANS.	EX.	ANS.	EX.	ANS.
121.	1	$\frac{5}{12}$	5	$\frac{3}{16}$	9	147	12	$\frac{41}{3080}$
121.	2	$\frac{7}{30}$	6	$\frac{75}{64}$	10	$8\frac{1}{6}$	13	$13\frac{21}{43}$
121.	3	$\frac{9}{14}$	7	1	11	$\frac{15}{416}$	—	——
121.	4	$\frac{5}{18}$	8	$35\frac{3}{4}$	—	——	—	——
122.	1	$\frac{63}{84}$; $\frac{348}{34}$; $\frac{72}{84}$	1	$\frac{9}{12}$; $\frac{7}{12}$; $\frac{6}{12}$; $\frac{10}{12}$				
122.	2	$\frac{126}{210}$; $\frac{140}{210}$; $\frac{30}{210}$; $\frac{525}{210}$	2	$\frac{18}{21}$; $\frac{8}{21}$; $\frac{14}{21}$				
122.	3	$\frac{570}{60}$; $\frac{260}{60}$; $\frac{165}{60}$; $\frac{48}{60}$	3	$\frac{84}{20}$; $\frac{18}{20}$; $\frac{145}{20}$				
122.	4	$\frac{16}{24}$; $\frac{21}{24}$; $\frac{20}{24}$; $\frac{12}{24}$; $\frac{54}{24}$	4	$\frac{190}{18}$; $\frac{15}{18}$; $\frac{132}{18}$				
122.	5	$\frac{4725}{630}$; $\frac{540}{630}$; $\frac{280}{630}$; $\frac{378}{630}$	5	$\frac{186}{30}$; $\frac{25}{30}$; $\frac{220}{30}$				
122.	6	$\frac{220}{42}$; $\frac{399}{42}$	6	$\frac{32}{40}$; $\frac{35}{40}$; $\frac{580}{40}$; $\frac{150}{40}$				
122.	7	$\frac{5}{28}$; $\frac{9}{28}$	7	$\frac{42}{72}$; $\frac{64}{72}$; $\frac{204}{72}$; $\frac{99}{72}$				
122.	8	$\frac{88}{18}$; $\frac{42}{18}$; $\frac{99}{18}$; $\frac{108}{18}$	8	$\frac{36}{42}$; $\frac{7}{42}$; $\frac{32}{42}$; $\frac{28}{42}$				
122.	9	$\frac{156}{30}$; $\frac{36}{30}$; $\frac{105}{30}$; $\frac{110}{30}$	9	$\frac{36}{44}$; $\frac{33}{44}$; $\frac{38}{44}$; $\frac{22}{44}$				
122.	10	$\frac{672}{168}$; $\frac{264}{168}$; $\frac{833}{168}$	10	$\frac{150}{60}$; $\frac{310}{60}$; $\frac{54}{60}$; $\frac{265}{60}$				
122.	11	$\frac{266}{21}$; $\frac{9}{21}$; $\frac{129}{21}$; $\frac{7}{21}$	—	——				
123.	1	$\frac{63}{168}$; $\frac{96}{168}$; $\frac{70}{168}$	4	$\frac{129}{24}$; $\frac{106}{24}$; $\frac{7}{24}$				
123.	2	$\frac{15}{42}$; $\frac{18}{42}$; $\frac{32}{42}$	5	$\frac{254}{30}$; $\frac{12}{30}$; $\frac{7}{30}$				
123.	3	$\frac{88}{32}$; $\frac{10}{32}$; $\frac{9}{32}$	6	$\frac{642}{66}$; $\frac{9}{66}$; $\frac{10}{66}$				
124.	7	$\frac{105}{42}$; $\frac{136}{42}$; $\frac{3}{42}$	12	$\frac{128}{20}$; $\frac{174}{20}$; $\frac{49}{20}$				
124.	8	$\frac{164}{48}$; $\frac{56}{48}$; $\frac{18}{48}$; $\frac{27}{48}$	13	$\frac{354}{66}$; $\frac{405}{66}$; $\frac{4}{66}$				
124.	9	$\frac{96}{108}$; $\frac{20}{108}$; $\frac{21}{108}$	14	$\frac{36}{68}$; $\frac{142}{68}$; $\frac{73}{68}$				
124.	10	$\frac{348}{78}$; $\frac{555}{78}$; $\frac{10}{78}$	15	$\frac{416}{72}$; $\frac{452}{72}$; $\frac{14}{72}$; $\frac{1}{72}$				
124.	11	$\frac{124}{36}$; $\frac{226}{36}$; $\frac{37}{36}$	—	——				
125.	1	$5\frac{2}{5}$	7	$1\frac{11}{20}$	12	$2\frac{21}{64}$		
125.	2	$2\frac{8}{9}$	8	$2\frac{23}{30}$	13	$1\frac{187}{1008}$		
125.	3	$2\frac{7}{11}$	9	$2\frac{17}{56}$	14	$4\frac{14}{15}$		
125.	4	$4\frac{1}{15}$	10	$2\frac{133}{204}$	15	$4\frac{20}{33}$		
125.	5	$6\frac{5}{7}$	11	$3\frac{97}{144}$	16	$2\frac{451}{2040}$		
125.	6	$2\frac{3}{4}$	—	——	—	——		
126.	18	$\frac{5}{6}$; $\frac{7}{10}$; $\frac{16}{63}$; $\frac{19}{90}$	21	$9\frac{25}{28}$	24	$26\frac{11}{84}$		
126.	19	$\frac{22}{120}$; $\frac{31}{240}$; $\frac{15}{54}$; $\frac{13}{40}$	22	$15\frac{337}{600}$	25	$10\frac{53}{112}$		
126.	20	$39\frac{103}{105}$	23	$11\frac{9}{20}$	26	$4\frac{23}{24}$		

P.	EX.	ANS.	EX.	ANS.	EX.	ANS.
126.	27	$6\frac{41}{110}$	31	$170\frac{79}{168}$	35	$212\frac{7}{32}$ *pounds.*
126.	28	$6\frac{43}{88}$	32	$80\frac{41}{126}$	36	$\$16\frac{17}{18}$
126.	29	$24\frac{489}{616}$	33	$\$18\frac{5}{24}$	37	$65\frac{61}{120}$ *pounds.*
126.	30	$13\frac{113}{168}$	34	$89\frac{5}{2\text{[illegible]}}$*mi.*	38	$100\frac{13}{90}$*cwt.*

P.	EX.	ANS.	EX.	ANS.
127.	39	$89\frac{16}{45}$ *acres.*	2	2*da.* $14\frac{1}{2}$*hr.*
127.	40	$347\frac{11}{12}$ *bushels.*	3	1*cwt.* 2*qr.* 2*lb.* 13*oz.*
127.	40	$\$417\frac{313}{360}$.	4	2*oz.* 10*pwt.* 12*gr.*
127.	1	$14\frac{1}{18}$ *inches.*	—	———

P.	EX.	ANS.	EX.	ANS.
128.	5	9*cwt.* 1*qr.* 5*lb.* $8\frac{8}{9}$*oz.*	16	7*oz.* 7*pwt.* 23*grs.*
128.	6	20*bu.* 1*pk.* $5\frac{5}{7}$*qt.*	17	5 *signs* 16° 16′ $40\frac{20}{133}$″.
128.	7	3*hhd.* 37*gal.* 1*qt.* $1\frac{1}{5}$*pt.*	18	1*yd.* 0*qr.* $2\frac{5}{6}$ *na,*
128.	8	1*mo.*3*w.*1*d.*18*hr.*23*m.*30*s.*	19	1 *cord ft.* 11*c.ft.* $219\frac{3}{7}$*c.in.*
128.	9	2*R.*20*P.*11*sq.ft.*$58\frac{1}{35}$*sq.in.*	20	2 *cords,* 3 *cord ft.* 8*c. ft.*
128.	10	7 *inches.*	21	3*yd.* 2*qr.* $0\frac{2}{3}$*na.*
128.	11	13*s.* $10\frac{2}{3}$*d.*	22	3*A.* 2*R.* $31\frac{1}{4}$*P.*
128.	12	2*da.* 2*hr.* 30*m.* 45*sec.*	23	11*cwt.*3*qr.*18*lb.*11*oz.*$1\frac{1}{3}$*dr.*
128.	13	7*fur.* 2*ft.* 9*in.*	24	3*fur.* 0*rd.* 2*ft.* 6*in.*
128.	14	7*mo.* 1*wk.* 1*da.* 1*hr.* 24*m.*	25	2*wk.* 3*da.* 3*hr.* 58*m.* 36*sec.*
128.	15	12*cwt.* 1*qr.* 1*lb.* 2*oz.* $11\frac{3}{7}$*dr.*	—	———

P.	EX.	ANS.	EX.	ANS.	EX.	ANS.
130.	1	$\frac{2}{7}$	10	$\frac{43}{72}$	19	$9\frac{14}{15}$
130.	2	$\frac{3}{19}$	11	$24\frac{4}{15}$	20	$6\frac{29}{40}$
130.	3	$\frac{4}{25}$	12	$1\frac{27}{135}$	21	$8\frac{7}{8}$
130.	4	$\frac{100}{305}$	13	$\frac{1}{8}$	22	$\frac{2}{9}$ *sold;* $\frac{7}{9}$ *left.*
130.	5	$\frac{2}{35}$	14	$3\frac{1}{24}$	23	$72
130.	6	$\frac{5}{48}$	15	$7\frac{3}{5}$	24	$\$\frac{3}{8}$
130.	7	$\frac{2}{195}$	16	$14\frac{3}{5}$	25	$18\frac{11}{14}$ *gallons.*
130.	8	$35\frac{71}{90}$	17	$\frac{1}{40}$	26	$18\frac{13}{18}$ *cords.*
130.	9	$\frac{7}{36}$	18	$3\frac{2}{9}$	27	$33\frac{3}{20}$ *pounds.*

P.	EX.	ANS.	EX.	ANS.	EX.	ANS.	EX.	ANS.
131.	28	$\$22\frac{3}{16}$	30	$18\frac{2}{3}$ *yards.*	2	$76\frac{1}{8}$	4	$6\frac{33}{38}$
131.	29	$\$\frac{3}{10}$	1	$2\frac{34}{133}$	3	$73\frac{31}{32}$	5	$182\frac{83}{100}$

P.	EX.	ANS.	EX.	ANS.
132.	1	9*oz.* 7*pwt.* 12*grs.*	7	563*A.* 0*R.* $35\frac{5}{9}$*P.*
132.	2	7*cwt.* 1*qr.* 24*lb.* 8*oz.*	8	10*cwt.* 1*qr.* 15*lb.* $7\frac{13}{21}$*oz.*
132.	3	29*gal.* $3\frac{5}{8}$*qt.*	9	1*lb.* 8*oz.* 16*pwt.* 16*gr.*
132.	4	1*mi.* 1*fur.* 16*rd.*	10	2 *cords,* 2 *cord ft.* 4*c. ft.*
132.	5	1*s.* 3*d.*	11	5*in.* 1*bar.*
132.	6	38′ $34\frac{2}{7}$″.	—	———

P.	EX.	ANS.	EX.	ANS.	EX.	ANS.	EX.	ANS.
133.	12	4 ℥ 3 ʒ 2 ℈ 4*gr*	13	1*pwt.* $18\frac{1}{2}$*gr.*				
134.	1	$3\frac{3}{7}$.	15	5405.	11	$\frac{1}{24}$.		
134.	2	$1\frac{7}{25}$.	16	6975.	12	$\frac{3}{20}$.		
134.	3	$7\frac{1}{5}$.	17	11725.	13	18.		
134.	4	$11\frac{1}{19}$.	1	$3\frac{5}{9}$.	14	$15\frac{3}{25}$		
134.	5	$9\frac{3}{8}$.	2	$12\frac{6}{7}$.	15	$130\frac{1}{2}$.		
134.	6	16.	3	$5\frac{13}{15}$.	16	$\frac{10}{66}$.		
134.	7	$32\frac{2}{3}$.	4	63.	17	14.		
134.	8	70.	5	$178\frac{1}{8}$.	18	$6316\frac{7}{8}$.		
134.	9	44.	6	$14\frac{124}{231}$.	19	$\frac{10}{21}$.		
134.	10	1584.	7	$19\frac{4}{9}$.	20	$6\frac{48}{55}$.		
134.	11	$608\frac{7}{12}$.	8	$\frac{9}{20}$.	21	$\frac{4}{15}$.		
134.	13	$5987\frac{5}{9}$.	9	$\frac{21}{40}$.	22	$1\frac{4}{5}$.		
134.	14	4536.	10	$\frac{5}{96}$.	—	——		
135.	23	$\frac{4}{7}$.	31	$\$5\frac{1}{4}$.	39	$\$8\frac{7}{16}$.		
135.	24	$\frac{1}{15}$.	32	$\$14\frac{4}{7}$.	40	55 *cents.*		
135.	25	$2\frac{2}{3}$.	33	$\$\frac{2}{3}$.	41	$\$34\frac{1}{3}$.		
135.	26	20.	34	$11\frac{1}{4}$ *tons.*	42	\$36.		
135.	27	$\frac{53}{56}$.	35	$\$22\frac{1}{2}$.	43	$43\frac{3}{4}$ *shillings.*		
135.	28	$\frac{23}{84}$.	36	$\$3\frac{1}{8}$.	44	\$325.		
135.	29	$2\frac{8}{21}$.	37	$\$14\frac{2}{3}$.	45	$\$\frac{5}{9}$.		
135.	30	540.	38	$\$7\frac{21}{32}$.	46	$\frac{3}{10}$.		
136.	47	$\$\frac{3}{8}$.	55	$228\frac{1}{8}$ *cents.*				
136.	48	$\$\frac{11}{24}$.	56	$\$63\frac{23}{24}$.				
136.	49	$\$20\frac{4}{5}$.	57	$\$10\frac{67}{80}$.				
136.	50	$12\frac{1}{2}$ *days.*	58	\$12000.				
136.	51	$5\frac{1}{3}$ *hours.*	59	$\frac{9}{35}$, *A's*; $\frac{6}{35}$, *B's.*				
136.	52	$\$66\frac{4}{5}$.	60	$\frac{1}{3}$, *D's.*				
136.	53	34 *miles.*	61	120 *acres, A's*; 80 *acres, B's*; 20 *acres, C's.*				
136.	54	$456\frac{1}{4}$*cts.*						
138.	1	$\frac{1}{5}$.	7	36.	13	$\frac{2}{5}$.	19	$1\frac{19}{58}$.
138.	2	$\frac{3}{28}$.	8	$\frac{7}{8}$.	14	$61\frac{1}{11}$.	20	$2\frac{13}{21}$.
138.	3	$\frac{13}{135}$.	9	$2\frac{2}{5}$.	15	$1662\frac{1}{2}$.	21	$9\frac{8}{13}$.
138.	4	$\frac{3}{319}$.	10	$1\frac{23}{40}$.	16	1363.	22	$48\frac{1}{35}$.
138.	5	$\frac{23}{832}$.	11	$\frac{112}{135}$.	17	$\frac{7}{8}$.	23	$\frac{40}{133}$.
138.	6	$7\frac{1}{7}$.	12	$1\frac{7}{128}$.	18	$\frac{26}{33}$.	24	$1\frac{37}{200}$.

P.	EX.	ANS.	EX.	ANS.	EX.	ANS.
138.	25	$\frac{7}{121}$.	34	$1\frac{1}{5}$.	43	$\frac{56}{189}$.
138.	26	$\frac{3}{17}$.	35	$\frac{243}{500}$.	44	$\frac{1}{3}$.
138.	27	$\frac{4}{27}$.	36	$\frac{112}{135}$.	45	$22\frac{6}{7}$.
138.	28	$\frac{1}{10}$.	37	$\frac{112}{135}$.	46	$68\frac{653}{781}$.
138.	29	$\frac{9}{521}$.	38	$1\frac{1}{2}$.	47	$3\frac{1}{3}$.
138.	30	$\frac{2}{125}$.	39	$825\frac{1}{2}$.	48	$1\frac{239}{1195}$.
138.	31	40.	40	$4193\frac{7}{17}$.	49	$9\frac{5}{9}$.
138.	32	1120.	41	$16046\frac{3}{32}$.	50	$72\frac{53}{180}$.
138.	33	$1\frac{1}{5}$.	42	$\frac{7}{9}$.	—	——
139.	51	$5\frac{1}{4}$*lbs.*	57	$\$1\frac{1}{15}$.	63	$\frac{56}{363}$.
139.	52	$1\frac{3}{32}$*lbs.*	58	6 *gallons.*	64	$\$\frac{2}{25}$.
139.	53	$1\frac{1}{2}$*bush.*	59	$\frac{5}{6}$ *of the whole.*	65	$\$\frac{2}{49}$.
139.	54	$4\frac{1}{2}$ *hours.*	60	21.	66	$\$5\frac{1}{4}$.
139.	55	$\$\frac{7}{10}$.	61	$27\frac{3}{8}$.	67	$19\frac{23}{51}$*lbs.*
139.	56	$\$3\frac{3}{35}$.	62	$14\frac{12}{123}$.	68	$14\frac{4}{5}$*bar.*
140.	69	$\$\frac{4}{9}$.	75	$24\frac{1}{2}$ *bottles.*	80	$13\frac{1}{8}$ *times.*
140.	70	$\$\frac{61}{64}$.	76	$1\frac{4}{9}$ *days.*	81	$\$\frac{4}{5}$.
140.	71	$\$13\frac{1}{3}$.	77	$\$10\frac{1}{2}$.	82	$6096.
140.	72	$108\frac{9}{16}$*bush.*	78	$4\frac{1}{2}$.	83	$\$43\frac{1}{3}$.
140.	73	$\frac{3}{4}$*yd.*	79	6.	84	$17\frac{7}{15}$.
140.	74	4 *days.*	—	——	—	——
142.	1	$1\frac{1}{24}$.	9	$\frac{350}{549}$.	4	$42\frac{479}{702}$.
142.	2	$\frac{128}{135}$.	10	$2\frac{68}{119}$.	5	$\frac{41}{840}$.
142.	3	$2\frac{62}{81}$.	11	$53\frac{1}{7}$.	6	$\$26\frac{7}{18}$.
142.	4	100.	12	$\frac{9}{64}$.	7	15 *yards.*
142.	5	$\frac{16}{81}$.	1	$15.	8	$\$16\frac{1}{3}$
142.	6	$\frac{5}{7}$.	2	$\$17\frac{3}{5}$	9	$\frac{101}{112}$.
142.	7	$1\frac{1}{4}$.	3	$7\frac{13}{35}$*miles.*	10	$\frac{4670593}{6242920}$ *sum,* $\frac{1717137}{6242920}$ *diff.*
142.	8	35.	—	——	—	—— ——
143.	11	$33\frac{1}{3}$*bush.*	19	$\$\frac{3}{5}$.		
143.	12	1*mi.* 2*fur.* 16*rd.*	20	14 *bushels.*		
143.	13	1*qr.* 23*lb.* 4*oz.* $11\frac{5}{21}$*dr.*	21	$20\frac{41}{44}$.		
143.	14	4*mi.* 7*fur.* 19*rd.* 2*yd.* $3\frac{27}{35}$*ft.*	22	$2700, *A's share.* $2800, *B's* " $ 800, *C's* "		
143.	15	$\$20\frac{1}{3}$.				
143.	16	$\$12\frac{1}{3}$.				
143.	17	272 *sheep.*	23	40		
143.	18	13 *yards.*	24	2*da.* 14*hr.* 30*m.*		

P.	EX.	ANS.	EX.	ANS.
143.	25	£7 17*s.* 5*d.* 0$\frac{4}{7}$*far.*	27	285$\frac{5}{7}$ *acres.*
143.	26	24 *marbles to John.*	28	*A*, 80; *B*, 24; *C*, 30;
143.		32 " *to James.*		*D*, 40; 66 *remain.*

P.	EX.	ANS.	EX.	ANS.
144.	29	\$467$\frac{2}{3}$.	32	7$\frac{1}{2}$ *bushels.*
144.		\$2$\frac{1}{16}$, *what it sold for.*	33	\$1724$\frac{1}{3}$, *A's.*
144.	30	\$$\frac{42}{208}$ = *first one's gain.*		\$1231$\frac{2}{3}$, *B's.*
144.		\$$\frac{49}{208}$ = *second* "	34	165 *sheep.*
144.	31	\$257$\frac{19}{28}$.	—	— —

P.	EX.	ANS.	EX.	ANS.
145.	1	7*ft.* 2′.	6	2*ft.* 7′ 3″.
145.	2	5*ft* 2′ 6″.	7	15*ft.* 4′ 10″ 4‴.
145.	3	21*ft.* 4′ 11″ 4‴.	8	3*ft.* 6′ 5″ 5‴.
145.	4	5*ft.* 7′.	9	87*ft.* 10′ 7″ 4‴.
145.	5	3′ 3″ 2‴.	10	183*ft.* 5′ 6″ 2‴.

P.	EX.	ANS.	EX.	ANS.
146.	11	223*ft.* 8′ 4″ 9‴.	14	107*ft.* 8′ 9″ 2‴.
146.	12	87*ft.* 2′ 7″ 9‴ 6⁗.	15	1721*ft.* 10′ 9″ 11‴ *sum.*
146.	13	317*ft.* 11′ 0″ 4‴.		280*ft.* 1′ 3″ 9‴ *diff.*

P.	EX.	ANS.	EX.	ANS.
147.	2	43*ft.* 6′ 6″.	7	194*ft.* 4′ 3″ 6‴.
147.	3	82*ft.* 9′ 4″.	8	[illegible]9*ft.* 11′ 2′ 3‴.
147.	4	347*ft.* 10′ 3″.	9	296*ft.* 10′ 6″.
147.	5	554*ft.* 7′ 8″ 8‴ 3⁗.	10	866*ft.* 8′ 3″.
147.	6	2917*ft.* 0′ 0″ 7‴ 4⁗.	—	— —

P.	EX.	ANS.	EX.	ANS.	EX.	ANS.
148.	11	\$208,01$\frac{1}{4}$.	14	849*ft.* 8′ 8″.	16	275$\frac{46}{243}$*c. yds.*
148.	12	89*ft.* 3′.	15	\$15,403+.	17	\$19,80$\frac{8}{9}$.
148.	13	\$18,49$\frac{1}{3}$.	—	—	—	—

P.	EX.	ANS.	EX.	ANS.	EX.	ANS.	EX.	ANS.
149.	1	4*ft.* 7′.	3	48*ft.* 6′.	5	12*ft.* 6′.	7	1*ft.* 7′.
149.	2	5*ft.* 3′ 3″.	4	8*ft.* 7′.	6	37*ft.* 3′.	8	8*ft.*

P.	EX.	ANS.	EX.	ANS.	EX.	ANS.
153.	1	.06	7	7.008	3	99.0027
153.	2	1.7	8	9.05	4	.320
153.	3	.005	9	11.50	5	200.000320
153.	4	.27	10	44.7	6	.3610
153.	5	.047	1	27.4	7	5.000003
153.	6	6.41	2	36.015	8	40.0000009

P.	EX.	ANS.	EX.	ANS.	EX.	ANS.
154.	9	.4900	14	105.0000095	4	\$275.005
154.	10	59.0067	1	\$37.265	5	\$9.008
154.	11	.0469	2	\$17.005	6	\$15.069
154.	12	79.000415	3	\$215.08	7	\$27.182
154.	13	67.0227	—	—	—	—

P.	EX.	ANS.	EX.	ANS.	EX.	ANS.
156.	1	1306.1805	4	1.5415	6	27.2087
156.	2	528.697893	5	446.0924	7	88.76257
156.	3	159.37	—	—	—	—

P.	EX.	ANS.	EX.	ANS.	EX.	ANS.
157.	8	71.01	13	204.0278277	18	3.8896 *tons.*
157.	9	1835.599	14	400.33269960	19	$427.835
157.	10	397.547	15	.1008879	20	$19.215
157.	11	31.02464	16	$85.463	21	$670.875
157.	12	1.110129	17	$1065.19	—	———
158.	22	$30.286	3	9.888899	6	1571.85
158.	1	3277.9121	4	51.722	7	.6946
158.	2	249.60401	5	2.7696	—	———
159.	8	.89575	16	6314.9	24	103.0150
159.	9	603.925	17	365.007495	25	.4232
159.	10	1379.25922	18	20.9942	26	171.925 *acres.*
159.	11	99.706	19	260.3608953	27	$82.625
159.	12	17.949	20	10.030181	28	$26.60
159.	13	.699993	21	2.0294	29	126.84194 *tons.*
159.	14	328.9992	22	999.999	30	$761.18
159.	15	.999	23	2499.75	31	1781.725 *pounds.*
160.	1	.796875	4	1.50050	6	10376.283913
160.	2	.263872	5	26.99178	7	165235.5195
160.	3	.0000500	—	———	—	———
161.	8	.0206211250	16	.00715248	24	148.28125 *acres.*
161.	9	28033.797099	17	.608785264	25	12.13035*ft.*
161.	10	175.26788356	18	.02860992	26	$24.0625
161.	11	.000432045770	19	2.435141056	27	$3191.805625
161.	12	216.94165850	20	1296	28	$210.03125
161.	13	.000000000294	21	312.5	29	$708.901875
161.	14	18616.74	22	.375	30	$2.06525 *gained.*
161.	15	933.8253150762	23	.0036	—	———

163.	2	258.13007	3	162.521	4	2757.89786	5	3566159.

165.	1	2.22	13	25.05068	15	41.622
165.	2	.852		250.5068		416.22
165.	3	33.331		2505.068		4162.2
165.	4	1.0001		25050.68		41622.
165.	5	12420.5		250506.8		416220.
165.	6	.005				4162200.
165.	7	4.25	14	48.65961	16	254.7347748
165.	8	.007		4865.961		25473.47748
165.	9	.75		48659.61		254734.7748
165.	10	1.27		486596.1		2547347.748
165.	11	.015		4865961.		25473477.48
165.	12	17.008				254734774.8

P.	EX.	ANS.	EX.	ANS.	EX.	ANS.
165.	17	.13956463+	21	69.7125	25	6.165*c. yd.*
165.	18	1918.515+	22	1.36832+	26	$9.875
165.	19	.004735	23	12976.81+	27	$2.15
165.	20	174.412033+	24	.004958+	28	$0.62
166.	29	18 *pounds.*	33	269 *acres,* $13574.204 *cost.*		
166.	30	8 *suits.*		$50.4617+, *average price per acre.*		
166.	31	14 *days.*	34	$7631.8855, *elder's share.*		
166.	32	55.5 *bush.*		$5723.914375, *each of others.*		
167.	2	10970	5	100		
167.	3	60200	6	10; 100; 1000; 30; 20; 2000; 12; 1200; 500000.		
167.	4	1000				

P.	EX.	ANS.	EX.	ANS.	EX.	ANS.	EX.	ANS.
168.	3	8.311+	4	1.563+	5	1.1604+	6	16.119+

P.	EX.	ANS.	EX.	ANS.	EX.	ANS.
170.	2	79.1188	4	.015625; .2666+	11	.536; .372
170.	3	35.2843	5	.125; .003	12	.9
170.	4	11.58340366	6	.2571+; .4411+	13	.7333+
170.	5	3202.8869	7	.23903+	14	.48375
170.	1	.25; .5; .75	8	.07157+	15	.5128
170.	2	.8; .875; .3125	9	.4375; .078125	16	.5375; 5606+
170.	3	.375; .04	10	.00448	17	.1666+
171.	18	1.555+	23	.25	4	$\frac{1603}{2000}$; $\frac{3021}{5000}$
171.	19	.15909+	24	2.8412	5	$\frac{547}{800}$
171.	20	$100.80	1	$\frac{1}{4}$; $\frac{3}{4}$	6	$\frac{3}{160}$
171.	21	$17.85	2	$\frac{1}{8}$; $\frac{5}{8}$	7	$\frac{301}{1600}$
171.	22	30.611+	3	$\frac{21}{200}$; $\frac{1}{400}$	8	$\frac{17}{64}$
172.	1	.0546875*lb.*	7	.15375 *tons.*	13	.875*yd.*
172.	2	£.325.	8	£.1225	14	.01587+ *hhd.*
172.	3	3.9375*pk.*	9	.26175*A.*	15	.7129975*da.*
172.	4	.375*da.*	10	.100511+ *mi.*	16	.2325 *tons.*
172.	5	71.1511+ *mi.*	11	.64*cwt.*	17	£.9729+
172.	6	.6625*lb.*	12	.91111+ *lb.*	—	——
173.	18	.48125*A.*	25	.7833+ *yr.*	32	.02142*yr.*
173.	19	.55*E.E.*	26	.9111*lb.*	33	.3489 ℔
173.	20	.0016186 *mi.*	27	.3375	34	.01537+ *hhd.*
173.	21	.25625°.	28	.3125 *chal.*	35	.005*A.*
173.	22	.041956 *tons.*	29	.0409*mi.*	36	.5725*yd.*
173.	23	.10416 *chal.*	30	.01875 *ream.*	37	3.390625*ft.*
173.	24	.00994318*mi.*	31	.02026*rd.*	—	——
174.	1	2*qr.* 17*lb.* 4*oz.*	3	16*s.* 7*d.* 2.99*far.*		
174.	2	1*hhd.* 13*gal.* 3.44*qt.*	4	2*gal.* 1*qt.*		

P.	EX.	ANS.	EX.	ANS.
174.	5	1*wk.* 4*da.* 23*hr.* 59*m.* 56.5*sec.*	14	32*mi.* 1*fur.* 14*rd.* 14*ft.* 6.24*in.*
174.	6	8.*P.*	15	2*ft.* 7.5*in.*
174.	7	6*cwt.* 3*qr.*	16	4 ℥ 1 ʒ 1 ℈ 9.6*gr.*
174.	8	1*hhd.* 47*gal.* 1*qt.*	17	3*R.* 5*P.* 13.31*sq. yd.*
174.	9	20*gal.* 1*qt.*+	18	9 *sheets.*
174.	10	10*oz.* 18*pwt.* 15.99*gr.*	19	11*lbs.*
174.	11	3*qr.* 1.3*na.*	20	7*d.* 2*far.*
174.	12	5*ft.* 11.9+ *in.*	21	1*R.* 14*P.*
174.	13	24*sq.rd.* 23*sq.yd.* 5*sq. ft.* 82.4*sq. in.*	22	286*da.* 17*hr.* 18*m.*
174.			—	—— ——
176.	1	.06	5	.029729+
176.	2	.09285+	6	.034
176.	3	.034375	7	.028
176.	4	.013281+	8	.043056+
179.	3	$\frac{2}{3}$; $\frac{6}{37}$; $\frac{4070}{5291}$; $\frac{35}{37}$; $\frac{1}{11}$	4	$\frac{85}{143}$; $\frac{4}{11}$; $\frac{1}{7}$
180.	4	$\frac{5}{36}$; $7\frac{269}{495}$; $\frac{29}{666}$; $37\frac{49}{90}$; $\frac{223}{330}$; $\frac{75434}{99999}$		
180.	5	$\frac{34}{45}$; $\frac{217}{495}$; $\frac{7}{75}$; $4\frac{1256}{1665}$; $\frac{163}{16500}$; $\frac{41}{90}$		

P.	EX.	ANS.	EX.	ANS.	EX.	ANS.
182.	2	.1875′.	3	.0‵0344827+	4	.‵097560′; .‵592′; .5‵3
183.	2	2.4‵181818′	1	165.16‵416416′	2	.5‵353′
183.		.5‵925925′		.04‵040404′		.4‵754′
183.		.008‵497133′		.03‵703703′		1.7‵577′

P.	EX.	ANS.	EX.	ANS.
184.	2	95.2‵829647′	5	47.4‵754481′
184.	3	69.74‵203112′	6	216.2‵5428763′
184.	4	55.6‵209780437503′	—	——
185.	2	45.7‵755′	6	4.38‵20′
185.	3	2.9‵957′	7	4.619‵525′
185.	4	5.09	8	1.0923‵7
185.	5	.65‵370016280906′	9	1.3462‵937′
186.	2	5.53705‵5	2	13.570413‵961038′
186.	3	1.093‵086′	3	35.028‵1+
186.	4	1.6411‵7	4	7.719‵54′
186.	5	1.7183‵39′	5	26.7837‵42857‵1
186.	6	1.4710‵037′	6	3.1‵45′
186.	7	6.16‵566′	7	3.‵8235294117647058′
186.	8	11.‵068735402′	8	1.2‵6
186.	9	.81654‵168350′	9	15.48‵423′

P. 188.

EX.	ANS.
1	$\frac{21}{39}=\cfrac{1}{1+\cfrac{1}{1+\frac{1}{6}}}$
2	$\frac{47}{65}=\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\frac{1}{3}}}}}}}$
3	$\frac{17}{27}=\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{2+\frac{1}{3}}}}}$
4	$\frac{67}{85}=\cfrac{1}{1+\cfrac{1}{3+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{1+\frac{1}{2}}}}}}}$
5	$\frac{37}{87}=\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{5+\frac{1}{2}}}}}$

P.	EX.	ANS.	EX.	ANS.	EX.	ANS.	EX.	ANS.	EX.	ANS.
191.	1	38	2	56	3	12	4	40	5	96

P.	EX.	ANS.	EX.	ANS.	EX.	ANS.
193.	2	$\frac{2}{3}$	6	$\frac{1}{6}$	9	$\frac{1}{5}$
193.	3	$\frac{1}{2}$	7	$\frac{2}{3}$	10	$\frac{1}{11}$
193.	4	$\frac{1}{2}$	8	$\frac{1}{3}$	11	$\frac{9}{11}$
193.	5	$\frac{2}{3}$	—	—	—	—

P.	EX.	ANS.	EX.	ANS.	EX.	ANS.
197.	1	\$330	10	1400 *pounds.*	19	\$382,85
197.	2	\$90	11	16485 *miles.*	20	\$63
197.	3	504 *miles.*	12	\$121,87$\frac{1}{2}$.	21	\$0,04
197.	4	\$208	13	216 *shillings.*	22	\$2,52
197.	5	\$875	14	7$\frac{1}{5}$.	23	\$1,925
197.	6	99 *pounds.*	15	\$3533,932+	24	\$2,10
197.	7	\$2762,50	16	\$86.62	25	\$52,50
197.	8	\$20	17	£39679.10*s.*	26	\$$\frac{36}{49}$
197.	9	\$122,85	18	\$39,37$\frac{1}{2}$.	—	—

P.	EX.	ANS.	EX.	ANS.	EX.	ANS.
198.	27	\$7200	31	\$18,66$\frac{2}{3}$	35	\$112,86
198.	28	\$37,909+	32	\$56,355	36	\$5427
198.	29	\$67,113+	33	106$\frac{2}{3}$ *yards.*	37	21*gal. water.*
198.	30	112$\frac{1}{2}$*bu.*	34	40 *weeks.*	—	—

P.	EX.	ANS.	EX.	ANS.
199.	38	*A*, \$2142; *B*, \$1125.	43	93$\frac{3}{4}$ *gallons.*
199.	39	\$0,62$\frac{1}{2}$.	44	552 *miles.*
199.	40	6$\frac{2}{3}$ *bottles.*	45	\$17444.
199.	41	126$\frac{14}{18}$ *shillings.*	46	6*hr.* 32*m.* 43$\frac{7}{11}$*sec.*
199.	42	168 *pounds.*	47	140*ft.*

P.	EX.	ANS.	EX.	ANS.
199.	48	*A*, 155 *miles*; *B*, 124 *miles*.	50	$22\frac{1}{2}$ *days*.
199.	49	$1\frac{1}{3}$ *days*.	—	——
200.	51	*A's*, $88,40; *B's*, $77,35.	54	$$24,66\frac{2}{3}$.
200.	52	10*hr*. 40*m*. $36\frac{7}{48}$*sec*.	55	$16\frac{1}{2}$ *times*.
200.	53	48*m*. $7\frac{1}{2}$*sec*.	—	——

P.	EX.	ANS.	EX.	ANS.	EX.	ANS.
202.	1	9 *yards*.	4	$7\frac{1}{5}$ *days*.	6	920
202.	2	$8\frac{2}{5}$ *rods*.	5	10	7	54 *days*.
202.	3	160 *yards*.	—	——	—	——

P.	EX.	ANS.	EX.	ANS.	EX.	ANS.	EX.	ANS.
203.	8	225 *days*.	11	588000*lb*.	14	12 *days*.	18	120*mo*.
203.	9	13 *ounces*.		14 *ounces*.	15	6 "	19	$5\frac{1}{3}$ *days*.
203.	10	588000*lb*.	12	20 *days*.	16	160 "	—	——
203.		546000*lb*.	13	54 *days*.	17	40.47.	—	——

P.	EX.	ANS.	EX.	ANS.	EX.	ANS.
204.	20	45 *men*.	24	$10\frac{2}{3}$ *rods*.	28	6 *horses*.
204.	21	$13\frac{3}{5}$ *ounces*.	25	$13\frac{1}{3}$ *days*.	29	$8571\frac{3}{7}$ *planks*.
204.	22	$13\frac{7}{11}$ *days*.	26	$8\frac{1}{10}$ *cwt*.	30	3 *hours*.
204.	23	$20\frac{4}{7}$ *days*.	27	36 *men*.	—	——
207.	1	$16\frac{1}{5}$ *days*.	5	10 *days*.	9	96 *tailors*.
207.	2	7200 *men*.	6	$92\frac{1}{4}$ *days*.	10	9600 *mcn*.
207.	3	$187\frac{1}{2}$ *miles*.	7	$36.	11	50 *men*.
207.	4	72 *acres*.	8	$12.5 *gal*.	—	——

P.	EX.	ANS.	EX.	ANS.	EX.	ANS.	EX.	ANS.
208.	12	$471,04.	15	600.	18	$97\frac{1}{5}$*lb*.	21	5 *men*.
208.	13	$3\frac{15}{16}$*da*.	16	$14\frac{2}{5}$*da*.	19	32 *days*.	22	132 *days*.
208	14	180.	17	$7\frac{1}{2}$*oz*.	20	32 *horses*.	—	——

P.	EX.	ANS.	EX.	ANS.	EX.	ANS.
210.	1	$1000, *A's*.	4	$5000 *A's*.	6	$3000, *B's*.
210.		$1200, *B's*.		$2500 *B's*.		$3000. *C's*.
210.		$ 800, *C's*.		$$3333\frac{1}{3}$, *C's*.		$$2666\frac{2}{3}$, *D's*.
210.	2	$$1714,28\frac{4}{7}$, *A's*.		$2500, *D's*.	7	$1500, *son's sh*.
210.		$$285,71\frac{3}{7}$, *B's*.		$$6666\frac{2}{3}$, *E's*.		$3000, *mo's sh*.
210.	3	£4030, *A's*.	5	100, *A's*.	8	$12961,50 *A's*.
210.		£3980, *B's*.		140. *B's*.		$15737,25 *B's*.
210.		£3980, *C's*.		200, *C's*.		$10802,25 *C's*.
210.		£4010, *D's*.	6	$$3333\frac{1}{3}$, *A's*.		$1833, *D's gain*

P.	EX.	ANS.	EX.	ANS.
211.	9	$450.	11	$125,4375 *C*
211.		$600.		$70, *D's*.
211.		$750.	12	$12720.
211.	10	*A's*, $4242,50 *stock*: $1697 *gain*.	13	$87,831+ *A*
211.		*B's*, $5939,50 " $2375,80 "		$65,06+ *B*.
211.		*C's*, $6788 " $2715,20 "		$48,795+ *C*
211.	11	$237,75, *A's*.		$68,313+ *D*
211.		$181,0625, *B's*.	—	——

P.	EX.	ANS.	EX.	ANS.	EX.	ANS.	EX.	ANS.
211.		$1015,33$\frac{1}{3}$, *the first.*						
211.	14	$1523, " *second.*						
211.		$2030,66$\frac{2}{3}$, " *third.*						
212.		$16,38, *A's.*		6577.23\frac{543}{659}$, *A's.*				
212.	1	$35,10, *B's.*	3	$1822,76$\frac{116}{659}$, *B's.*				
212.		$18,72, *C's.*		$288, *A's.*				
212.	2	$7.	4	$270, *B's.*				
212.	—	——		$240, *C's.*				
213.	5	$280, *D's.*		$84, *A's.*				
213.		$168, *C's.*		$90, *B's.*				
213.		$1309,43$\frac{377}{761}$, *officers.*	8	$82,50, *C's.*				
213.	6	$2946,22$\frac{658}{761}$, *midshipmen.*		$90, *D's.*				
213.		$16504,33$\frac{487}{761}$, *sailors.*		$12, 1*st grade.*				
213.		$2648,86$\frac{4}{11}$, *A's.*	9	$12, 2*d* "				
213.	7	$2901,13$\frac{7}{11}$, *B's.*		$ 3, 3*d* "				
213.		$1850, *C's.*	10	$800, *B's stock.*				
213.	—	—— ——		13 *mos. C's time.*				
214.	1	.095 ; .0875.	3	2.08 ; 3.75 ; .95.				
214.	2	.125 ; .09875.	4	.666$\frac{2}{3}$.				
215.	2	$3,14	9	16.74 *miles.*	16	$4344,35		
215.	3	$4,7825	10	47.725 *sheep.*	17	2625*bar.*		
215.	4	3.5625*yds.*	11	27.54 *tons.*	18	$5144,625		
215.	5	2.839375*cwt.*	12	$300,365.	19	$12500		
215.	6	1.002*lbs.*	13	15,75 *cows.*	20	$3867,01875		
215.	7	12*bu.*	14	160 *bales.*	21	$15000		
215.	8	$90.	15	478.125*yds.*	22	$65		
216.	23	742,85 *gallons.*	26	$6093,75.				
216.	24	205 *boxes.*	27	$196,59375.				
216.	25	.42$\frac{1}{2}$; $10625.	—	——				
217.	1	.20	5	.25	9	.01375	13	.05
217.	2	.125	6	.875	10	.33$\frac{1}{3}$	14	.38$\frac{8}{9}$
217.	3	.075	7	.625	11	.375	15	.72
217.	4	.136	8	.0075	12	.125	—	—
218.	1	$4 *per head.*	2	$5425	3	$50000	4	$5000
220.	2	$60,9875	7	$283,8438	12	$373,2495		
220.	3	$224,91	8	$422,8976	13	$735		
220.	4	$360,2832	9	$1112,90	14	$916,075		
220.	5	$473,844	10	$265,2345	15	$120,80		
220.	6	$1312,50	11	$1893,75	16	$5796		

P.	EX.	ANS.	EX.	ANS.	EX.	ANS.
221.	1	$20,909	4	$1979,5013	7	$64,0625
221.	2	$26,313	5	$5618,75	8	$157,65625
221.	3	$458,88	6	$628,416$\frac{2}{3}$	—	——
223.	2	$42,2432+	6	$11,0415	10	1334,2187+
223.	3	$420,2531+	7	$132,7707+	11	$120,069+
223.	4	$213	8	$26,9586+	12	$40,0968
223.	5	$181,25	9	$416,1673+	—	——
224.	13	$81,6778+	19	$84,6855	25	$160,4408+
224.	14	$162[illegible]+	20	$55,6685+	26	$12,954+
224.	15	$221,266	21	$32,666+	27	$82,036+
224.	16	$389,2466	22	$8590,832+	28	$70,964
224.	17	$135,3714	23	$36	29	$879,467
224.	18	$42,9404+	24	$93,7843+	30	$800,6618
225.	31	$932,777+	34	$5085	1	$394,325+
225.	32	$499,339+	35	$403,891	2	$697,986
225.	33	$140,644+	36	$9337,50	—	——
226.	3	$219,613	5	$4640,532+		
226.	4	$823,902+	6	$1976,63+		
227.	2	£45 8*s.* 1$\frac{3}{4}$*d.*	5	£1133 10*s.* 9$\frac{1}{4}$*d.*+		
227.	3	£45 12*s.* 4$\frac{1}{2}$*d.*	6	£199 6*s.* 3$\frac{3}{4}$*d.*		
227.	4	£154 7.04*s.*+	7	£6 16*s.* 5*d.*		
228.	1	$3976,777+	5	$952,576+	8	.10
228.	2	$439,80	6	.07	9	.05$\frac{1}{2}$
228.	3	$6234,831+	7	.09	10	.12$\frac{1}{2}$
228.	4	$30000	—	——	—	——
229.	11	2*yr.* 6*mo.*	13	1*yr.* 4*mo.*	15	5*yr.* 4*mo.*
229.	12	3*mo.* 18*da.*	14	16*yr.* 8*mo.*	16	1*yr.*
231.	2	$5359,366+	4	$1127,041		
231.	3	$8925.544+	5	$190,758		
232.	6	$156,20+				
233.	2	$25,3575	5	$73,015	7	$845,826
233.	3	$291,72	6	$83,20	8	$48165,91
233.	4	$57,303	—	——	—	——
234.	9	$14523,555	11	$8501984,9+		
234.	10	$926,744	12	$124,1614		
235.	13	$151,5811	14	$16,3875	15	$445,857
236.	1	$562.52	2	$184,497+	3	$21

P.	EX.	ANS.	EX.	ANS.	EX.	ANS.
236.	4	$5000	10	$3538,083 *cash val.*		
236.	5	$1902,587+		$388,083 *gain.*		
236.	6	$2764,4673 *pr. val.*;	11	$9890,239		
236.		$235,5327 *disc't.*	12	$10,890 *loss.*		
236.	7	$4820,537	13	.00414, *at* 7½*cts.*		
236.	8	$4800,011+	14	$12		
236.	9	$1379,6123+	15	$2369,2617 *cash val*		
236.	—	——		$61,9883 *diff.*		
240.	1	$6,15	5	$4,374		
240.	2	$7,65	6	$204,82 *gain.*		
240.	3	23,2913 *discount.*	7	$21,447 *difference.*		
240.		$476,708+ *pres. val.*	8	$86,617 *difference.*		
240.	4	$1225,3557 *pres. val.*	9	$981,21 *cash value.*		
241.	2	$296,50	3	$697,20	4	$474,625
242.	5	$3522,092	——	——		
243.	3	$38,8375	8	$25320,19		
243.	4	$98,7187	9	$6835,283		
243.	5	$5769,249+	10	$935		
243.	6	$163,80 *commission.*	11	$63,6299 *com'n.*		
243.		$4340,70 *whole cost.*		4544.62+ *bushels.*		
243.	7	$115,39+	——	——		
244.	12	$2558,16	15	15 *tons.*		
244.	13	158 *barrels.*	16	$70		
244.		$2412,66	17	20 *shares.*		
244.	14	$420,922	18	$55743,289		
246.	1	$5320	3	$59110		
246.	2	$666	4	$21375		
247.	5	$7999,6875	7	$300		
247.	6	$213500	1	$3529,41+		
248.	2	56 *shares.*	5	$8000		
248.	3	$4000	6	$10432,432+		
248.	4	$7235,142+	—	——		
249.	1	.08	3	.08	5	.5833+
249.	2	.20	4	.08	6	.05
250.	2	7 *per cent. the best.*	4	$166,66⅔		
250.	3	8 *per cent. the best.*	—	——		
251.	1	$33,75	2	$0,56	3	$236,25

P.	EX.	ANS.	EX.	ANS.	EX.	ANS.
252.	1	$170	3	$6,0053	4	$70 *gain.*
252.	2	548,60				
253.	1	$0,90	4	$5,70	7	$1,80
253.	2	$3,20	5	$18,03	—	——
253.	3	$0,96	6	$0,66	—	——
254.	1	.18	6	$160,34375 *whole gain.*		
254.	2	.25		.046+ *per cent.*		
254.	3	*the same.*	7	.45		
254.	4	.80	8	$25,65 *lost.*		
254.	5	.25	—	——		
255.	9	.05 *per cent loss.*	11	$.10\frac{2}{5}$		
255.	10	.04	12	$508,50		
256.	1	$5168,59	4	$300	7	$1252,125
256.	2	$158,40	5	$89,55	8	$163,80
256.	3	$126	6	$17,8125	—	——
257.	9	$16481,25 *loss.*	12	$.04\frac{1}{5}$	15	$127,4625
257.	10	$.05\frac{1}{2}$	13	$24000	16	$17043,125
257.	11	$.01\frac{3}{5}$	14	$9020	—	——
259.	1	$121,72	4	$20	7	$103,885
259.	2	$232,50	5	$98,20	—	——
259.	3	$262,50	6	$120	—	——
260.	1	$311,15	3	$1227,395		
260.	2	$757,988	4	$1318,94		
262.	1	$7051,63415	4	$16355,52		
262.	2	$9049,53795	5	$2160,90		
262.	3	$23058,6765	—	——		
263.	6	$2159,63+	—	——		
265.	1	$\frac{2}{5}$ *per cent.*	3	$1\frac{1}{2}$ *per cent.*		
265.	2	$37901125		$82,25		
265.				$56,9075		
266.	4	$\frac{3}{4}$ *per ct.*; $15,50	8	.015 on $1		
266.	5	$5820		$112,50		
266.	6	$22236,197		$18		
266.	7	$4656,05 *whole tax.*	9	$7,40		
266.		.005 on $1 ; $27		$9,225		
266.		$6,8775 *G's*; $12,78 *H's.*				
269.	3	$260,9932	4	$713,37		

P.	EX.	ANS.	EX.	ANS.	EX.	ANS.
270.	5	6*T.* 14*cwt.* 1*qr.* 16.58*lb.*	9	$1196,343+		
270.	6	3*T.* 7*cwt.*	10	$744,546		
270.	7	6*T.* 13*cwt.* 2*qr.* 4*lb.*	11	$255,835		
270.		$308,4774	12	$4,09+		
270.	8	$792,612	13	$125,18$\frac{3}{4}$		
271.	14	$386,219+	20	$1512		
271.	15	$466,278	21	$423,36		
271.	16	$1101,24 ; $0,14	22	$251,453+		
271.	17	$7936,50	23	$1457,75		
271.	18	$820,4625	24	22.605*cwt. tare.*		
271.	19	$16,206+		$68,5856 *duty.*		
272.	1	9*mo.* 15*da.*	—	———		
273.	2	9*mo.*	6	21+ *days.*		
273.	3	8$\frac{2}{9}$*mo.*	7	6*mo.* 6*da.*		
273.	4	7*mo.* 3*da.*	8	26$\frac{1}{2}$*da.*, or on July 28		
273.	5	6$\frac{1}{4}$*mo.*	—	———		
274.	9	48$\frac{702}{1771}$*da.* or on Sept. 19	11	51$\frac{4}{17}$*da.*, or Sept. 22.		
274.	10	28$\frac{155}{213}$*da.* or Dec. 30th	12	57+*da.*, or July 29.		
275.	1	$0,50	—	———		
276.	2	$0,66	5	75°	8	$0,30
276.	3	$0,49	6	19 carats.	–	
276.	4	$1,00	7	$0,13$\frac{1}{2}$	–	———
278.	1	1*lb.* at 8*cts.* 1*lb.* at	3	1 calf, 2 cows, 1 ox, 1 colt.		
278.		10*cts.* 3*lb.* at 14*cts.*	4	3 gallons of water.		
278.	2	4*lb.* each.	–	———		
279.	1	20 pounds of each.	2	75 pounds of each.		
280.	3	36*gal.* at 7*s.*, 24*gal.* at 7*s.* 6*d.* and at 9*s.* 6*d.*, 12*gal.* at 9*s.*				
280.	4	10 at $2, 15 at $$\frac{3}{4}$.				
280.	5	25*lb.* at 5 and 7, 100 at 7$\frac{1}{2}$*cts.*, 37$\frac{1}{2}$ at 9$\frac{1}{3}$, and 50 at 10*cts.*				
281.	1	22 pound of each.				
281.	2	9*gal.* of water, 40$\frac{1}{2}$ at $2,50, 13$\frac{1}{3}$ at $3.				
281.	3	12 calves, 12 sheep, 16 lambs.				
281.	4	8 at $6, 8 at $7, 4 at $19.				
281.	5	90*gal.* at 4*s.* and 10*gal.* each at 6*s.* 8*s.* and 10*s.*				
281.	6	6 vests, 12 pants, 6 coats.				
281.	7	30 at 15 carats, and 4 each of 20*c.*, 22*c.*, 24*c.*				
281.	8	10 at $$\frac{1}{2}$, 15 at $1, 10 at $5.				
288.	1	$8591,975.				

P.	EX.	ANS.	EX.	ANS.	EX.	ANS.	EX.	ANS.
289.	2	\$8637,168+	2	\$176204,4729				
289.	3	\$9777,636	3	£14014 18*s.* 2*d.*+				
290.	4	\$6005,368	2	.07 *per cent above par.*				
290.	5	\$807,874+	—	———				
291.	3	\$12286,06	2	\$1250,52				
291.	4	84597 *f'ncs*66 *centimes.*		.03 *per cent nearly, below par.*				
291.	1	\$6657,693	–	———				
299.	1	$225\frac{9}{19}$ *tons.*	3	$729\frac{97}{855}$ *tons.*	5	1006,57 *T.*		
299.	2	$438\frac{45}{74}$ *tons.*	4	300.14 *T.*	—	———		
301.	1	16	15	$\frac{169}{196}$	29	76,765625		
301.	2	225	16	$\frac{1225}{7056}$	30	10,4976		
301.	3	676	17	$\frac{15625}{61009}$	31	34012,224		
301.	4	20164	18	7,84	32	.0184528125		
301.	5	214369	19	58,140625	33	$\frac{3375}{4096}$		
301.	6	1795600	20	$250\frac{26}{121}$	34	$\frac{125}{216}$		
301.	7	605,16	21	51030,81	35	$\frac{81}{4096}$		
301.	8	.276676	22	216	36	$3154\frac{26}{27}$		
301.	9	9,765625	23	13824	37	$57\frac{681}{1024}$		
301.	10	.00274576	24	373248	38	$\frac{390625}{531441}$		
301.	11	60639,0625	25	1953125	39	14886,936		
301.	12	$\frac{9}{16}$	26	2515456	40	,000244296875		
301.	13	$\frac{36}{49}$	27	20736	41	2893640,625		
301.	14	$\frac{49}{81}$	28	59049	42	10,272025		
307.	1	7	8	.25	15	.1581+	22	$\frac{3}{7}$
307.	2	12	9	16,7	16	.779+	23	.2828+
307.	3	15	10	2505	17	.149	24	11.618+
307.	4	48	11	89,409+	18	5,01	25	137,84
307.	5	$\frac{6}{9}$	12	.453	19	14,015	26	.885+
307.	6	$\frac{15}{48}$	13	.93+	20	1,2247+	27	75.15
307.	7	.14	14	.9682+	21	$\frac{53}{79}$	28	400.06
309.	1	30*ft.*	2	221 *stones.*				
309.	1	343	3	$21\frac{9}{11}$ *rods.*				
310.	4	60*rds. wide.*	8	94,708*ft.*				
310.		180*rds. long.*	9	$53.33\frac{1}{3}$ *ft.*				
310.	5	10*A.* 0*R.* 29*P.* $168\frac{3}{4}$*sq. ft.*	10	$8,66\frac{2}{3}$ *ft.*				
310.	6	75*ft.*	11	825,8 *miles.*				
310.	7	135*ft.*	12	\$100				

P.	EX.	ANS.	EX.	ANS.
311.	13	75*ft.*		4.405+*in.*, 1st man's share.
311.	14	28.28+*ft.*	17	5.739+*in.*, 2d " "
311.	15	6*in.*		13.856+*in.*, 3d " "
311.	16	11.041*rds.*	—	———

P.	EX.	ANS.	EX.	ANS.	EX.	ANS.	EX.	ANS.
315.	1	12	5	179	1	2,026+	5	.729+
315.	2	49	6	364	2	12,0014+	6	.015
315.	3	36	7	439	3	.232+	7	.185+
315.	4	247	8	3072	4	27,0001+	8	4,339+

P.	EX.	ANS.	EX.	ANS.
316.	1	$\frac{4}{5}$	1	27*ft.*
316.	2	$\frac{7}{9}$	2	19*ft.*, length of each side.
316.	3	$3\frac{1}{7}$		2166*sq. ft.*, area
316.	4	$3\frac{1}{2}$	3	36*ft.*, length of each side.
316.	5	$\frac{7}{8}$	4	8.57+*ft.*
316.	6	$\frac{7}{25}$	5	9.77*ft.*, length and breadth.
316.	7	$\frac{27}{64}$		19.54+*ft.*, height.
316.	8	$\frac{24}{35}$	6	10.125 *cu. ft.*
316.	9	1.97+	7	45 cents per yard.
316.	10	3.83+		2025 whole number of yards.

P.	EX.	ANS.
317.	9	64*lbs.*
317.	10	8*ft.*, length of each side.
317.	11	8 globes.
317.	12	$1331
317.	13	12*in.* long, 6*in.* wide, 1*in.* thick.
317.	14	24*ft.* long, 20*ft.* wide, 9*ft.* deep.
317.	15	20 feet.
317.		.54+*in.*, 1st woman's share.
317.	16	.69+*in.*, 2d " "
317.		1*in.*, 3d " " 4th, 3.76*in.*

P.	EX.	ANS.	EX.	ANS.	EX.	ANS.
319.	1	89	2	$80	3	$396

P.	EX.	ANS.	EX.	ANS.
320.	4	$17\frac{1}{3}$*rds.*	1	5 miles.
320.	5	200*ft.*, to bring back the nearest.	2	$2

P.	EX.	ANS.	EX.	ANS.
321.	3	$\frac{3}{4}$*in.*	3	$791\frac{1}{4}$*mi.*
321.	1	$2730	4	10*mi.*, 7*fur.*, 27*rds.*, $1\frac{1}{2}$*yd.*
321.	2	$64,96	—	———

P.	EX.	ANS.	EX.	ANS.	EX.	ANS.
322.	1	5551*bu.*	2	13*da.*; 312*mi.*	3	6

P.	EX.	ANS.	EX.	ANS.
324.	2	3125000	3	$\frac{1}{27}$

P.	EX.	ANS.	EX.	ANS.
325.	4	100000000000000	6	$18000
325.	5	$3200	7	$327,68

P.	EX.	ANS.	EX.	ANS.
326.	1	118081	4	$42949672,95
326.	2	2044	5	938249922+ ships.
326.	3	$11184810	—	———

341.	24	$4166,40	28	$2\frac{19}{28}$ hours.	32	117*lb.*	36	$2100
341.	25	$9\frac{5}{8}$ days.	29	\$$1,33\frac{1}{3}$	33	$693	37	\$$0,07\frac{1}{5}$
341.	26	36 feet.	30	$100	34	11 cents.	38	$126
341.	27	$1770	31	$5\frac{1}{2}$ yards.	35	$56	39	$44,10

342.	40	$43	47	60 the first.
342.	41	$246,75 gain.		100 " second.
342.	42	*5hr. 27mi.* $16\frac{4}{11}$*sec.*		140 " third.
342.	43	40 yards.		180 " fourth.
342.	44	10 hours.	48	$16\frac{4}{5}$ inches.
342.	45	36 days.	49	$2\frac{5}{14}$ months.
342.	46	$1\frac{13}{47}$ days.	50	$35\frac{1}{4}$ yards baize.
342.	—	———		$2,20 per yard.

343.	51	$12	58	$2317,15 *A's.*
443.	52	$1,20		$1853,72 *B's.*
443.	53	$78,652+discount.		$2317,15 *C's.*
343.	54	$129,60		$2780,58 *D's.*
343.	55	$19,375 most advan-	59	$95,10 *A's.*
343.		tageous for cash.		$95,10 *B's.*
343.	56	$292.823 gain.		$133,14 *C's*
343.	57	$122,70 *A's.*		$152,16 *D's.*
343.		$163,60 *B's.*	60	$7\frac{1}{7}$ ounces.
343.		$196,32 *C's.*	61	$8\frac{8}{9}$ days.
343.	—	———	62	17 times.

344.	63	$4\frac{12}{17}$ days.	67	$62,223+ more advanta-
344.	64	5 months 24 days.		geous in bond and mort-
344.	65	68 days.		gage.
344.	66	126 gallons.	68	$3312,417+
344.	—	———	69	$42,60

345.	70	4 yards high.	77	$36000
345.	71	20 hours ; 140 miles.	78	41,13+ bushels.
345.	72	$2 the first.	79	137.942+ feet.
345.		$6 the second.	80	The second, 10 days
345.	73	100 thousand feet.		after the first; the
345.	74	$21\frac{39}{50}$		third, 8 days after
345.	75	$3825		the second, or 18
345.	76	$144, better to pay cash.		days after the 1st.

P.	EX.	ANS.	EX.	ANS.
346.	81	$6890	86	72*lbs.* soap.
346.	82	$1316, whole cost.	87	5 o'clock 20*m.* P. M.
346.		$7, cost per acre.	88	$\$\frac{2}{3}=\$0.66\frac{2}{3}$.
346.	83	32 days, or Mar. 16.	89	24 chickens.
346.	84	$516\frac{4}{9}$ slabs;		36 turkies.
346.		$\$302,22\frac{2}{9}$, cost.		
346.	85	$350 A's; $297,50 B's;	—	—
346.		$210 C's; $175 D's;		
346.		$122,50 E's.		
347.	90	8 days.	94	$\frac{8}{9}$ of a week.
347.	91	$1797,50 first,	95	$11\frac{2}{3}$*hr.*; $134\frac{1}{6}$ miles.
347.		$2157 second,	96	$309,677+ A's.
347.		$2516,50 third.		$1032,258+ B's.
347.	92	$960 stock;		$258,064+ C's.
347.		$180 gain, first.	97	$36\frac{1}{2}$ days.
347.		$640 stock;	98	84461.14+ feet.
347.		$120 gain, 2d.	99	$206,06 in favor of 1st invest.
347.	93	49.945+ feet.	—	—
348.	100	23599680 cubic yards.	102	$4004,338+ 5th.
348.	101	$4646,363	103	2160 men.
348.	102	$1555,017+ 1st.	104	$57,142 A's.
348.		$4354,717+ 2d.		$42,857 B's.
348.		$4304,663+ 3d.	105	$31
348.		$5781,263+ 4th.	106	8 hours.
349.	107	$30 com. diff.	111	$97\frac{1}{5}$ pounds.
349.		$2460 whole cost.	112	$\frac{3}{4}$ of a cent cost;
349.	108	$144,03 A's		$\frac{4}{5}$ of a cent sold for;
349.		$ 90,12 B's		$\frac{1}{20}$ gain on each;
349.		$ 63,45 C's		80 eggs sold.
349.		$168,35 D's	113	84 years.
349.	109	.06397	114	943.0587+ cubic feet.
349.	110	$14467,505	—	—
350.	115	155*A.* 3*R.* 38.72*P.*	118	$6\frac{6}{19}$ hours.
350.	116	A, 25 days,	119	$108\frac{7}{75}$ planks.
350.		B, 30 days,	120	5 inches.
350.		C, $37\frac{1}{2}$ days.	121	$4006,54+
350.	117	$360,56 nearly.	—	—
352.	2	36 acres.	4	135 acres.
352.	3	5*A.* 1*R.* 15*P.*	—	—

P.	EX.	ANS.	EX.	ANS.	EX.	ANS.	EX.	ANS.
353.	1	437A. 2R. 34P+	5	40A.				
353.	2	291A. 2R. 16P.	6	15A.				
353.	3	35A. 0R. 25P.	7	24A. 1R. 8P.				
353.	4	20A.	8	27A. 0R. 16P.				
354.	2	21A. 0R. 39.824P.	5	60A. 3R. 12.8P.				
354.	3	921.875*sq. ft.*	6	270A. 1R. 24P.				
354.	4	704.125*sq. yd.*	—	——				
355.	2	584.3376	4	179.0712	3	4360.835+		
355.	3	125.664	2	7418	—	——		
356.	2	19.635+	3	153.9384	4	1.069+*sq. yd.*		
357.	2	615.7536	4	196996571.722104*sq. mi*				
357.	3	4071.5136	—	——				
358.	2	268.0832	5	904.7808 *c. ft.*				
358.	3	2144.6656 *c. in.*	1	9100*sq. ft.*				
358.	4	259992792079.869+ *c. mi.*	2	1440 *sq. ft.*				
359.	2	110592 *c. in.*	4	$315\frac{65}{67}$ gallons.				
359.	3	$42\frac{2}{9}$ *c. ft.*	5	13820 *c. ft.*				
360.	2	$233.33\frac{1}{3}$ *sq. ft.*	3	13571.712				
360.	3	2827.44 *sq. in.*	4	9650.9952				
360.	4	6283.2*sq. ft.*	5	7363.125				
360.	2	36442.56	—	——				
361.	2	4380	4	5620	6	14400		
361.	3	2484	5	5760	7	1800		
362.	2	9160.9056	3	8659.035	4	2827.44		
364.	2	32.4938*in.*	8	28.2574*in.*		——		
365.	1	197.459+ gal. wine.	3	144.856+ wine gal. 119.293+ beer gal.				
365.	2	162.613+ gal. beer.						
365.	–	——	4	149.23+ wine gal.				
367.	1	40*lb.*	2	25*lb.*	3	50*lb.*	4	20*lb.*
368.	5	40*lb.*	7	64*lb.*				
368.	6	1*in.*, $1\frac{1}{2}$*in.*, 2*in.*, 4*in.*	8	150*lb.*				
370.	1	60*lb.*	2	40*lb.*	3	25*lb.*		
371.	1	$7\frac{1}{2}$*ft.*	2	$1\frac{1}{2}$*ft.*		——		
372.	1	40*lb.*	2	100*lb.*	3	60*lb.*	1	576*lb.*
373.	2	2250*lb.*	—		—		—	
374.	1	259200*lb.*	2	1.47+lb.	3	1.1+lb.	4	1.21+in.

P.	EX.	ANS.	EX.	ANS.
375.	1	23*mi.* 2760*ft.*	7	6*da.* $11\frac{3}{7}$*hr.*
375.	2	5760*ft.*	8	$13\frac{1}{3}$*ft.*
875.	3	2*hr.* 56*m.*	9	6*mi.* 3595.1*ft.*
375.	4	8*ft.*	10	3*hr.* 12*m.* $12\frac{12}{19}$*sec.*
375.	5	$26\frac{2}{3}$*sec.*	11	8*m.* 16.6*sec.*
375.	6	$4\frac{4}{9}$*ft.*	12	16428.5*mi.*
377.	1	$369\frac{1}{12}$*ft.*	5	$1608\frac{1}{3}$*ft.*
377.		2316*ft.*		$321\frac{2}{3}$ velocity.
377.	2	$3618\frac{3}{4}$*ft.*	6	2*mi.* $4984\frac{8}{193}$*ft.*
377.		$482\frac{1}{2}$*ft.*	7	164.69*ft.*
377.	3	$223\frac{4}{5}$*ft.*	8	100.52*ft.*
377.	4	$2\frac{1}{2}$*sec.* nearly.	9	$4\frac{188}{193}$*sec.*
378.	10	$402\frac{193}{2316}$*ft.*	13	14.28+
378.	11	1447.5*ft.*	14	$15050\frac{118}{193}$*ft.*
378.	12	61.24*sec.*	15	2316*ft.*
380.	1	8.857	7	7.234
380.	2	$38\frac{138}{185}$ *c. ft.*	8	.786
380.	3	.980	9	.875
380.	4	2*ft.* 11.383*in.*	10	177*lb.* 5*oz.*
380.	5	190*T.* 709*lb.*	11	1.103
380.	6	2.75	—	——
381.	12	4.23*in.*	14	4.572*oz.*
381.	13	3.172	15	1418*lb.* 3.3841*oz.*
381.		.504	—	——
382.	1	3.49*qts.*	4	.5319
382.	2	37.5*lb.*	5	1.88
382.	3	2.46*gr.*	6	18.526*qt.*

$\cancel{\frac{1}{2}} \div \frac{1}{6} = \frac{2}{12} \quad \frac{6}{12} \quad \frac{8}{12}$

$\frac{2}{1} \times \frac{1}{6} = \frac{1}{3}$

$\cancel{\frac{15}{3}} + \frac{25}{6}$

$\frac{45}{6} + \frac{25}{6} = \frac{70}{6}$